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BOOK REVIEW

SEMIGROUPS (Serbian)
by Stojan M. Bogdanović and Miroslav D. Ćirić
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The book is an advanced course in semigroup theory. It is intended for the specialists in this area and for those who want to become such specialists. The book contains nine chapters. Each of them is equipped with its own summary, and every section ends with selected exercises.

The topics covered in the book belong mainly to the General structure theory of semigroups. There are three main subjects of the book: semilattice decompositions, that are studied through Chapters 5 and 6, and partly in Chapter 7, decompositions of semigroups with zero, which are the subject of Chapter 8, and band compositions, treated in Chapter 9. Particular attention is paid to decompositions of π-regular semigroups.

Chapter 1: "*Introduction*". This chapter is devoted to the fundamental concepts of semigroup theory, such as subsemigroups, congruence relations, homomorphisms, ideals and ideal extensions, Green's equivalences, free semigroups etc.

Chapter 2: "π*-Regular Semigroups*". The subject of this chapter are some general properties of semigroups whose any element has a power which is (completely) regular. Various decompositions of such semigroups will be studied systematically throughout the entire book.

Chapter 3: "*(0-) Archimedean Semigroups*". This chapter deals with semigroups in which for any two nonzero elements, each of them divides some power of another one, that are natural generalizations of simple and 0-simple semigroups. Various structural descriptions of completely (0-) Archimedean semigroups, as well as those of completely (0-) simple semigroups, are given.

Chapter 4: "*Semigroups With a Completely Simple Kernel*". The authors present here the Clifford's construction of semigroups having a completely simple kernel. They apply this construction to some particular classes of semigroups.

Chapter 5: "*Theory of Semilattice Decompositions*". Semilattice decompositions, that is the decompositions determined by homomorphisms of a semigroup onto semilattices, have been the subject of interest of semigroup theorists since

1941, when A. H. Clifford introduced them. The fundamental results concerning them are the theorem of T. Tamura and N. Kimura from 1954, which asserts that every semigroup has a greatest semilattice decomposition, and the theorem of T. Tamura from 1956, which asserts that the components of the greatest semilattice decomposition are semilattice indecomposable semigroups. These results initiated an intensive study of greatest semilattice decompositions and many characterizations of them appeared. These were due to M. Yamada, G. Thierrin, T. Tamura, M. Petrich, M. S. Putcha, R. Šulka and others. In this book the authors treat this topic using a new approach that joins the former results from this area. They first introduce the concept of a principal radical of a semigroup, that they use to characterize the greatest semilattice homomorphic image of a semigroup. On the other hand, introducing a system of equivalence relations that generalize the Green's equivalences, they generalize the archimedeaness and give a complete study of the situation of semilattice components between archimedeaness and semilattice indecomposability. These results can be also viewed as a generalization of the Croisot theory. Particular attention is paid to semilattices of Archimedean and simple semigroups.

Chapter 6: "*Semilattices of Completely Archimedean Semigroups*". Decomposition methods developed in the preceding chapter are applied here to π-regular semigroups. The authors collect various structural and indicatorial characterizations of semigroups having a semilattice decomposition into completely Archimedean components, or equivalently, of π-regular semigroups each of whose regular elements is a group element. Certain more significant particular types of them are also treated. The most important results presented here were announced by L. N. Shevrin in 1977 and later, but their proofs have not been published until 1994, so the authors had to give their own proofs of these results.

Chapter 7: "*Nil-extensions of a Union of Groups*". The subject of the present chapter are the semigroups mentioned in the title. They are one of the most important particular types of semigroups considered in the previous chapter. The authors present mainly their own results, published recently, that give some criteria for nil-extensions and retractive nil-extensions of various significant types of completely regular semigroups. In particular, retractive nil-extensions of a regular semigroup K are characterized as subdirect products of a nil-semigroup and K. The authors also describe all identities that induce the structure of a nil-extension of a union of groups in semigroups satisfying them.

Chapter 8: "*Theory of Decompositions of Semigroups With Zero*". Semigroups with zero have the specific structure and many "classical" decomposition methods do not give good results when they are applied to such semigroups. This motivated many authors to investigate some decomposition methods that are more suitable for them. The authors present here two such methods: orthogonal decompositions, introduced by E. S. Lyapin in 1950 and Š. Schwarz in 1951, and decompositions into a right sum of semigroups, that appear here for the first time. The authors approach differs from the one used by J. Dieudonné in ring theory and Š. Schwarz in semigroup theory, whose main tool were 0-minimal ideals, and than the one developed by G. Lallement and M. Petrich, who studied decompositions induced by homomorphisms onto 0-rectangular bands. The main tool of the authors are the elements of the center of the lattice of ideals (in the case of orthogonal decompositions) and the lattice of left ideals (in the case of decompositions into a right sum). This approach connects orthogonal decompositions (decompositions into a right sum) of a semigroup with zero with decompositions of the lattice of its ideals (left ideals) into a direct product. It is shown that every semigroup with zero has both the greatest orthogonal decomposition and the greatest decomposition into a right sum, and that the summands in the greatest orthogonal decomposition are orthogonally indecomposable, what does not hold for decompositions into a right sum.

Chapter 9: "*Band Compositions*". This chapter deals with the following problem: If a family of semigroups indexed by a band is given, how to define a multiplication on the union of this family in order that it might be a semigroup such that the given band is its homomorphic image. That general problem was first stated by A. H. Clifford in 1941, and he and many other authors gave several composition methods that solve this problem in various particular cases. The concept that the authors introduce here generalizes two known concepts: the concept of the Lallement semilattice sum and the concept given by B. M. Schein, that the authors call a strong band of semigroups. This concept they use to characterize certain decompositions of semigroups into subdirect and spined products. They also present constructions of all bands of monoids and groups.

The comprehensive bibliography includes more than 500 items. The book gives a survey of fundamental results concerning the structure of semigroups. Many of them appeared recently. The book may be recommended to all who are interested in semigroup theory and want to have a deeper knowledge of it. It could be also useful to all the specialists in other branches who intend to use the results of semigroup theory. The book contains the wealth of promising ideas, efficient general methods as well as plenty of inspiration for further investigations. It would be worth translating it into English.