

BOOK REVIEW

Polugrupe [Semigroups]

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This is the eleventh monograph in the general algebraic theory of semigroups. Previous ten books (see [1]–[10]) appeared in four different languages (Russian, English, Japanese, and Romanian—in the order, in which they were published), and this book is in the fifth language, Serbian. After its publication two more books (see [11] and [12]) appeared. I should have included a much earlier book [13] of Suschkewitsch, but it was published “too early,” and, as many others, I have been conditioned to think that semigroup theory was born in the late forties, if not later, and only a few relevant papers had been published before that. Also, I excluded from my count numerous books on topological and other semigroups, as well as books devoted to special aspects of the algebraic theory of semigroups.

The authors of this book work at the University of Niš** in Serbia (Yugoslavia), they published numerous papers on various aspects of semigroup theory, especially on semilattice decompositions, and their research interests are reflected in their book.

The book is a systematic exposition of three major directions in the theory of semigroups. One of them is decompositions of semigroups (especially, semilattice decompositions). These decompositions were considered in books of Petrich more than twenty years ago (see [4] and [5]), but many new results, approaches, and methods have appeared since that time, they are included in this book.

Another direction, which attracted less attention in semigroup literature, is decompositions of semigroups with zero. Of course, all results on decompositions of arbitrary semigroups can be applied to semigroups with zero, and yet semigroups with zero merit a special consideration and special decompositions.

The third direction considered in the book is composition (or synthesis) of semigroups: how new semigroups can be built from a family of known semigroup “building blocks.”

The book consists of a short preface, nine chapters, references to almost 550 books and papers, an English translation of the preface and the list of contents, and an index of symbols followed by a terminological index. It is designed as an advanced course of semigroup theory. Here is a brief description of the chapters. Each chapter is subdivided into sections, each section ends with a list of problems and references to literature.

** Just in case (and for those who need guidance), English approximation for the “ć” and “š” sounds are, respectively, “ch” and “sh.”

Chapter 1 introduces main definitions and notions used throughout the book. Chapter 2 is devoted to the so-called π -regular and completely π -regular semigroups (a semigroup is called π -regular if each of its elements has a regular power, these semigroups are also known as “eventually regular” semigroups.) The subject of Chapter 3 is the structure of archimedean and 0-archimedean semigroups. Chapter 4 considers semigroups with completely simple kernel, thus it generalizes the famous results of Suschkewitsch and subsequent (1950) Clifford’s results.

Chapter 5 is devoted to semilattice decompositions (actually, to the greatest semilattice decomposition and various types of it), it is continued in Chapter 6 with semilattice decompositions of (completely) π -regular semigroups into completely archimedean components.

Chapter 7 is about nil extensions of unions of groups, especially retractive ones. In Chapter 8 the authors consider the greatest decompositions of semigroups with zero into right sums and orthogonal sums. These results are used in various special cases. Chapter 9 (the final one) treats band decompositions of semigroups and their connections with subdirect products (especially spined products).

This book is a welcome addition to semigroup literature. It is a valuable source of information both for graduate students interested in semigroups and for more mature mathematicians. It contains—in a systematic form—a great variety of results on decompositions and compositions of semigroups many of which are disseminated in various papers, sometimes in hardly available sources.

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