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Semigroups. (Polugrupe). (Serbian)

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The book is an advanced course in semigroup theory. It is written for specialists in semigroup theory and for those who want to become such specialists. The book has nine chapters; each of them is preceded by its summary.

The first chapter is an “Introduction”. Fundamental concepts of semigroup theory are explained, namely subsemigroups, congruences, homomorphisms, ideals, free semigroups etc. The second chapter has the title “ $\pi$ -regular semigroups”. A  $\pi$ -regular semigroup is a generalization of a regular semigroup; in such a semigroup  $S$  for each element  $a \in S$  there exists a positive integer  $n$  and an element  $x \in S$  such that  $a^n = a^n x a^n$ . The properties of  $\pi$ -regular semigroups are described; regular semigroups are treated as a particular case. Then the third chapter “(0-) Archimedean semigroups” follows. The title means that both Archimedean and 0-Archimedean semigroups are studied; a 0-Archimedean semigroup is a semigroup  $S$  with zero such that for any two non-zero elements  $a, b$  of  $S$  the element  $a$  is a divisor of some power of  $b$ . The fourth chapter is “Semigroups with a completely simple kernel”. It describes semigroups in which the intersection of all ideals is completely simple, i.e. has no proper ideal and contains a primitive idempotent (which is not zero and is not absorbed by any other idempotent). The fifth chapter has the title “Theory of semilattice decompositions”. It studies homomorphisms of a semigroup onto a semilattice. The classes of the congruence corresponding to such a homomorphism are subsemigroups and the semigroup is the union of them. Particular attention is paid to the cases when these semigroups are simple or Archimedean. An immediate continuation of this topic is the sixth chapter “Semilattices of completely Archimedean semigroups”. Then the seventh chapter “Nil-extensions of a union of groups” follows. Such an extension is a semigroup  $S$  having an ideal  $T$  which is a union of groups and has the property that some power  $a^n$  of each element  $a \in S$  is in  $T$ . The eighth chapter is “Theory of decompositions of semigroups with zero”. It describes decompositions into right sums and into orthogonal sums of subsemigroups. The ninth chapter has the title “Band compositions of semigroups”. It studies the following problem: If a family of semigroups indexed by elements of a band is given, how to define the multiplication on the union of this family in order that it might be a semigroup such that the given band is its homomorphic image.

A small mistake in the book can be mentioned. The authors have forgotten to define a rectangular band, although this concept is used throughout the whole book. But this may happen in a theory with such a rich terminology. As it is seen from the titles of the chapters, the book gives a survey of fundamental results concerning the structure of semigroups. It may be recommended to all who are interested in semigroup theory and want to have a deeper knowledge of it. I think that it would be worth of being translated into English.

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*Keywords* : nil-extensions; band compositions; congruences; homomorphisms; ideals; free semigroups;  $\pi$ -regular semigroups; 0-Archimedean semigroups; completely simple kernel; primitive idempotents; semilattice decompositions; union of groups; orthogonal sums

*Classification*:

- 20M10 General structure theory of semigroups
- 20Mxx Semigroups
- 20M17 Regular semigroups

- 20-02 Research monographs (group theory)