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Generalized weighted composition operators from Bloch spaces into Bers-type spaces

Xiangling Zhu[a]

[a] *Department of Mathematics, JiaYing University, 514015, Meizhou, GuangDong, China*

Abstract. New criteria for the boundedness and the compactness of the generalized weighted composition operators from Bloch spaces into Bers-type spaces are given in this paper.

1. Introduction

Let $\mathbb{D}$ be the unit disk of complex plane $\mathbb{C}$, and $H(\mathbb{D})$ the class of functions analytic in $\mathbb{D}$. We denote by $H^\infty = H^\infty(\mathbb{D})$ the bounded analytic function space on $\mathbb{D}$. Recall that an $f \in H(\mathbb{D})$ is said to belong to the Bloch space $\mathscr{B}$ if

$$\|f\|_b = \sup_{z\in\mathbb{D}}(1-|z|^2)|f'(z)| < \infty.$$

With the norm $\|f\|_{\mathscr{B}} = |f(0)| + \|f\|_b$, $\mathscr{B}$ is a Banach space. Let $\mathscr{B}_0$ be the space which consists of all $f \in \mathscr{B}$ satisfying

$$\lim_{|z|\to 1}(1-|z|^2)|f'(z)| = 0.$$

This space is called the little Bloch space. See [25] for more information on Bloch spaces.

Let $\alpha \geq 0$. The Bers-type space, denoted by H_α^∞, is a Banach space consisting of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{H_\alpha^\infty} = \sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha \left|f(z)\right| < \infty.$$

It is clear that $H_0^\infty = H^\infty$.

In this paper, let φ always denote an analytic self-map of $\mathbb{D}$. The composition operator C_φ, induced by φ, is defined by

$$C_\varphi f = f \circ \varphi, \qquad f \in H(\mathbb{D}).$$

A fundamental and interesting problem concerning composition operators is to relate function theoretic properties of φ to operator theoretic properties of C_φ on various spaces. See [3] for more topics about the composition operator.

Let $u \in H(\mathbb{D})$. The weighted composition operator uC_φ, induced by φ and u, is defined by

$$(uC_\varphi f)(z) = u(z) \cdot f(\varphi(z)), \;\; f \in H(\mathbb{D}), \;\; z \in \mathbb{D}.$$

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Email address: xiangling-zhu@163.com (Xiangling Zhu)

Let D be the differentiation operator and n be a nonnegative integer. Write

$$Df = f', \quad D^n f = f^{(n)}, \quad f \in H(\mathbb{D}).$$

The generalized weighted composition operator $D^n_{\varphi,u}$, which introduced by the author of this paper, is defined as follows (see, e.g., [26–28]).

$$(D^n_{\varphi,u} f)(z) = u(z) \cdot f^{(n)}(\varphi(z)), \quad f \in H(\mathbb{D}), \quad z \in \mathbb{D}.$$

When $n = 0$, then $D^n_{\varphi,u} = uC_\varphi$. When $n = 0$ and $u(z) \equiv 1$, then $D^n_{\varphi,u} = C_\varphi$. When $n = 1$, $u(z) = \varphi'(z)$, then $D^n_{\varphi,u} = DC_\varphi$. When $n = 1$ and $u(z) = 1$, then $D^n_{\varphi,u} = C_\varphi D$. The operators DC_φ and $C_\varphi D$ were studied, for example, in [7, 9, 12, 17, 20, 22].

Composition operators, weighted composition operators and generalized weighted composition operators between Bloch spaces and some other spaces in one and several complex variables were studied, for example, in [1, 2, 8, 10, 11, 13–15, 18–24, 27]. See [4–6, 16–19, 23, 26, 29] for corresponding operators between Bers-type spaces and some other spaces.

In this paper, motivated by [1, 2], we give some new criteria for the boundedness or compactness of the operator $D^n_{\varphi,u}$ from Bloch spaces to Bers-type spaces.

Throughout the paper, C denotes a positive constant which may differ from one occurrence to the other. The notation $A \asymp B$ means that there exists a positive constant C such that $B/C \le A \le CB$.

2. Main results and proofs

In this section we give our main results and proofs. For this purpose, we need the following lemma, which can be proved in a standard way (see, for example, Theorem 3.11 in [3]).

Lemma 2.1. *Let n be a nonnegative integer, $\alpha \ge 0$, $u \in H(\mathbb{D})$ and let φ be an analytic self-map of $\mathbb{D}$. Then $D^n_{\varphi,u} : \mathcal{B}(\text{or } \mathcal{B}_0) \to H^\infty_\alpha$ is compact if and only if $D^n_{\varphi,u} : \mathcal{B}(\text{or } \mathcal{B}_0) \to H^\infty_\alpha$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $\mathcal{B}(\text{or } \mathcal{B}_0)$ which converges to zero uniformly on compact subsets of $\mathbb{D}$, $D^n_{\varphi,u} f_k \to 0$ in H^∞_α as $k \to \infty$.*

For $w \in \mathbb{D}$, set

$$f_w(z) = \frac{1 - |w|^2}{1 - \overline{w}z}.$$

Next, we will this family functions and z^m to characterize the generalized weighted composition operator $D^n_{\varphi,u}$ from $\mathcal{B}$ and $\mathcal{B}_0$ into H^∞_α.

Theorem 2.2. *Let n be a positive integer, $\alpha > 0$, $u \in H(\mathbb{D})$ and φ be an analytic self-map of $\mathbb{D}$. Then the following statements are equivalent.*

(a) The operator $D^n_{\varphi,u} : \mathcal{B} \to H^\infty_\alpha$ is bounded;

(b) The operator $D^n_{\varphi,u} : \mathcal{B}_0 \to H^\infty_\alpha$ is bounded;

(c) $\sup_{m\ge n} \|D^n_{\varphi,u} I^m(z)\|_{H^\infty_\alpha} < \infty$, where $I^m(z) = z^m$;

(d) $u \in H^\infty_\alpha$ and $\sup_{w\in\mathbb{D}} \|D^n_{\varphi,u} f_{\varphi(w)}\|_{H^\infty_\alpha} < \infty$;

(e)

$$\sup_{z\in\mathbb{D}} \frac{(1 - |z|^2)^\alpha |u(z)|}{(1 - |\varphi(z)|^2)^n} < \infty .$$

Proof. $(a) \Rightarrow (b)$ This implication is obvious.

$(b) \Rightarrow (c)$ For $m \in \mathbb{N}$, the function I^m is bounded in $\mathcal{B}_0$ and $\|I^m\|_{\mathcal{B}} \le C$, here $C > 0$, independent of m. Therefore, by the boundedness of $D^n_{\varphi,u}$, we get

$$\|D^n_{\varphi,u} I^m(z)\|_{H^\infty_\alpha} \le C\|D^n_{\varphi,u}\| < \infty,$$

proving (c).

(c) ⇒ (d) Suppose (c) holds. It is easy to see that $(D^n_{\varphi,u}I^n)(z) = u(z)n!, z \in \mathbb{D}$, while for $k < n$, $(D^n_{\varphi,u}I^k)(z) = 0$. Thus,

$$\sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha|u(z)| \le \frac{1}{n!}\|D^n_{\varphi,u}I^n\|_{H^\infty_\alpha} \le \frac{1}{n!}\sup_{m\ge n}\|D^n_{\varphi,u}I^m\|_{H^\infty_\alpha} < \infty,$$

i.e. $u \in H^\infty_\alpha$. For any given $w \in \mathbb{D}$, it is easy to check that f_w is bounded in $\mathcal{B}$. Write

$$f_w(z) = (1-|w|^2)\sum_{k=0}^\infty \overline{w}^k z^k.$$

Using linearity, we get

$$\|D^n_{\varphi,u}f_w\|_{H^\infty_\alpha} \le (1-|w|^2)\sum_{k=0}^\infty |w|^k\|D^n_{\varphi,u}I^k\|_{H^\infty_\alpha} < \infty.$$

Therefore,

$$\sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}f_w\|_{H^\infty_\alpha} < \infty.$$

(d) ⇒ (e) For $\lambda \in \mathbb{D}$, it follows from the condition that

$$C \;\ge\; \|D^n_{\varphi,u}f_{\varphi(\lambda)}\|_{H^\infty_\alpha} \ge \frac{n!(1-|\lambda|^2)^\alpha|u(\lambda)||\varphi(\lambda)|^n}{(1-|\varphi(\lambda)|^2)^n}. \tag{1}$$

For any fixed $r \in (0, 1)$, from (1), we have

$$\sup_{|\varphi(\lambda)|>r}\frac{(1-|\lambda|^2)^\alpha|u(\lambda)|}{(1-|\varphi(\lambda)|^2)^n} \;\le\; \sup_{|\varphi(\lambda)|>r}\frac{|\varphi(\lambda)|^n}{r^n}\frac{(1-|\lambda|^2)^\alpha|u(\lambda)|}{(1-|\varphi(\lambda)|^2)^n} \le \frac{C}{r^n n!}. \tag{2}$$

From $u \in H^\infty_\alpha$, we have

$$\sup_{|\varphi(\lambda)|\le r}\frac{(1-|\lambda|^2)^\alpha|u(\lambda)|}{(1-|\varphi(\lambda)|^2)^n} \;\le\; \frac{1}{(1-r^2)^n}\sup_{|\varphi(\lambda)|\le r}(1-|\lambda|^2)^\alpha|u(\lambda)| < \infty. \tag{3}$$

Therefore, (2) and (3) yield the inequality of (e).

(e) ⇒ (a) By Theorem 5.1.5 of [25], if $f \in \mathcal{B}$ and $k \in \mathbb{N}$, then

$$B(f) \asymp |f'(0)| + \cdots + |f^{(k-1)}(0)| + \sup_{z\in\mathbb{D}}(1-|z|^2)^k|f^{(k)}(z)|,$$

which implies that

$$\sup_{z\in\mathbb{D}}(1-|z|^2)^k|f^{(k)}(z)| \le C_k\|f\|_{\mathcal{B}},$$

where C_k is a constant only depending on k. Therefore, for $z \in \mathbb{D}$, we have

$$(1-|z|^2)^\alpha|(D^n_{\varphi,u}f)(z)| = (1-|z|^2)^\alpha|u(z)||f^{(n)}(\varphi(z))| \le C\frac{(1-|z|^2)^\alpha|u(z)|}{(1-|\varphi(z)|^2)^n}\|f\|_{\mathcal{B}}, \tag{4}$$

where C is a suitable constant depending only on n. Taking the supremum in (4) over $\mathbb{D}$ and then using the condition (e) we see that $D^n_{\varphi,u} : \mathcal{B} \to H^\infty_\alpha$ is bounded. The proof is completed. □

Theorem 2.3. *Let n be a positive integer, $\alpha > 0$, $u \in H(\mathbb{D})$ and let φ be an analytic self-map of $\mathbb{D}$. If $D^n_{\varphi,u} : \mathcal{B} \to H^\infty_\alpha$ is bounded, then the following statements are equivalent.*

(a) The operator $D^n_{\varphi,u} : \mathcal{B} \to H^\infty_\alpha$ is compact;

(b) The operator $D^n_{\varphi,u} : \mathcal{B}_0 \to H^\infty_\alpha$ is compact;

(c) $\lim_{m\to\infty}\|D^n_{\varphi,u}I^m(z)\|_{H^\infty_\alpha} = 0$;

(d) $\lim_{|\varphi(w)|\to 1} \|D^n_{\varphi,u} f_{\varphi(w)}\|_{H^\infty_\alpha} = 0;$

(e)

$$\lim_{|\varphi(z)|\to 1} \frac{(1-|z|^2)^\alpha |u(z)|}{(1-|\varphi(z)|^2)^n} = 0.$$

Proof. $(a) \Rightarrow (b)$ This implication is clear.

$(b) \Rightarrow (c)$ Assume $D^n_{\varphi,u} : \mathscr{B}_0 \to H^\infty_\alpha$ is compact. Since the sequence $\{I^m\}$ is bounded in $\mathscr{B}_0$ and converges to 0 uniformly on compact subsets, by Lemma 2.1 it follows that $\|D^n_{\varphi,u} I^m\|_{H^\infty_\alpha} \to 0$ as $m \to \infty$.

$(c) \Rightarrow (d)$ Suppose (c) holds. For any given $\varepsilon > 0$, there exists an $N \in \mathbb{N}$ such that

$$\|D^n_{\varphi,u} I^j\|_{H^\infty_\alpha} < \varepsilon/2,$$

for all $j \geq N$. Write

$$f_{\varphi(z_k)}(z) = (1-|\varphi(z_k)|^2) \sum_{j=0}^{\infty} \overline{\varphi(z_k)}^j z^j, \quad z \in \mathbb{D}.$$

By linearity, we have

$$\begin{aligned} \|D^n_{\varphi,u} f_{\varphi(z_k)}\|_{H^\infty_\alpha} &\leq (1-|\varphi(z_k)|^2) \sum_{j=0}^{\infty} |\varphi(z_k)|^j \|D^n_{\varphi,u} I^j\|_{H^\infty_\alpha} \\ &= (1-|\varphi(z_k)|^2) \sum_{j=0}^{N-1} |\varphi(z_k)|^j \|D^n_{\varphi,u} I^j\|_{H^\infty_\alpha} + (1-|\varphi(z_k)|^2) \sum_{j=N}^{\infty} |\varphi(z_k)|^j \|D^n_{\varphi,u} I^j\|_{H^\infty_\alpha} \\ &\leq 2(1-|\varphi(z_k)|^N) M + \varepsilon, \end{aligned} \tag{5}$$

where $M = \sup_{0\leq j\leq N-1} \|D^n_{\varphi,u} I^j\|_{H^\infty_\alpha}$. Since $|\varphi(z_k)| \to 1$ as $k \to \infty$, from (5), we deduce that

$$\lim_{k\to\infty} \|D^n_{\varphi,u} f_{\varphi(z_k)}\|_{H^\infty_\alpha} \leq \varepsilon. \tag{6}$$

Since ε is an arbitrary positive number, we obtain the desired result.

$(d) \Rightarrow (e)$ Let $\{z_k\}_{k\in\mathbb{N}}$ be a sequence in $\mathbb{D}$ such that $\lim_{k\to\infty} |\varphi(z_k)| = 1$. Since the sequences $\{f_{\varphi(z_k)}\}$ are bounded in $\mathscr{B}$ and converge to 0 uniformly on compact subsets of $\mathbb{D}$, by (1) and Lemma 2.1, we have

$$\frac{n!(1-|z_k|^2)^\alpha |u(z_k)||\varphi(z_k)|^n}{(1-|\varphi(z_k)|^2)^n} \leq \|D^n_{\varphi,u} f_{\varphi(z_k)}\|_{H^\infty_\alpha} \to 0$$

as $k \to \infty$. Therefore

$$\lim_{k\to\infty} \frac{(1-|z_k|^2)^\alpha |u(z_k)|}{(1-|\varphi(z_k)|^2)^n} = \lim_{k\to\infty} \frac{(1-|z_k|^2)^\alpha |u(z_k)||\varphi(z_k)|^n}{(1-|\varphi(z_k)|^2)^n} = 0, \tag{7}$$

which implies (e).

$(e) \Rightarrow (a)$ Assume $(f_k)_{k\in\mathbb{N}}$ is a bounded sequence in $\mathscr{B}$ converging to 0 uniformly on compact subsets of $\mathbb{D}$. By the assumption, for any $\varepsilon > 0$, there exists a $\delta \in (0, 1)$ such that

$$\frac{(1-|z|^2)^\alpha |u(z)|}{(1-|\varphi(z)|^2)^n} < \varepsilon \tag{8}$$

when $\delta < |\varphi(z)| < 1$. Let $\Omega = \{z \in \mathbb{D} : |\varphi(z)| \leq \delta\}$. Since $D^n_{\varphi,u} : \mathscr{B} \to H^\infty_\alpha$ is bounded, as shown in the proof of Theorem 2.2,

$$C_1 := \sup_{z\in\mathbb{D}} (1-|z|^2)^\alpha |u(z)| < \infty. \tag{9}$$

By (8) and (9), we have

$$\begin{aligned}\|D^n_{\varphi,u}f_k\|_{H^\infty_\alpha} &= \sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha|(D^n_{\varphi,u}f_k)(z)| \\ &\leq \sup_{z\in\Omega}(1-|z|^2)^\alpha|u(z)||f_k^{(n)}(\varphi(z))| + C\sup_{z\in\mathbb{D}\backslash\Omega}\frac{(1-|z|^2)^\alpha|u(z)|}{(1-|\varphi(z)|^2)^n}\|f_k\|_{\mathscr{B}} \\ &\leq C_1\sup_{z\in\Omega}|f_k^{(n)}(\varphi(z))| + C\varepsilon\|f_k\|_{\mathscr{B}}. \end{aligned} \tag{10}$$

Since $(f_k)_{k\in\mathbb{N}}$ converges to 0 uniformly on compact subsets of $\mathbb{D}$, by Cauchy's estimates so do the sequences $(f_k^{(n)})$. From (10), letting $k\to\infty$ and using the fact that ε is an arbitrary positive number, we obtain $\lim_{k\to\infty}\|D^n_{\varphi,u}f_k\|_{H^\infty_\alpha}=0$. By Lemma 2.1, we deduce that the operator $D^n_{\varphi,u}:\mathscr{B}\to H^\infty_\alpha$ is compact. □

From Theorems 2.2 and 2.3, we can obtain the following corollaries, which give some new criteria for the boundedness and compactness of the operator $DC_\varphi:\mathscr{B}\to H^\infty_\alpha$. Partial results can be found in [12].

Corollary 2.4. *Let $\alpha>0$ and φ be an analytic self-map of $\mathbb{D}$. Then the following statements are equivalent.*

(a) The operator $DC_\varphi:\mathscr{B}\to H^\infty_\alpha$ is bounded;

(b) The operator $DC_\varphi:\mathscr{B}_0\to H^\infty_\alpha$ is bounded;

(c) $\sup_{m\geq n}\|DC_\varphi I^m(z)\|_{H^\infty_\alpha}<\infty$, where $I^m(z)=z^m$;

(d) $\varphi'\in H^\infty_\alpha$ and $\sup_{w\in\mathbb{D}}\|DC_\varphi f_{\varphi(w)}\|_{H^\infty_\alpha}<\infty$;

(e)

$$\sup_{z\in\mathbb{D}}\frac{(1-|z|^2)^\alpha|\varphi'(z)|}{1-|\varphi(z)|^2}<\infty\,.$$

Corollary 2.5. *Let $\alpha>0$ and φ an analytic self-map of $\mathbb{D}$. If $DC_\varphi:\mathscr{B}\to H^\infty_\alpha$ is bounded, then the following statements are equivalent.*

(a) The operator $DC_\varphi:\mathscr{B}\to H^\infty_\alpha$ is compact;

(b) The operator $DC_\varphi:\mathscr{B}_0\to H^\infty_\alpha$ is compact;

(c) $\lim_{m\to\infty}\|DC_\varphi I^m(z)\|_{H^\infty_\alpha}=0$;

(d) $\lim_{|\varphi(w)|\to1}\|DC_\varphi f_{\varphi(w)}\|_{H^\infty_\alpha}=0$;

(e)

$$\lim_{|\varphi(z)|\to1}\frac{(1-|z|^2)^\alpha|\varphi'(z)|}{1-|\varphi(z)|^2}=0.$$

Remark 1. When n is a positive integer, from the proof of Theorems 2.2 and 2.3, we see that $D^n_{\varphi,u}:\mathscr{B}\to H^\infty_\alpha$ is bounded if and only if $D^n_{\varphi,u}:H^\infty\to H^\infty_\alpha$ is bounded; $D^n_{\varphi,u}:\mathscr{B}\to H^\infty_\alpha$ is compact if and only if $D^n_{\varphi,u}:H^\infty\to H^\infty_\alpha$ is compact.

Next we consider the case $n=0$. For $w\in\mathbb{D}$, set

$$g_w(z)=\Big(\ln\frac{e}{1-\overline{w}z}\Big)^2\Big(\ln\frac{e}{1-|w|^2}\Big)^{-1},\quad z\in\mathbb{D}.$$

From [12], we see that $\{g_{\varphi(w)}\}$ are bounded in $\mathscr{B}_0$ for $w\in\mathbb{D}$, the sequences $\{g_{\varphi(z_k)}\}$ converge to 0 uniformly on compact subsets of $\mathbb{D}$ when $|\varphi(z_k)|\to1$. Using this family functions, we can obtain a new criterion for the boundedness and compactness of weighted composition operator $uC_\varphi:\mathscr{B}\to H^\infty_\alpha$. Since the proof is similar to the above, we omit the details.

Theorem 2.6. *Let $\alpha > 0$, $u \in H(\mathbb{D})$ and φ be an analytic self-map of $\mathbb{D}$. Then the following statements are equivalent.*

(a) The operator $uC_\varphi : \mathcal{B} \to H_\alpha^\infty$ is bounded;

(b) The operator $uC_\varphi : \mathcal{B}_0 \to H_\alpha^\infty$ is bounded;

(c) $u \in H_\alpha^\infty$ and $\sup_{w\in\mathbb{D}} \|uC_\varphi g_{\varphi(w)}\|_{H_\alpha^\infty} < \infty$;

(d) $u \in H_\alpha^\infty$ and

$$\sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha |u(z)| \ln \frac{e}{1-|\varphi(z)|^2} < \infty.$$

Theorem 2.7. *Let $\alpha > 0$, $u \in H(\mathbb{D})$ and φ be an analytic self-map of $\mathbb{D}$. If $uC_\varphi : \mathcal{B} \to H_\alpha^\infty$ is bounded, then the following statements are equivalent.*

(a) The operator $uC_\varphi : \mathcal{B} \to H_\alpha^\infty$ is compact;

(b) The operator $uC_\varphi : \mathcal{B}_0 \to H_\alpha^\infty$ is compact;

(c) $\lim_{|\varphi(w)|\to 1} \|uC_\varphi g_{\varphi(w)}\|_{H_\alpha^\infty} = 0$;

(d)

$$\lim_{|\varphi(z)|\to 1}(1-|z|^2)^\alpha |u(z)| \ln \frac{e}{1-|\varphi(z)|^2} = 0.$$

Remark 2. Partial results of Theorems 2.6 and 2.7 have been obtained, for example, in [23].

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