Faculty of Sciences and Mathematics, University of Niš, Serbia
Available at: http://www.pmf.ni.ac.yu/filomat

Filomat 22:2 (2008), 43–52

A COMMON FIXED POINT THEOREM IN PROBABILISTIC METRIC SPACE USING IMPLICIT RELATION

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Dedicated to Late Shri Ramdhari Singh Chauhan (Father of the first author) on his 53rd birthday anniversary.

ABSTRACT. In this paper, we prove a common fixed point theorem in a probabilistic metric space by combining the ideas of pointwise R-weak commutativity and reciprocal continuity of mappings satisfying contractive conditions with an implicit relation.

2000 Mathematics Subject Classification: 54H25, 47H10.
Keywords and phrases. Probabilistic metric space, metric space, reciprocal continuity, compatible mappings, R-weakly commuting mappings, implicit relation, fixed point.
Received: May 28, 2008
Communicated by Dragan S. Djordjević

1. Introduction

In 1942, K. Menger [6] introduced the notion of probabilistic metric space (briefly, PM-space) as a generalization of metric space. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physical quantities and physiological thresholds. It is also of fundamental importance in probabilistic functional analysis. The development of fixed point theory in PM-spaces was due to Schweizer and Sklar [14, 15].

In fixed point theory, contraction mapping theorems have been always an active area of research since 1922 with the celebrated Banach contraction fixed point theorem [1]. Sehgal [16] initiated the study of contraction mapping theorems in PM-spaces. Subsequently, several contraction mapping theorems for commuting mappings have been proved in PM-spaces; see for instance [5], [9], [18], [19], [20].

The notions of improving commutativity of mappings have been extended to PM-spaces by various mathematicians. For example, Singh and Pant [21] extended the notion of weak commutativity (introduced by Sessa [17] in metric spaces), Mishra [8] extended the notion of compatibility (introduced by Jungck [3] in metric spaces) and Ćirić and Milovanović-Arandjelović [2] extended the notion of poinwise R-weak commutativity (introduced by Pant [11] in metric spaces) to PM-spaces. These mathematicians have also proved some common fixed point theorems for contraction mappings by applying them in PM-spaces.

Most of the common fixed point theorems for contraction mappings invariably require a compatibility condition besides assuming continuity of at least one of the mappings. In 1999, Pant [12] noticed these criteria for fixed points of contraction mappings and introduced a new continuity condition, known as reciprocal continuity and obtained a common fixed point theorem by using the compatibility in metric spaces. He also showed that in the setting of common fixed point theorems for compatible mappings satisfying contraction conditions, the notion of reciprocal continuity is weaker than the continuity of one of the mappings.

Also, the notion of pointwise R-weakly commuting mappings made the scope of the study of common fixed point theorems from the class of compatible to the wider class of pointwise R-weakly commuting mappings. Using the ideas of pointwise R-weak commutativity and reciprocal continuity of mappings, Kumar and Chugh [4] established some common fixed point theorems in metric spaces. In 2005, Miheţ [7] established a fixed point theorem concerning probabilistic contractions satisfying an implicit relation.
The purpose of this paper is to prove a common fixed point theorem by combining the ideas of pointwise R-weak commutativity and reciprocal continuity of mappings satisfying contractive conditions with an implicit relation. Our result is an improved extension of the result of Kumar and Chugh [4] to PM-spaces.

2. Preliminaries

Definition 2.1[15] A mapping $F : R \to R^+$ is called a distribution function if it is non-decreasing and left continuous with $inf_{t\in R}F(t) = 0$ and $sup_{t\in R}F(t) = 1$.
We shall denote by $\Im$ the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & t \le 0; \\ 1, & t > 0. \end{cases}$$

Definition 2.2[15] A PM-space is an ordered pair (X, F), where X is a nonempty set of elements and F is a mapping from $X \times X$ to $\Im$, the collection of all distribution functions. The value of F at $(u,v) \in X \times X$ is represented by $F_{u,v}$. The functions $F_{u,v}$ are assumed to satisfy the following conditions:

(PM1) $F_{u,v}(t) = 1$ for all $t > 0$ iff $u = v$;

(PM2) $F_{u,v}(0) = 0$;

(PM3) $F_{u,v}(t) = F_{v,u}(t)$;

(PM4) if $F_{u,v}(t) = 1$ and $F_{v,w}(s) = 1$ then $F_{u,w}(t+s) = 1$
for all $u, v, w \in X$ and $t, s \ge 0$.

Definition 2.3[15] A mapping $\Delta : [0, 1] \times [0, 1] \to [0, 1]$ is called a triangular norm (briefly, t-norm) if the following conditions are satisfied:

(*i*) $\Delta(a, 1) = a$ for all $a \in [0, 1]$;

(*ii*) $\Delta(a, b) = \Delta(b, a)$;

(*iii*) $\Delta(c, d) \ge \Delta(a, b)$ for $c \ge a$, $d \ge b$;

(*iv*) $\Delta(\Delta(a, b), c) = \Delta(a, \Delta(b, c))$;
for all $a, b, c, d \in [0, 1]$.

Example 2.1. The following are the four basic t-norms:

(i) The *minimum* t*-norm*, Δ_M, is defined by

$\Delta_M(x, y) = min(x, y)$,

(ii) The *product* t*-norm*, Δ_P, is defined by

$\Delta_P(x, y) = x.y$,

(iii) The *Lukasiewicz* t*-norm*, Δ_L, is defined by

$\Delta_L(x, y) = max(x + y - 1, 0)$,

(iv) The weakest t-norm, the *drastic product*, Δ_D, is defined by

$$\Delta_D(x, y) = \begin{cases} min(x, y) & if \;\; max(x, y) = 1, \\ 0 & otherwise. \end{cases}$$

As regards the pointwise ordering, we have the inequalities

$\Delta_D < \Delta_L < \Delta_P < \Delta_M$.

Definition 2.4[15] A Menger space is a triplet (X, F, Δ), where (X, F) is a PM-space and t-norm Δ is such that the inequality

$F_{u,w}(t + s) \geq \Delta\{F_{u,v}(t), F_{v,w}(s)\}$

holds for all $u, v, w \in X$ and all $t, s \geq 0$.

Every metric space (X, d) can be realized as a PM-space by taking $F : X \times X \to \Im$ defined by $F_{u,v}(t) = H(t - d(u,v))$ for all u,v in X.

Definition 2.5[21] Two self-mappings A and S of a PM-space (X, F) are said to be weakly commuting if $F_{ASz, SAz}(t) \geq F_{Az,Sz}(t)$ for each z in X and $t > 0$.

Every pair of commuting self-maps is weakly commuting, but the reverse is not true. For this, refer to example in [10].

Definition 2.6[8] Two self-mappings A and S of a PM-space (X, F) will be called compatible if and only if $F_{ASu_n, SAu_n}(t) \to 1$ for all $t > 0$, whenever $\{u_n\}$ is a sequence in X such that $Au_n, Su_n \to z$ for some z in X.

Definition 2.7[2] Two self-mappings A and S of a PM-space (X, F) are said to be pointwise R-weakly commuting if given z in X there exist $R > 0$ such that $F_{ASz, SAz}(t) \geq F_{Az,Sz}(t / R)$ for $t > 0$.

Clearly, every pair of weakly commuting mappings is pointwise R-weakly commuting with $R = 1$.

Remark 2.1. It is obvious that A and S can fail to be pointwise R-weakly commuting only if there is some z in X such that $Az = Sz$ but $ASz \neq SAz$, that is, only if they posses a coincidence point at which they do not commute. This means that a contractive type mapping pair cannot posses a common fixed point without being pointwise R-

weakly commuting since a common fixed point is also a coincidence point at which the mappings commute, and contractive conditions exclude the possibility of two types of coincidence points. Also, compatible mappings are necessarily pointwise R-weakly commuting since compatible mappings commute at their coincidence points. However, pointwise R-weakly commuting mappings need not to be compatible as shown in the following example:

Example 2.2. Let $X = [2, 20]$ and let F be defined by

$$F_{u,\,v}(t) = \begin{cases} \dfrac{t}{t+|u-v|}, & if \;\; t>0; \\ 0, & if \;\; t=0. \end{cases}$$

Then (X, F) is a probabilistic metric space. Let A and S be self-mappings of X defined as

$$Au = \begin{cases} 2, & u=2 \;\; or \;\; u>5; \\ 8, & 2<u\le 5 \end{cases} \quad \text{and} \quad Su = \begin{cases} 2, & u=2; \\ 12+u, & 2<u\le 5; \\ u-3, & u>5. \end{cases}$$

It can be verified that A and S are pointwise R-weakly commuting mappings but not compatible. Also, neither A nor S is continuous, not even at their coincidence points.

The concept of reciprocal continuity of mappings in PM-spaces is as follows:

Definition 2.8. Two self-mappings A and S of a PM-space (X, F) will be called reciprocally continuous if $ASu_n \to Az$ and $SAu_n \to Sz$, whenever $\{u_n\}$ is a sequence such that $Au_n, Su_n \to z$ for some z in X.

If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of mappings satisfying contractive conditions, continuity of one of the mappings A and S implies their reciprocal continuity but not conversely.

Lemma 2.1 [20] Let $\{u_n\}$ be a sequence in a Menger space (X, F, Δ_M). If there exists a constant $h \in (0, 1)$ such that

$$F_{u_n,\,u_{n+1}}(ht) \ge F_{u_{n-1},\,u_n}(t), \quad n = 1,2,3...$$

then $\{u_n\}$ is a Cauchy sequence in X.

3. Implicit Relation

In [7], Mihet established a fixed point theorem concerning probabilistic contractions satisfying an implicit relation. This implicit relation is similar to that in [13]. In [13], Popa used the family F_4 of implicit real functions to find the fixed points of two pairs of semi-compatible mappings in a d-compatible topological space. Here, F_4 denotes the family of all real continuous functions $F:(R^+)^4 \to R$ satisfying the following properties:

(F_h) There exists $h \geq 1$ such that for every $u \geq 0, v \geq 0$ with $F(u,v,u,v) \geq 0$ or $F(u,v,v,u) \geq 0,$ we have $u \geq hv$.

(F_u) $F(u,u,0,0) < 0$ for all $u > 0$.

In our result, we deal with the class $\mathbf{\Phi}$ of all real continuous functions $\varphi:(R^+)^4 \to R$, non-decreasing in the first argument and satisfying the following conditions:

(3.I) For $u, v \geq 0$, $\varphi(u,v,u,v) \geq 0$ or $\varphi(u,v,v,u) \geq 0$ implies that $u \geq v$.

$(3.II)$ $\varphi(u,u,1,1) \geq 0$ for all $u \geq 1$.

Example 3.1. Define $\varphi(t_1, t_2, t_3, t_4) = at_1 + bt_2 + ct_3 + dt_4$, where $a, b, c, d \in R$ with $a+b+c+d=0$, $a>0,\ a+c>0,\ a+b>0$ and $a+d>0$. Then $\varphi \in \mathbf{\Phi}$.

Example 3.2. Define $\varphi(t_1, t_2, t_3, t_4) = 14t_1 - 12t_2 + 6t_3 - 8t_4$. Then $\varphi \in \mathbf{\Phi}$.

4. Common Fixed Point Theorem

Before proving the main result, we give following lemma:

Lemma 4.1. Let (X, F, Δ_M) be a complete Menger space. Further, let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of X satisfying

(4.1.1) $A(X) \subseteq T(X),\ B(X) \subseteq S(X);$

(4.1.2) $\varphi(F_{Au,Bv}(ht), F_{Su,Tv}(t), F_{Au,Su}(t), F_{Bv,Tv}(ht)) \geq 0;$

(4.1.3) $\varphi(F_{Au,Bv}(ht), F_{Su,Tv}(t), F_{Au,Su}(ht), F_{Bv,Tv}(t)) \geq 0,$

for all $u, v \in X$, $t > 0$, $h \in (0,1)$ and for some $\varphi \in \mathbf{\Phi}$. Then the continuity of one of the mappings in compatible pair (A, S) or (B, T) on (X, F, Δ) implies their reciprocal continuity.

Proof. First, assume that A and S are compatible and S is continuous. We show that A and S are reciprocally continuous. Let $\{u_n\}$ be a sequence such that $Au_n \to z$ and $Su_n \to z$ for some $z \in X$ as $n \to \infty$. Since S is continuous, we have $SAu_n \to Sz$ and $SSu_n \to Sz$ as $n \to \infty$ and since (A, S) is compatible, we have

$F_{ASu_n,\, SAu_n}(t) \to 1$. This implies that $F_{ASu_n,\, Sz}(t) \to 1$ that is, $ASu_n \to Sz$ as $n \to \infty$. By (4.1.1), for each n, there exists v_n in X such that $ASu_n = Tv_n$. Thus, we have $SSu_n \to Sz$, $SAu_n \to Sz$, $ASu_n \to Sz$ and $Tv_n \to Sz$ as $n \to \infty$ whenever $ASu_n = Tv_n$.

Now we claim that $Bv_n \to Sz$ as $n \to \infty$. Suppose not, then by (4.1.2),

$$\varphi(F_{ASu_n, Bv_n}(ht),\, F_{SSu_n, Tv_n}(t),\, F_{ASu_n, SSu_n}(t),\, F_{Bv_n, Tv_n}(ht)) \geq 0.$$

Letting $n \to \infty$,

$$\varphi(F_{Sz, Bv_n}(ht),\, F_{Sz, Sz}(t),\, F_{Sz, Sz}(t),\, F_{Bv_n, Sz}(ht)) \geq 0,$$

that is, $\varphi(F_{Bv_n,\, Sz}(ht), 1, 1, F_{Bv_n, Sz}(ht)) \geq 0.$

Using $(3.I)$, we get $F_{Bv_n,\, Sz}(ht) \geq 1$ for all $t > 0$. Hence, $F_{Bv_n,\, Sz}(ht) = 1$. Thus, $Bv_n \to Sz.$

Again by (4.1.2),

$$\varphi(F_{Az, Bv_n}(ht),\, F_{Sz, Tv_n}(t),\, F_{Az, Sz}(t),\, F_{Bv_n, Tv_n}(ht)) \geq 0.$$

Letting $n \to \infty$,

$$\varphi(F_{Az, Sz}(ht), 1,\; F_{Az, Sz}(t), 1) \geq 0.$$

As φ is non-decreasing in the first argument, we have

$$\varphi(F_{Az, Sz}(t), 1,\; F_{Az, Sz}(t), 1) \geq 0.$$

Using $(3.I)$, we get $F_{Az, Sz}(t) \geq 1$ for all $t > 0$. This gives $F_{Az, Sz}(t) = 1$. Thus, $Az = Sz.$

Thus, $SAu_n \to Sz$ and $ASu_n \to Sz = Az$ as $n \to \infty$.

Therefore, A and S are reciprocally continuous on X. If the pair (B, T) is assumed to be compatible and T is continuous, the proof is similar.

Theorem 4.1. Let (X, F, Δ_M) be a complete Menger space. Further, let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of X satisfying (4.1.1), (4.1.2) and (4.1.3). If one of the mappings in compatible pair (A, S) or (B, T) is continuous, then A, B, S and T have a unique common fixed point.

Proof. Let $u_0 \in X$. By (4.1.1), we define the sequences $\{u_n\}$ and $\{v_n\}$ in X such that for all $n = 0, 1, 2...$

(4.1.4) $\quad v_{2n+1} = Au_{2n} = Tu_{2n+1}$, $v_{2n+2} = Bu_{2n+1} = Su_{2n+2}$.

By (4.1.2),

$$\varphi(F_{Au_{2n}, Bu_{2n+1}}(ht),\, F_{Su_{2n}, Tu_{2n+1}}(t),\, F_{Au_{2n}, Su_{2n}}(t),\, F_{Bu_{2n+1}, Tu_{2n+1}}(ht)) \geq 0,$$

that is, $\varphi(F_{v_{2n+1},v_{2n+2}}(ht),\ F_{v_{2n},v_{2n+1}}(t),\ F_{v_{2n+1},v_{2n}}(t),\ F_{v_{2n+2},v_{2n+1}}(ht)) \geq 0.$

Using $(3.I)$, we get

(4.1.5) $F_{v_{2n+1},v_{2n+2}}(ht) \geq F_{v_{2n},v_{2n+1}}(t).$

Similarly, by (4.1.3) and then by using $(3.I)$, we have

(4.1.6) $F_{v_{2n+2},v_{2n+3}}(ht) \geq F_{v_{2n+1},v_{2n+2}}(t).$

Thus, for any n and t, we have

$$F_{v_n,v_{n+1}}(ht) \geq F_{v_{n-1},v_n}(t).$$

Hence by Lemma 2.1, $\{v_n\}$ is a Cauchy sequence in X. Since X is complete, $\{v_n\}$ converges to z. Its subsequences $\{Au_{2n}\}$, $\{Bu_{2n+1}\}$, $\{Su_{2n}\}$ and $\{Tu_{2n+1}\}$ also converge to z.

Now suppose that (A, S) is a compatible pair and S is continuous. Then by Lemma 4.1, A and S are reciprocally continuous, then $ASu_{2n} \to Az$ and $SAu_{2n} \to Sz$. Compatibility of A and S gives $F_{ASu_{2n},\, SAu_{2n}}(t) \to 1$ i.e. $F_{Az,\, Sz}(t) \to 1$ as $n \to \infty$. Hence, $Az = Sz$.

Since $A(X) \subseteq T(X)$, there exists a point p in X such that $Az = Tp$.

By (4.1.2),

$$\varphi(F_{Az,\, Bp}(ht),\ F_{Sz,\, Tp}(t),\ F_{Az,\, Sz}(t),\ F_{Bp,\, Tp}(ht)) \geq 0,$$

that is, $\varphi(F_{Az,\, Bp}(ht),\ 1,\ 1,\ F_{Bp,\, Az}(ht)) \geq 0.$

Using $(3.I)$, we get $F_{Az,Bp}(ht) \geq 1$ for all $t > 0$, which gives $F_{Az,Bp}(ht) = 1.$

Hence, $Az = Bp$.

Thus, $Az = Sz = Bp = Tp$. Since A and S are pointwise R-weakly commuting mappings, there exists $R > 0$ such that

$$F_{ASz,SAz}(t) \geq F_{Az,Sz}(t/R) = 1.$$

That is $ASz = SAz$ and $AAz = ASz = SAz = SSz$.

Similarly, since B and T are pointwise R-weakly commuting mappings, we have

$$BBp = BTp = TBp = TTp.$$

Again by (4.1.2),

$$\varphi(F_{AAz,Bp}(ht),\ F_{SAz,Tp}(t),\ F_{AAz,SAz}(t),\ F_{Bp,Tp}(ht)) \geq 0,$$

that is, $\varphi(F_{AAz,\, Az}(ht),\ F_{AAz,\, Az}(t),\ 1,\ 1) \geq 0.$

As φ is non-decreasing in the first argument, we have

$$\varphi(F_{AAz,\, Az}(t),\ F_{AAz,\, Az}(t),\ 1,\ 1) \geq 0.$$

Using $(3.II)$, we have $F_{AAz,Az}(t) \geq 1$ for all $t > 0$. This gives $F_{AAz,Az}(t) = 1$ implying $AAz = Az$ and $Az = AAz = SAz$. Thus, Az is a common fixed point of A and S. Similarly by (4.1.2), we have that $Bp(= Az)$ is a common fixed point of B and T. Thus, Az is a common fixed point of A, B, S and T.

Finally, suppose that $Ap(\neq Az)$ is another common fixed point of A, B, S and T. Then by (4.1.2),

$$\varphi(F_{AAz,BAp}(ht), F_{SAz,TAp}(t), F_{AAz,SAz}(t), F_{BAp,TAp}(ht)) \geq 0,$$

that is, $\varphi(F_{Az,Ap}(ht), F_{Az,Ap}(t), 1, 1) \geq 0.$

As φ is non-decreasing in the first argument, we have

$$\varphi(F_{Az,Ap}(t), F_{Az,Ap}(t), 1, 1) \geq 0.$$

Using $(3.II)$, we have $F_{Az,Ap}(t) \geq 1$, for all $t > 0$, which gives $F_{Az,Ap}(t) = 1$ implying $Az = Ap.$

Thus Az is a unique common fixed point of A, B, S and T.

This completes the proof of the theorem.

Remark 4.1. Theorem 4.1 is an improved extension of the result of Kumar and Chugh [4, Theorem 3.2] to PM-spaces.

Taking $S = T = I_X$ (identity mapping) in Theorem 4.1, we have the following result:

Corollary 4.1. Let (X, F, Δ_M) be a complete Menger space. Further, let A and B be self-mappings of X satisfying

$$(4.1.a)\quad \varphi(F_{Au,Bv}(ht), F_{u,v}(t), F_{Au,u}(t), F_{Bv,v}(ht)) \geq 0;$$

$$(4.1.b)\quad \varphi(F_{Au,Bv}(ht), F_{u,v}(t), F_{Au,u}(ht), F_{Bv,v}(t)) \geq 0$$

for all $u, v \in X$, $t > 0$, $h \in (0, 1)$ and for some $\varphi \in \mathbf{\Phi}$. If A and B are reciprocally continuous mappings, then A and B have a unique common fixed point.

The following example illustrates Theorem 4.1.

Example 4.1. Let $X = R^+$ and let F be defined by

$$F_{u,v}(t) = \begin{cases} \dfrac{t}{t + |u - v|}, & if \;\; t > 0 \\ 0, & if \;\; t = 0 \end{cases}$$

Then (X, F) is a probabilistic metric space. Let A, B, S and T be self-mappings of X defined as

$A0 = 0,\ Au = 1$ if $u > 0$

$Bu = 0$ if $u = 0$ or $u > 6$, $Bu = 2$ if $0 < u \leq 6$

$S0 = 0,\ Su = 2$ if $u > 0$

$T0 = 0,\ Tu = 4$ if $0 < u \le 6,\ Tu = u - 6$ if $u > 6.$

Then A, B, S and T satisfy all the conditions of Theorem 4.1 with $h \in (0, 1)$ and have a unique common fixed point $u = 0$. Clearly A and S are reciprocally continuous compatible mappings. However, A and S are not continuous, not even at the common fixed point. The mappings B and T are non-compatible because if we suppose that $\{u_n\}$ be a sequence defined as $u_n = 6 + \frac{1}{n},\ n \ge 1,$ then $Bu_n = 0,$ $Tu_n \to 0,\ TBu_n = 0$ and $BTu_n = 2$, hence B and T are non-compatible, but pointwise R-weakly commuting since they commute at their coincidence points.

Acknowledgement: The authors thank the referee for the critical reading of the paper and valuable suggestions to improve the paper.

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