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C - (m,n) - I D E A L   S E M I G R O U P S

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ABSTRACT: In this paper we consider semigroups in which every cyclic subsemigroup is an (m,n)-ideal.

INTRODUCTION: The generalization of the ideal of semigroups are given by S. Lajos, by a notion of an (m,n)-ideal of a semigroup [4]. P. Protić and S. Bogdanović considered (m,n)-ideal semigroups in which every subsemigroup is an (m,n)-ideal [5]. This class of semigroups are described by P. Protić and S. Bogdanović [5,6]. Bi-ideal semigroups, as a special case of (m,n)-ideal semigroups, are described by B. Trpenovski [7], S. Bogdanović, P. Kržovski, B. Trpenovski and P. Protić [8]. The construction of the (m,n)-ideal semigroup is given by S. Bogdanović and S. Milić [9].

Here, we consider c-(m,n)-ideal semigroups in which every cyclic subsemigroup is an (m,n)-ideal. In Theorem 1.5. c-(m,n)-ideal semigroups are described by an ideal extension. In Theorem 4.1. we have a construction of a c-(m,n)-ideal semigroup, where results of Theorem 1.1. [10] are used (see also the book of S. Bogdanović [1], Chapter VIII).

1. C-(m,n)-IDEAL SEMIGROUPS

A subsemigroup A of a semigroup S is an (m,n)-*ideal* of S if $A^mSA^n \subseteq A$, where $m,n \in N \cup \{0\}$ ($A^oS=SA^o=S$) [4].

This paper is in final form and no version of it will be submitted for publication elsewhere.

S is an (m,n)-ideal semigroup if every subsemigroup of S is an (m,n)-ideal of S [5]. Every (1,1)-ideal semigroup we call bi-ideal semigroup.

S is a c-(m,n)-ideal semigroup if every cyclic subsemigroup of S is an (m,n)-ideal of S. It is clear that the class of all (m,n)-ideal semigroups is a subclass of the class of all c-(m,n)-ideal semigroups.

The (m,n)-ideal of S generated by nonempty subset C of S is $[C]_{m,n}= C\cup C^2\cup\ldots\cup C^{m+n}\cup C^mSC^n$. If $C=\{a\}$ we obtain the principal (m,n)-ideal of S generated by element a which is $[a]_{m,n}= a\cup a^2\cup\ldots\cup a^{m+n}\cup a^mSa^n$.

A subset R of a partial semigroup Q is a partial subsemigroup of Q if for $x,y\in R$, $xy\in Q$ implies $xy\in R$. A partial subsemigroup R of a partial semigroup Q is an (m,n)-ideal of Q if $R^mQR^n\subseteq Q$ implies $R^mQR^n\subseteq R$. Q is an (m,n)-ideal partial semigroup if every partial subsemigroup of Q is an (m,n)-ideal of Q [5]. Q is a c-(m,n)-ideal partial semigroup if every partial cyclic subsemigroup of Q is an (m,n)-ideal of Q.

Let S be a semigroup with zero 0, then S is a nil-semigroup if for every $a\in S$ there exists $k\in N$ such that $a^k=0$ [8]. A partial semigroup Q is a partial nil-semigroup if for every $a\in Q$ there exists $k\in N$ such that $a^k\notin Q$ (In [6] it is called the power breaking partial semigroup).

THEOREM 1.1. The following conditions on a semigroup S are equivalent:

(i) S is c-(m,n)-ideal

(ii) $(\forall a\in S)\ a^mSa^n\subseteq\langle a\rangle$

(iii) $(\forall a\in S)\ [a]_{m,n}=\langle a\rangle$

Proof: Let $a\in S$ and let S be a c-(m,n)-ideal semigroup. Then $a^mSa^n\subseteq\langle a\rangle^mS\langle a\rangle^n\subseteq\langle a\rangle$. Conversely, let (ii) holds and let $\langle a\rangle$ be a cyclic subsemigroup of S. Since $\langle a\rangle^m=\{a^p: p\geq m\}$, then every element from $\langle a\rangle^mS\langle a\rangle^n$ is of the form a^psa^q where $p\geq m$ and $q\geq n$, whence $a^psa^q=a^{p-m}a^msa^na^{q-n}\in\langle a\rangle\langle a\rangle\langle a\rangle\subseteq\subseteq\langle a\rangle$. Thus (i)$\Leftrightarrow$(ii).

(ii)$\Rightarrow$(iii). Then $[a]_{m,n}= a\cup a^2\cup\ldots\cup a^{m+n}\cup a^mSa^n\subseteq$

$\subseteq\langle a\rangle\cup\langle a\rangle\cup\ldots\cup\langle a\rangle=\langle a\rangle$. Conversely, $a^mSa^n\subseteq[a]_{m,n}=\langle a\rangle$ i.e. (ii)$\Leftrightarrow$(iii).□

THEOREM 1.2. Let S be a c-(m,n)-ideal semigroup. Then:

(i) S is periodic

(ii) the set E of all idempotents of S is a rectangular band and it is an ideal of S

(iii) $S\setminus E$ is a c-(m,n)-ideal partial nil-semigroup

(iv) $(\forall a\in S)\ |\langle a\rangle|\leq 2m+2n+1$

(v) S is a disjoint union of the maximal unipotent c-(m,n)-ideal semigroups $S_e=\{x\in S:\ (\exists p\in N)\ x^p=e\}$, $e\in E$ and e is a zero in S_e

(vi) $(\forall a,b\in S)\ e_ae_b\in\langle a^mb^n\rangle$, where e_a and e_b are the idempotents from $\langle a\rangle$ and $\langle b\rangle$.

Proof:

(i) Let $a\in S$. Let $\langle a\rangle$ be an infinite semigroup. Then $B=\langle a^2\rangle=\{a^{2k}:k\in N\}$ is a subsemigroup of S and $a^{2m}aa^{2n}\in$ $\in B^mSB^n\subseteq B$ which is impossible. Hence, for every $a\in S$, $\langle a\rangle$ is a finite semigroup and so $E\neq\emptyset$.

(ii) For $e\in E$ we have that $eSe\subseteq\langle e\rangle=\{e\}$, whence for every $x\in S$ is $exe=e$ and by Proposition 3.2. [3] E is a rectangular band. Also, for every $e\in S$ and every $x\in S$ from $exe=e$ we have that $ex=exex$ and $xe=xexe$ whence ex and xe are elements from E and E is an ideal of S.

(iii) From (i), for every $a\in S\setminus E$ the cyclic subsemigroup $\langle a\rangle$ contains an idempotent and $S\setminus E$ is a partial nil-semigroup. Let A be a cyclic partial subsemigroup of $S\setminus E$ generated by element $a\in S\setminus E$. Then $a^mSa^n\subseteq S\setminus E$ implies that $a^mSa^n\subseteq A$ whence A is an (m,n)-ideal of $S\setminus E$ and $S\setminus E$ is a partial c-(m,n)-ideal semigroup.

(iv) Let $a\in S$ and $p\in N$ be the smallest natural number such that $a^p\in E$ and let $a^p=e$. Then $a^{p+1}=aa^p$ and $a^{p+1}=a^pa$ implies that $a^{p+1}=ea=ae\in E$ since E is an ideal of S. Every finite cyclic semigroup is unipotent, whence $a^{p+1}=e$, e is a zero in $\langle a\rangle$ and $\langle a\rangle=\{a,a^2,\ldots,a^p=e\}$. Let $p>2m+2n+1$. Then for the semigroup B from (i) we have the same contradiction as like in (i). Hence $2m+2n+1\geq p$, where $p=|\langle a\rangle|$.

(v) Let $x,y\in S_e$. Then, there exists $p,q\in N$ such that $x^p=y^q=e$ and e is a zero for x and y. By Theorem 1.1. we have that $(xy)^m e(xy)^n\in\langle xy\rangle$ i.e. $e\in\langle xy\rangle$. Hence, there exists $r\in N$ such that $e=(xy)^r$.

(vi) Let $a,b\in S$ and $e_a\in\langle a\rangle$, $e_b\in\langle b\rangle$. Let $g\in\langle a^m b^n\rangle$ i.e. $(a^m b^n)^k=g$. Then $ge_a=a^m b^n(a^m b^n)^{k-1}e_a\in a^m Se_a\subseteq\langle a\rangle$, whence $ge_a=e_a$. Also, $e_b g=e_b(a^m b^n)^{k-1}a^m b^n\in e_b Sb^n\subseteq\langle b\rangle$, whence $e_b g=e_b$. From $ge_a=e_a$ and $e_b g=e_b$ we have that $g=ge_a e_b g=e_a e_b$ i.e. $e_a e_b\in\in\langle a^m b^n\rangle$.□

THEOREM 1.3. Let Q be a periodic partial c-(m,n)-ideal semigroup, E be a rectangular band and $Q\cap E=\emptyset$. Let $f\colon S=Q\cup E\to E$ such that $f(e)=e$ for every $e\in E$ and $f_{/Q}$ is a partial homomorphism. We define an operation on S by

$$xy=\begin{cases} xy & \text{as in } Q, \text{ if } x,y\in Q \text{ and } xy \text{ is definied in } Q\\ f(x)f(y) & \text{otherwise}\end{cases}$$

Then S is a c-(m,n)-ideal semigroup.

Proof: Let $a,s\in S$. Then $a^m sa^n\in Q$ implies $a^m sa^n=a^k\in\in\langle a\rangle$. Let $a^m sa^n\notin Q$. Then $a^m sa^n=f(a^m)f(s)f(a^n)=f(a)^m f(s)f(a)^n==f(a)f(s)f(a)$. If $p\in N$ is the smallest number such that $a^p\notin Q$, then $a^p=aa^{p-1}=f(a)f(a^{p-1})=f(a)f(a)^{p-1}=f(a)f(a)=f(a)$. Hence, $a^m sa^n=f(a)\in\langle a\rangle$.□

THEOREM 1.4. S is a c-(m,n)-ideal semigroup with zero if and only if S is a c-(m,n)-ideal nil-semigroup.

Proof: Let S be a c-(m,n)-ideal semigroup with zero 0 and let $a\in S$. Then $a^m 0a^n\in\langle a\rangle$ i.e. $0\in\langle a\rangle$. Hence, S is a nil-semigroup. If e is an idempotent from S then $0\in\langle e\rangle=\{e\}$, whence $e=0$ and S is unipotent. Conversely follovs immediately.□

Let M and T be the disjoint semigroups and T contains a zero 0. The semigroup S is called ideal extension of a semigroup M by T if M is an ideal in S and the Rees quotient semigroup $S_{/M}$ is isomorphic to T [2].

THEOREM 1.5. S is a c-(m,n)-ideal semigroup if and only if S is an ideal extension of the rectangular band E by a c-(m,n)-ideal nil-semigroup.

Proof: Let S be a c-(m,n)-ideal semigroup. By Theorem 1.2.(iii) we have that $S\setminus E$ is a c-(m,n)-ideal partial nil-semigroup and we can get $S/_E$ from $S\setminus E$ by the extension by 0 as like in Theorem 1.3.. From this Theorem $S/_E$ is a c-(m,n)-ideal semigroup.

Conversely, let S is an ideal extension of the rectangular band by a c-(m,n)-ideal nil-semigroup. For $a\in S\setminus E$ we have that $(a\rho)^m S/_E (a\rho)^n \subseteq \langle\langle a\rho\rangle\rangle = \langle\{a\}\rangle$, where $a\rho$ is the class of the element a of modE. Hence for all $b\in S$ we have that $(a\rho)^m(b\rho)(a\rho)^n\subseteq\langle a\rangle$, whence $a^m b a^n\in\langle a\rangle$. Also, for $e\in E$ we have that $e^m S e^n = eSe = e(eSe)e = eEe = e$.

Hence, for every $a\in S$ we have that $a^m S a^n\in\langle a\rangle$ and by Theorem 1.1. it implies that S is a c-(m,n)-ideal semigroup.□

2. (m,n)-IDEAL SEMIGROUPS

THEOREM 2.1. <u>The following conditions on a semigroup S are equivalent:</u>

(i) S <u>is</u> (m,n)-<u>ideal</u>

(ii) $C^m S C^n\subseteq\langle C\rangle$ <u>for every nonempty</u> $C\subset S$

(iii) $[C]_{m,n}=\langle C\rangle$ <u>for every nonempty</u> $C\subset S$

<u>Proof</u>: Let (i) holds. Then for every nonempty subset C of S we have that $\langle C\rangle^m S\langle C\rangle^n\subseteq\langle C\rangle$ i.e. $C^m S C^n\subseteq\langle C\rangle^m S\langle C\rangle^n\subseteq\langle C\rangle$. Conversely, if (ii) holds, let B be a subsemigroup of S and C is the set of generators of B. Then $x\in B^m$ is of the form $x=a_1\dots a_m$ where $a_i\in B$ and $y\in B^n$ is of the form $y=b_1\dots b_n$ where $b_j\in B$. Since $a_i, b_j\in B$ and C generates B, we have that $a_i\in C^{k_i}(k_i\geq 1)$, $b_j\in C^{r_j}(r_j\geq 1)$. Then $xsy\in B^m S B^n$ is of the form $xsy=a_1\dots a_m s b_1\dots b_n\in C^{k_1}\dots C^{k_m} S C^{r_1}\dots C^{r_n}=C^{k_1+\dots+k_m} S C^{r_1+\dots+r_n}=C^{k_1+\dots+k_m-m} C^m S C^n C^{r_1+\dots+r_n-n}\in\langle C\rangle\langle C\rangle\langle C\rangle\subseteq\langle C\rangle=B$. Hence, we have that (i)⇔(ii).

Let (ii) holds. Then $[C]_{m,n}=C\cup C^2\cup\dots\cup C^{m+n}\cup\cup C^m S C^n\subseteq\langle C\rangle\cup\dots\cup\langle C\rangle=\langle C\rangle$. Conversely, let (iii) holds. Then $C^m S C^n\subseteq[C]_{m,n}=\langle C\rangle$, whence (ii)⇔(iii).□

THEOREM 2.2. <u>If</u> S <u>is an</u> (m,n)-<u>ideal semigroup, then</u> S <u>is an ideal extension of the rectangular band</u> E <u>by a</u> (m,n)-<u>ideal nil-semigroup</u>.

Proof: $S/_E$ is a semigroup with zero. By Theorem 3.1. [5] $S\setminus E$ is a partial (m,n)-ideal semigroup and we can get $S/_E$ from $S\setminus E$ by an extension by the zero as like in Theorem 3.2. [5]. From this Theorem $S/_E$ is an (m,n)-ideal semigroup and by Theorem 1.4. it is a nil-semigroup.□

3. BI-IDEAL SEMIGROUPS

COROLLARY 3.1. The following conditions on a semigroup S are equivalent:

(i) S is bi-ideal

(ii) $CSC \subseteq \langle C \rangle$ for every nonempty $C \subset S$

(iii) $B[C] = \langle C \rangle$ for every nonempty $C \subset S$

(iv) $(\forall a,b \in S)\ aSb \subseteq \langle a,b \rangle$

(v) $(\forall a,b \in S)\ \{a,b\}S\{a,b\} \subseteq \langle a,b \rangle$

(vi) $(\forall a,b \in S)\ B[\{a,b\}] = \langle a,b \rangle$

Proof: From Theorem 5. [8] we have that (i)⇔(ii)⇔⇔(iii). Also, it is clear that (iii)⇒(vi).

(vi)⇒(v) since $\{a,b\}S\{a,b\} \subseteq B[\{a,b\}] = \langle a,b \rangle$

(v)⇒(iv) since $aSb \subseteq \{a,b\}S\{a,b\} \subseteq \langle a,b \rangle$

(iv)⇒(ii) since for every $asb \in CSC$ we have that $asb \in aSb \subseteq \langle a,b \rangle \subseteq \langle C \rangle$.□

The following Theorem 3.3. and Lemma are got from Theorem 1.5. and Theorem 1.2.(vi) for the c-bi-ideal semigroup.

THEOREM 3.3. S is a c-bi-ideal semigroup if and only if S is an ideal extension of the rectangular band E by a c-bi-ideal nil-semigroup.

LEMMA: Let S is a c-bi-ideal semigroup. Then for every $a,b \in S$ $e_a e_b \in \langle ab \rangle$, where e_a and e_b are idempotents from $\langle a \rangle$ and $\langle b \rangle$.

COROLLARY 3.4. S is a bi-ideal semigroup if and only if S is an ideal extension of the rectangular band E by a bi-ideal nil-semigroup.

Proof: Let S be a bi-ideal semigroup. From Theorem 2.2. we have that $S/_E$ is a bi-ideal semigroup and it is a nil-semigroup.

Conversely, let S be an ideal extension of the rectangular band E by a bi-ideal nil-semigroup S/E. Then S/E is a c-bi-ideal nil-semigroup and by Theorem 3.3. S is a c-bi-ideal semigroup and the condition of Lemma holds.

Let $a,b,s\in S$. If $asb\in E$, by Lemma we have that $asb=e_ae_se_b=e_ae_b\in\langle a,b\rangle$. If $asb\in S\setminus E$ i.e. $asb=0$ in S/E, then by Corollary 3.1. we have that $asb\in\langle a,b\rangle$ in S/E since S/E is a bi-ideal semigroup, whence $asb\in\langle a,b\rangle$ in S. Hence, the condition $aSb\subseteq\langle a,b\rangle$ holds for every $a,b\in S$ and by Corollary 3.1. S is a bi-ideal semigroup.□

4. THE CONSTRUCTION OF A C-(m,n)-IDEAL SEMIGROUP

CONSTRUCTION: Let $E=I\times J$ be a rectangular band and let Q be a partial semigroup such that $E\cap Q=\emptyset$.

Let $\Phi: p\rightarrow\Phi_p$ be a mapping from Q into the semigroup $\mathcal{T}(I)$ of all mappings from I into itself and, also, let $\Psi: p\rightarrow\Psi_p$ be a mapping from Q into $\mathcal{T}(J)$.

For all $p,q\in Q$ let:

(i) $pq\in Q \Rightarrow \Phi_{pq}=\Phi_q\Phi_p \quad , \quad \Psi_{pq}=\Psi_p\Psi_q$

(ii) $pq\notin Q \Rightarrow \Phi_q\Phi_p=\text{const.} \quad , \quad \Psi_p\Psi_q=\text{const.}$

Let us define a multiplication on $S=E\cup Q$ with:

(1) $(i,j)(k,l)=(i,l)$

(2) $p(i,j)=(i\Phi_p,j)$

(3) $(i,j)p=(i,j\Psi_p)$

(4) $pq=r\in Q \Rightarrow pq=r\in S$

(5) $pq\notin Q \Rightarrow pq=(i\Phi_q\Phi_p, j\Psi_p\Psi_q)$

Then S with this multiplication is a semigroup [10, 5,1-VIII]. A semigroup which is constructed in this way will be denoted by $\Sigma(I,J,Q,\Phi,\Psi)$.

THEOREM 4.1. S is a c-(m,n)-ideal semigroup if and only if S is isomorphic to a semigroup $\Sigma(I,J,Q,\Phi,\Psi)$ where Q is a partial c-(m,n)-ideal semigroup $(m,n\geq 1)$.

Proof: Let S is a c-(m,n)-ideal semigroup. Then by Theorem 1.5. S is an ideal extension of a rectangular band E by a c-(m,n)-ideal nil-semigroup $T=S/E$. Let $Q=T\setminus 0=S\setminus E$. Then Q is a c-(m,n)-ideal partial semigroup.

From Theorem 4.20.[2] we have that S is a subsemigroup of an ideal extension $\bar{S}$ of a translational hull $\Omega(E)$ of E by T. Since translational hull $\Omega(E)$ is a semigroup with identity, then by Theorem 4.19.[2] we have that the multiplication on $\bar{S}$ is determined by a partial homomorphism $f:Q\to\Omega(E)$ with:

$$ab=\begin{cases} ab & \text{if } ab\in Q \\ f(a)f(b) & \text{if } ab\notin Q \end{cases}$$

$$ua=uf(a), \quad au=f(a)u , \quad uv=uv$$

for all $a,b\in Q$ and all $u,v\in\Omega(E)$.

The translational hull $\Omega(E)$ of a rectangular band $E=I\times J$ is isomorphic to a Cartesian product $\mathcal{T}(I)\times\mathcal{T}(J)$ where the multiplication is given with:

$$(\Phi_1, \Psi_1)(\Phi_2,\Psi_2)=(\Phi_2\Phi_1,\Psi_1\Psi_2)$$

for all $(\Phi_1,\Psi_1),(\Phi_2,\Psi_2)\in\Omega(E)$ [3].

Also, E is an ideal of $\Omega(E)$ and elements from E are of the form (Φ^i, Ψ^j) where Φ^i and Ψ^j are constant mappings i.e. for every $k\in I$ and every $\ell\in J$

$$k\Phi^i=i \quad \text{and} \quad \ell\Psi^j=j .$$

Then, we can write $(i,j)=(\Phi^i, \Psi^j)$. Then

$$(i,j)(\Phi,\Psi)=(\Phi^i,\Psi^j)(\Phi,\Psi)=(\Phi\Phi^i,\Psi^j\Psi)=(i,j\Psi)\in E$$
$$(\Phi,\Psi)(i,j)=(\Phi,\Psi)(\Phi^i,\Psi^j)=(\Phi^i\Phi,\Psi\Psi^j)=(i\Psi,j)\in E$$

For $a\in Q$ let $f(a)=(\Phi_a,\Psi_a)$. Since f is a partial homomorphism, for $a,b\in Q$, $ab\in Q$ we have that

$$(\Phi_{ab},\Psi_{ab})=f(ab)=f(a)f(b)=(\Phi_a,\Psi_a)(\Phi_b,\Psi_b)=(\Phi_b\Phi_a, ,\Psi_a\Psi_b)$$ whence $\Phi_{ab}=\Phi_b\Phi_a$ and $\Psi_{ab}=\Psi_a\Psi_b$.

Since S is a subsemigroup of $\bar{S}$, we have that $ab=f(a)f(b)\in E$ i.e. $(\Phi_b\Phi_a,\Psi_a\Psi_b)\in E$ for $a,b\in Q$, $ab\notin Q$, whence $\Phi_b\Phi_a=\text{const.}$ and $\Psi_a\Psi_b=\text{const.}$

Hence conditions (i) and (ii) hold.

From the definition of a multiplication on $\bar{S}$ we have that conditions (1) and (4) hold and

$$a(i,j)=f(a)(i,j)=(\Phi_a,\Psi_a)(i,j)=(i\Phi_a,j)$$
$$(i,j)a=(i,j)f(a)=(i,j)(\Phi_a,\Psi_a)=(i,j\Psi_a)$$

For $a,b\in Q$, $ab\notin Q$

$$ab=f(a)f(b)=(\Phi_b\Phi_a,\Psi_a\Psi_b)=(i\Phi_b\Phi_a,j\Psi_a\Psi_b)$$

since $\Phi_b\Phi_a=\text{const}$ and $\Psi_a\Psi_b=\text{const}$. Hence, conditions (2),

(3) and (5) hold.

Conversely, let $S=\Sigma(I,J,Q,\Phi,\Psi)$ where Q is a partial c-(m,n)-ideal semigroup. It is clear that $Q\cup\{0\}$ is a c-(m,n)-ideal nil-semigroup and S is an ideal extension of a rectangular band E by $Q\cup\{0\}$. Hence, by Theorem 1.5. we have that S is a c-(m,n)-ideal semigroup.□

THEOREM 4.2. S is a c-(m,0)-ideal (c-(0,n)-ideal) semigroup if and only if S is an ideal extension of a left zero (right zero) semigroup E by a c-(m,0)-ideal (c-(0,n)-ideal) nil-semigroup. $(m\geq 1(n\geq 1))$

Proof: Let S is a c-(m,0)-ideal semigroup. Then for all $a\in S$, $\langle a\rangle$ must be a finite semigroup, since $a^{2m}a\in\langle a^2\rangle^m S\subseteq\langle a^2\rangle$. Then S is periodic and the set E of all idempotents from S is nonempty set. For all $e\in E$ and $s\in S$ we have that $es=e^m s\in\langle e\rangle^m S\subseteq\langle e\rangle=\{e\}$, i.e. es=e and, also, (se)(se)=sese==s(es)e=see=se. Then $se,es\in E$ i.e. E is an ideal of S. Now, it is clear that $S/_E$ is a c-(m,0)-ideal nil-semigroup.

Conversely, if S is an ideal extension of a left zero semigroup E by a c-(m,0)-ideal nil-semigroup, then for $a\in S\setminus E$ we have that $(a\rho)^m S/_E\subseteq\langle(a\rho)\rangle=\langle a\rangle$, where $a\rho$ is the class of the element a of modE. Hence, for all $b\in S$ we have that $(a\rho)^m(b\rho)\subseteq\langle a\rangle$ i.e. $a^m b\in\langle a\rangle$. Also, for $e\in E$ we have that $e^m S=eS=e(eS)=eE=e$. Hence S is a c-(m,0)-ideal semigroup.□

COROLLARY 4.3. S is a (m,0)-ideal ((0,n)-ideal) semigroup if and only if S is an ideal extension of a left zero (right zero) semigroup E by a (m,0)-ideal ((0,n)-ideal) nil-semigroup. $(m\geq 1(n\geq 1))$

COROLLARY 4.4. S is a c-(m,0)-ideal (c-(0,n)-ideal) semigroup if and only if S is isomorphic to a semigroup $\Sigma(I,J,Q,\Phi,\Psi)$ where $|I|=1$ and Q is a partial c-(m,0)-ideal semigroup ($|J|=1$ and Q is a partial c-(0,n)-ideal semigroup). $(m\geq 1(n\geq 1))$

COROLLARY 4.5. S is a (m,0)-ideal ((0,n)-ideal) semigroup if and only if S is isomorphic to a semigroup $\Sigma(I,J,Q,\Phi,\Psi)$ where $|I|=1$ and Q is a partial (m,0)-ideal semigroup ($|J|=1$ and Q is a partial (0,n)-ideal semigroup). $(m\geq 1(n\geq 1))$

COROLLARY 4.6. S is a bi-ideal semigroup if and only if S is isomorphic to a semigroup $\Sigma(I,J,Q,\Phi,\Psi)$ where Q is a partial bi-ideal semigroup.

Proof: By Theorem 1. [9] and Corollary 3.4.□

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