Faculty of Sciences and Mathematics, University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Filomat **23:3** (2009), 251–255 DOI:10.2298/FIL0903251V

A NOTE ON ACHROMATIC COLORING OF STAR GRAPH FAMILIES

Vivin J. Vernold, M. Venkatachalam and Ali M.M. Akbar

Abstract

In this paper, we find the achromatic number of central graph, middle graph and total graph of star graph, denoted by $C(K_{1,n}), M(K_{1,n})$ and $T(K_{1,n})$ respectively.

1 Introduction

For a given graph $G = (V, E)$ we do an operation on G, by subdividing each edge exactly once and joining all the non adjacent vertices of G. The graph obtained by this process is called central graph [14] of G denoted by $C(G)$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The middle graph [4] of G, denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one the following holds: (i) x, y are in $E(G)$ and x, y are adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [4, 5] of G, denoted by $T(G)$ is defined as follows. The vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one the following holds: (i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y are adjacent in G. (iii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

An achromatic coloring [1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15] of a graph G is a proper vertex coloring of G in which every pair of colors appears on at least one pair of adjacent vertices. The achromatic number of G denoted $\chi_c(G)$, is the greatest number of colors in an achromatic coloring of G.

The achromatic number was introduced by Harary, Hedetniemi and Prins [6]. They considered homomorphisms from a graph G onto a complete graph K_n. A homomorphism from a graph G to a graph G' is a function $\phi : V(G) \to V(G')$

2000 *Mathematics Subject Classifications.* 05C15.
Key words and Phrases. Central graph, middle graph, total graph and achromatic coloring.
Received: April 3, 2009
Communicated by Dragan Stevanović

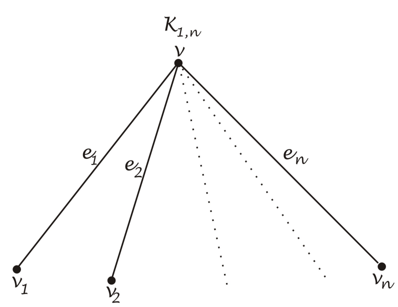
such that whenever u and v are adjacent in G, $u\phi$ and $v\phi$ are adjacent in G'. They show that, for every (complete) n-coloring τ of a graph G there exists a (complete) homomorphism ϕ of G onto K_n and conversely. They noted that the smallest n for which such a complete homomorphism exists is just the chromatic number $\chi = \chi(G)$ of G. They considered the largest n for which such a homomorphism exists. This was later named as the achromatic number $\psi(G)$ by Harary and Hedetniemi [6]. In the first paper [6] it is shown that there is a complete homomorphism from G onto K_n if and if only $\chi(G) \leq n \leq \psi(G)$.

2 Achromatic coloring of central, middle and total graph of star graphs

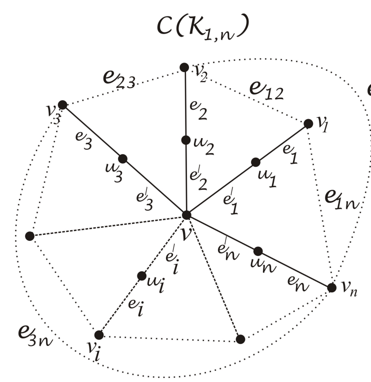
Theorem 2.1. *For any star graph $K_{1,n}$, the achromatic number,*

$$\chi_c[C(K_{1,n})] = n + 1.$$

Proof. Let $v_1, v_2, \cdots, v_n$ be the pendant vertices of $K_{1,n}$ and let v be the vertex of $K_{1,n}$ adjacent to $v_i (1 \leq i \leq n)$. Obviously, $deg(v) = n$. Let the edge vv_i be subdivided by the vertex $u_i (1 \leq i \leq n)$ in $C(K_{1,n})$, and let $V = \{v_1, v_2, \cdots v_n\}, V' = \{u_1, u_2, \cdots u_n\}$. Clearly $V[C(K_{1,n})] = V \cup V' \cup \{v\}$. The number of edges in $C(K_{1,n})$ is $\binom{n+1}{2} + n = \frac{n^2+3n}{2} < \binom{n+2}{2}$. Hence $\chi_c[C(K_{1,n})] \leq n+1$. Note that in $C(K_{1,n})$, the induced subgraph $\langle v_1, v_2, \cdots v_n \rangle$ is complete, and $\{u_1, u_2, \cdots u_n\}$ is independent set.



Star graph $K_{1,n}$
Figure 1(a)

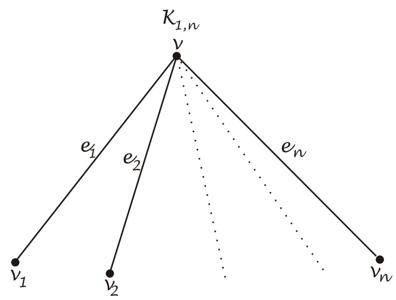


Central graph of Star graph $K_{1,n}$
Figure 1(b)

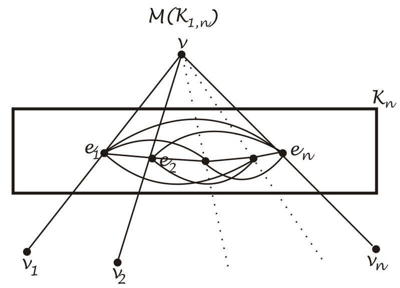
The following $(n+1)$-coloring for $C(K_{1,n})$ is achromatic: For $(1 \leq i \leq n)$, assign the color c_i for v_i. Assign color c_{n+1} for all $u_i (1 \leq i \leq n)$. Assign color c_1 for v. Thus we have $\chi_c[C(K_{1,n})] = n + 1$. □

Theorem 2.2. *For any star graph $K_{1,n}$ the achromatic number,*

$$\chi_c[M(K_{1,n})] = n + 1.$$



Star graph $K_{1,n}$
Figure 2(a)

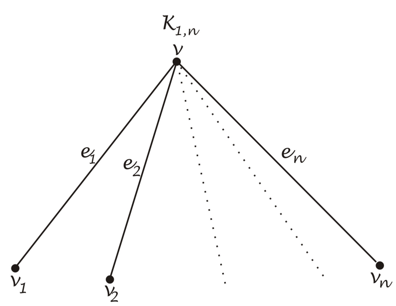


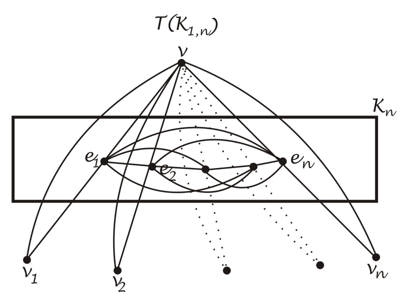
Middle graph of Star graph $K_{1,n}$
Figure 2(b)

Proof. Let $V(K_{1,n}) = \{v, v_1, v_2. \cdots, v_n\}$. By the definition of middle graph, each edge of $vv_i, (1 \leq i \leq n)$ of $K_{1,n}$ is subdivided by the vertex e_i in $M(K_{1,n})$ and the vertices $v, e_1, e_2, \cdots, e_n$ induce a clique of order $(n+1)$ in $M(K_{1,n})$.i.e.,$V[M(K_{1,n})] = \{v\} \cup \{v_i/1 \leq i \leq n\} \cup \{e_i/1 \leq i \leq n\}$. Now consider the color class $C = \{c_1, c_2, \cdots c_n, c_{n+1}\}$, and assign the achromatic coloring to $M(K_{1,n})$ as follows: For $(1 \leq i \leq n)$, assign the color c_i for e_i and assign color c_{n+1} to v. For $(2 \leq i \leq n-1)$, assign color c_1 for v_i and assign color c_n to v_1. Thus we have $\chi_c[M(K_{1,n})] \geq n + 1$. As the number of edges in $M(K_{1,n}) = \dfrac{n^2 + 3n}{2} < \dbinom{n+2}{2}$. Therefore $\chi_c[M(K_{1,n})] \leq n + 1$. Hence $\chi_c[M(K_{1,n})] = n + 1$. $\square$

Theorem 2.3. *For any star graph $K_{1,n}$ the achromatic number,*

$$\chi_c[T(K_{1,n})] = n + 2.$$





Star graph $K_{1,n}$
Figure 3(a)

Total graph of Star graph $K_{1,n}$
Figure 3(b)

Proof. Let $V(K_{1,n}) = \{v, v_1, v_2, \cdots, v_n\}$ and $E(K_{1,n}) = \{e_1, e_2, \cdots, e_n\}$. By the definition of total graph, we have $V[T(K_{1,n})] = \{v\} \cup \{e_i/1 \leq i \leq n\} \cup \{v_i/1 \leq i \leq n\}$, in which the vertices $v, e_1, e_2, \cdots, e_n$ induce a clique of order $(n+1)$. As the number of edges in $T(K_{1,n}) = \dfrac{n^2+5n}{2} < \dbinom{n+3}{2}$. Hence $\chi_c[T(K_{1,n})] \leq n+2$. The following $(n+2)$-coloring for $T(K_{1,n})$ is achromatic: For $(1 \leq i \leq n)$, assign the color c_i for e_i and assign color c_{n+1} to v. For $(1 \leq i \leq n)$, assign color c_{n+2} for v_i. Thus we have $\chi_c[T(K_{1,n})] = n+2$. □

Theorem 2.4. *For any star graph* $K_{1,n}$, $\chi_c[C(K_{1,n})] = \chi_c[M(K_{1,n})] = \chi[M(K_{1,n})] = \chi[T(K_{1,n})] = n+1$.

3 Observations

We observe that the achromatic number of middle graph of cycles and paths are as follows.

(i) The achromatic number of middle graph of cycle C_n, $\chi_c[M(C_n)] \geq n$.

(ii) The achromatic number of middle graph of path P_n, $\chi_c[M(P_n)] \geq n$.

Acknowledgement. The present version of this paper owes much to the comments, corrections, suggestions, typographical errors and kind remarks of the learned referee.

References

[1] V.N.Bhat-Nayak and M.Shanti, *Achromatic numbers of a graph and its complement*, Bulletin of the Bombay Mathematical Colloquium 6(1989), 9-14.

[2] N.Cairnie and K.J.Edwards, *Some results on the achromatic number*, Journal of Graph Theory 26(1997), 129-136.

[3] N.Cairnie and K.J.Edwards, *The achromatic number of bounded degree trees*, Discrete Mathematics 188(1998), 87-97.

[4] Danuta Michalak, *On middle and total graphs with coarseness number equal 1,* Graph theory, Proc. Conf., Lagów/Pol. 1981, Lect. Notes Math. 1018, Springer, Berlin 1983, 139-150.

[5] Frank Harary, Graph Theory, Narosa Publishing, Bombay, 1969.

[6] F.Harary and S.T.Hedetniemi, *The achromatic number of a graph*, Journal of Combinatorial Theory 8(1970), 154-161.

[7] P.Hell and D.J.Miller, *Graph with given achromatic number*, Discrete Mathematics 16(1976), 195-207.

[8] P.Hell and D.J.Miller, *Achromatic numbers and graph operations*, Discrete Mathematics 108(1992), 297-305.

[9] M.Hornak, *Achromatic index of* $K_{1,2}$, Ars Combinatoria 45(1997), 271-275.

[10] Jaeun Lee and Young-hee Shin, *The achromatic number of the union of cycles*, Discrete Applied Mathematics 143 (2004), 330-335.

[11] G.MacGillivray and A.Rodriguez, *The achromatic number of the union of paths*, Discrete Mathematics 231 (2001), 331-335.

[12] M.F.Mammana, *On the achromatic number of* $P(\alpha, K_n), P(\alpha, K_{1,n})$ *and* $P(\alpha, K_{m,n})$, Utilitas Mathematica 62 (2002),61-70.

[13] D.Marcu, *Note on the n-cycles and their achromatic numbers*, Computer Science Journal of Moldova 10 (2002), 329-332.

[14] Vernold Vivin J., *Harmonious Coloring of Total Graphs,* $n-$*Leaf, Central graphs and Circumdetic Graphs*, Ph.D Thesis, Bharathiar University (2007),Coimbatore, India.

[15] Wikipedia, The free encyclopedia.

Department of Mathematics, Sri Shakthi Institute of Engineering and Technology, Coimbatore-641 062, Tamil Nadu, India.
Department of Mathematics, SSK College of Engineering and Technology, Coimbatore-641 105, Tamil Nadu, India.
E-mails: vernold_vivin@yahoo.com,vernoldvivin@yahoo.in (V.J. Vernold),
venkatmaths@gmail.com (M. Venkatachalam),
um_akbar@yahoo.co.in (A.M.M. Akbar).