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Stability analysis of uncertain stochastic systems with interval time-varying delays and nonlinear uncertainties via augmented Lyapunov functional

R.Jeetendra[a], Vernold Vivin.J[b]

[a]Department of Mathematics, KSR College of Engineering, Tiruchengode - 637 215, Tamil Nadu, India.
[b]Department of Mathematics, University College of Engineering Nagercoil, Anna University of Technology Tirunelveli–(Nagercoil Campus), Nagercoil- 629 004, Tamil Nadu, India.

Abstract. In this work, the problem of delay-dependent stability for uncertain stochastic systems with interval time-varying delays and nonlinear uncertainties is addressed. The parameter uncertainties are assumed to be norm bounded and the delay is assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. By constructing an augmented Lyapunov functional, a new delay interval-dependent stability criterion for the system is obtained in terms of Linear Matrix Inequalities (LMIs). Comparisons are made through numerical examples and less conservatism results are reported.

1. Introduction

The stability analysis of stochastic time delay systems have been an active research area in the past years [1]-[4], since many practical systems can be modeled to stochastic differential equations with time-delays. Based on the Lyapunov theory of stability, many stability conditions have been obtained by means of linear matrix inequalities. Recently, model transformation and cross term bounding techniques [5]-[6], input-output method [7] and integral inequality approach [8] have been applied to reduce the conservatism of the stability criteria for stochastic delay systems. However, free-weighting matrix method plays a key role in reducing conservatism. In this paper some free-weighting matrices are added and less conservative results are obtained. It is worth mentioning that the exogenous nonlinear disturbance input has been dealt within many papers, since it may result from the linearization process of an originally highly nonlinear plant or may be an external nonlinear input, which could cause the instability of the system. The stability issue and the performance of uncertain stochastic systems with interval time-varying delays and nonlinear perturbations are therefore, both of theoretical and practical importance and have been attracted by considerable number of researches [9]-[13]. The authors of [14] proposed an augmented Lyapunov-Krasovskii functional for analyzing uncertain neutral systems with time-varying delays and has showed the proposed stability criterion provides larger feasible region. To the best of the author's knowledge, the stability analysis of uncertain stochastic system with interval time-varying delays and nonlinear uncertainties via augmented

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Email addresses: jee4maths@gmail.com, jee4maths@yahoo.com (R.Jeetendra), vernoldvivin@yahoo.in, vernold_vivin@yahoo.com (Vernold Vivin.J)

Lyapunov functional approach has not been investigated. So, we hope to apply the augmented Lyapunov functional approach to stochastic delay systems such that the conservatism of stability conditions could be reduced. The delay is assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. This paper discusses the delay-dependent stability criterion of uncertain stochastic delay systems with nonlinear uncertainties in terms of LMIs which can be solved efficiently by using the interior-point algorithms [15]. Compared to other methods, the proposed method overcomes some of the main sources of conservatism and has its own advantages. Finally, two numerical examples are given and the corresponding simulation by MATLAB is provided to illustrate the effectiveness of the proposed method.

2. Notations:

Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. The superscript T denotes the transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I denotes the identity matrix of appropriate dimension. The notation $*$ always denotes the symmetric block in one symmetric matrix.

3. Problem description and Preliminaries

Consider the following uncertain stochastic system with interval time-varying delays and nonlinear uncertainties:

$$\begin{aligned} dx(t) &= \Big[(A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - \tau(t))\Big]dt \\ &\quad + \Big[g(t, x(t), x(t - \tau(t)))\Big]dw(t) \\ x(t) &= \phi(t), \quad t \in [-h_2, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, A and B are known real constant matrices with appropriate dimensions, $w(t)$ is an m-dimensional Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ satisfying $\mathcal{E}\{dw(t)\} = 0$ and $\mathcal{E}\{dw^2(t)\} = dt$, where $\mathcal{E}\{\cdot\}$ is the mathematical expectation. $\phi(t)$ is the initial condition for all $t \in [-h_2, 0]$. $\tau(t)$ denotes the time-varying interval delay and is assumed to satisfy the following conditions:

$$0 \leq h_1 \leq \tau(t) \leq h_2, \quad \dot{\tau}(t) \leq \mu < \infty,$$

where h_1 and h_2 are the lower and upper bounds of $\tau(t)$ respectively. $g(t, x(t), x(t-\tau(t))) \in \mathbb{R}^{n\times m}$ is a nonlinear function satisfying

$$trace\{g^T(t, x(t), x(t - \tau(t)))g(t, x(t), x(t - \tau(t)))\} \leq \|G_1x(t)\|^2 + \|G_2x(t - \tau(t))\|^2, \tag{2}$$

where $G_1, G_2 \in \mathbb{R}^{n\times n}$ are known matrices of appropriate dimensions. $\Delta A(t)$ and $\Delta B(t)$ are the parametric uncertainties of the form:

$$[\Delta A(t) \quad \Delta B(t)] = HF(t)[T_1 \quad T_2], \tag{3}$$

where H, T_1, T_2 are constant matrices with compatible dimensions and $F(t)$ is an unknown time-varying matrix function satisfying,

$$F^T(t)F(t) \leq I. \tag{4}$$

Now, (1) can be rewritten as

$$dx(t) = \Big[Ax(t) + Bx(t-\tau(t) + Hp(t)\Big]dt + \Big[g(t, x(t), x(t-\tau(t)))\Big]dw(t), \tag{5}$$

$$p(t) = F(t)q(t), \tag{6}$$

$$q(t) = T_1x(t) + T_2x(t-\tau(t)). \tag{7}$$

For convenience, we set

$$y(t) = Ax(t) + Bx(t-\tau(t)) + Hp(t), \tag{8}$$

$$g(t) = g(t, x(t), x(t-\tau(t))). \tag{9}$$

Throughout this paper we shall use the following definition for system (1).

Definition 3.1. *The stochastic time-delay system (1) is said to be robustly asymptotically mean-square stable if for all admissible uncertainties (3) the following holds for any initial condition:*

$$\lim_{t\to\infty} \mathcal{E}\{\|x(t)\|^2\} = 0.$$

The following lemmas will be essential for the proof of main result.

Lemma 3.2. *(Schur Complement) Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2 > 0$, then*

$$\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

Lemma 3.3. *[16] For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t)$: $[a, b] \to \mathbb{R}^n$ such that the integrals concerned as well defined, then the following holds*

$$\Big[\int_a^b x(s)ds\Big]^T M\Big[\int_a^b x(s)ds\Big] \le (b-a)\int_a^b x^T(s)Mx(s)ds.$$

4. Main Result

In this section, we propose a new stability criterion for uncertain stochastic system (1) with interval time-varying delays and nonlinear uncertainties.

Theorem 4.1. *For given scalars $h_2 > h_1 \ge 0$ and μ, system (1) is globally asymptotically stable in the mean square if there exist positive definite symmetric matrices P, R_l $(l = 1, 2, 3)$, Q_i, Z_i $(i = 1, 2)$, U_j, $(j = 1, 5, 8, 10)$, any matrices U_2, U_3, U_4, U_6, U_7, U_9, N_k, M_k $(k = 1, \cdots, 8)$, X and a positive scalar W satisfying the following LMIs:*

$$\begin{bmatrix} U_1 & U_2 & U_3 & U_4 \\ * & U_5 & U_6 & U_7 \\ * & * & U_8 & U_9 \\ * & * & * & U_{10} \end{bmatrix} > 0, \tag{10}$$

$$\Pi = \begin{bmatrix} \Sigma & \Xi_1^T X & -N & -M & -S & \Xi_2^T W \\ * & \Omega_{2,2} & 0 & 0 & 0 & 0 \\ * & * & \Omega_{3,3} & 0 & 0 & 0 \\ * & * & * & \Omega_{4,4} & 0 & 0 \\ * & * & * & * & \Omega_{5,5} & 0 \\ * & * & * & * & * & -W \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned}
\Sigma &= (\Phi_{i,j})_{8\times 8}, \\
\Phi_{1,1} &= N_1 + N_1^T + PA + A^T P + R_1 + R_2 + R_3 + h_2 Z_1 + (h_2 - h_1)Z_2 + U_1 A + A^T U_1 + U_2 + U_2^T \\
&\quad + G_1^T P G_1, \quad \Phi_{1,2} = N_2^T - N_1 + M_1 - S_1 + PB + U_1 B - (1-\mu)U_2 + (1-\mu)U_3 \\
&\quad -(1-\mu)U_4, \quad \Phi_{1,3} = N_3^T + S_1 + U_4, \quad \Phi_{1,4} = N_4^T - M_1 - U_3, \\
\Phi_{1,5} &= N_5^T + A^T U_2 + U_5, \quad \Phi_{1,6} = N_6^T + A^T U_3 + U_6, \quad \Phi_{1,7} = N_7^T + A^T U_4 + U_7, \\
\Phi_{1,8} &= N_8^T + PH - U_1 H, \quad \Phi_{2,2} = -N_2 - N_2^T + M_2 + M_2^T - S_2 - S_2^T - (1-\mu)R_2 + G_2^T P G_2 \\
\Phi_{2,3} &= -N_3^T + M_3^T + S_2 - S_3^T, \quad \Phi_{2,4} = -N_4^T + M_4^T - M_2 - S_4^T, \\
\Phi_{2,5} &= -N_5^T + M_5^T - S_5^T + B^T U_2 - (1-\mu)U_5 + (1-\mu)U_6^T - (1-\mu)U_7^T, \\
\Phi_{2,6} &= -N_6^T + M_6^T - S_6^T + B^T U_3 - (1-\mu)U_6 + (1-\mu)U_8 - (1-\mu)S_9^T, \\
\Phi_{2,7} &= -N_7^T + M_7^T - S_7^T + B^T U_4 - (1-\mu)U_7 + (1-\mu)U_9 - (1-\mu)U_{10}, \\
\Phi_{2,8} &= -N_8^T + M_8^T - S_8^T \quad \Phi_{3,3} = S_3 + S_3^T - R_1, \quad \Phi_{3,4} = -M_3 + S_4^T, \quad \Phi_{3,5} = S_5^T + U_7^T, \\
\Phi_{3,6} &= S_6^T + U_9^T, \quad \Phi_{3,7} = S_7^T + U_{10}, \quad \Phi_{3,8} = S_8^T, \quad \Phi_{4,4} = -M_4 - M_4^T - R_3, \\
\Phi_{4,5} &= -M_5^T - U_6^T, \quad \Phi_{4,6} = -M_6^T - U_8, \quad \Phi_{4,7} = -M_7^T - U_9, \quad \Phi_{4,8} = -M_8^T, \quad \Phi_{5,5} = -\frac{1}{h_2} Z_1, \\
\Phi_{5,6} &= 0, \quad \Phi_{5,7} = 0, \quad \Phi_{5,8} = -U_2^T H, \quad \Phi_{6,6} = -\frac{1}{h_2 - h_1}(Z_1 + Z_2), \quad \Phi_{6,7} = 0, \quad \Phi_{6,8} = -U_3^T H, \\
\Phi_{7,7} &= -\frac{1}{h_2 - h_1} Z_2, \quad \Phi_{7,8} = -U_4^T H, \quad \Phi_{8,8} = -W, \quad \Omega_{2,2} = h_2 Q_1 + (h_2 - h_1)Q_2 - X - X^T \\
\Omega_{3,3} &= -\frac{1}{h_2} Q_1, \quad \Omega_{4,4} = -\frac{1}{h_2 - h_1}(Q_1 + Q_2), \quad \Omega_{5,5} = -\frac{1}{h_2 - h_1} Q_2.
\end{aligned}$$

$$\begin{aligned}
N &= \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T & N_7^T & N_8^T \end{bmatrix}^T, \\
M &= \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T & M_7^T & M_8^T \end{bmatrix}^T, \\
S &= \begin{bmatrix} S_1^T & S_2^T & S_3^T & S_4^T & S_5^T & S_6^T & S_7^T & S_8^T \end{bmatrix}^T, \\
\Xi_1 &= \begin{bmatrix} A & B & 0 & 0 & 0 & 0 & 0 & H \end{bmatrix}, \\
\Xi_2 &= \begin{bmatrix} T_1 & T_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

Proof. Let

$$\xi^T(t) = \Big[x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-h_1) \quad x^T(t-h_2) \quad \Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T \quad \Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T \quad \Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T \quad p^T(t)\Big].$$

Using Newton-Leibnitz formula, we have

$$\begin{aligned}
\eta_1(t) &= 2\xi^T(t)N\Big[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds - \int_{t-\tau(t)}^{t} g(s)dw(s)\Big] = 0,\\
\eta_2(t) &= 2\xi^T(t)M\Big[x(t-\tau(t)) - x(t-h_2) - \int_{t-h_2}^{t-\tau(t)} y(s)ds - \int_{t-h_2}^{t-\tau(t)} g(s)dw(s)\Big] = 0,\\
\eta_3(t) &= 2\xi^T(t)S\Big[x(t-h_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-h_1} y(s)ds - \int_{t-\tau(t)}^{t-h_1} g(s)dw(s)\Big] = 0.
\end{aligned}$$

On the other hand, from (8), the following equation holds for any matrix $X \in \mathbb{R}^{n\times n}$

$$\eta_4(t) = 2y^T(t)X^T\Big[Ax(t) + Bx(t-\tau(t)) + Hp(t) - y(t)\Big] = 0.$$

Let us define the Lyapunov functional candidate as

$$V(x_t, t) = \sum_{i=1}^{5} V_i(x_t, t) \tag{12}$$

where

$$\begin{aligned}
V_1(x_t,t) &= x^T(t)Px(t),\\
V_2(x_t,t) &= \int_{t-h_1}^{t} x^T(s)R_1x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)R_2x(s)ds + \int_{t-h_2}^{t} x^T(s)R_3x(s)ds,\\
V_3(x_t,t) &= \int_{-h_2}^{0}\int_{t+\theta}^{t} x^T(s)Z_1x(s)dsd\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x^T(s)Z_2x(s)dsd\theta,\\
V_4(x_t,t) &= \int_{-h_2}^{0}\int_{t+\theta}^{t} y^T(s)Q_1y(s)dsd\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} y^T(s)Q_2y(s)dsd\theta,\\
V_5(x_t,t) &= \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-h_2}^{t-\tau(t)} x(s)ds \\ \int_{t-\tau(t)}^{t-h_1} x(s)ds \end{bmatrix}^T \begin{bmatrix} U_1 & U_2 & U_3 & U_4 \\ * & U_5 & U_6 & U_7 \\ * & * & U_8 & U_9 \\ * & * & * & U_{10} \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-h_2}^{t-\tau(t)} x(s)ds \\ \int_{t-\tau(t)}^{t-h_1} x(s)ds, \end{bmatrix}.
\end{aligned}$$

Then, it can be obtained by Ito's formula that

$$dV(x_t,t) = \mathcal{L}V(x_t,t)dt + 2x^T(t)Pg(t)dw(t). \tag{13}$$

$$\mathcal{L}V(x_t,t) = 2x^T(t)P[Ax(t)+Bx(t-\tau(t))+Hp(t)] + trace\{g^T(t)Pg(t)\} + \sum_{i=2}^{5}\mathcal{L}V_i(x_t,t) + \sum_{j=1}^{4}\eta_j(t)$$

with

$$\mathcal{L}V_2(x_t,t) \le x^T(t)(R_1+R_2+R_3)x(t) - x^T(t-h_1)R_1x(t-h_1) - (1-\mu)x^T(t-\tau(t))R_2x(t-\tau(t)) \tag{14}$$

$$-x^T(t-h_2)R_3x(t-h_2), \tag{15}$$

$$\mathcal{L}V_3(x_t,t) = h_2x^T(t)Z_1x(t) - \int_{t-h_2}^{t} x^T(s)Z_1x(s)ds + (h_2-h_1)x^T(t)Z_2x(t) - \int_{t-h_2}^{t-h_1} x^T(s)Z_2x(s)ds, \tag{16}$$

$$\mathcal{L}V_4(x_t,t) = h_2y^T(t)Q_1y(t) - \int_{t-h_2}^{t} y^T(s)Q_1y(s)ds + (h_2-h_1)y^T(t)Q_2y(t) - \int_{t-h_2}^{t-h_1} y^T(s)Q_2y(s)ds \tag{17}$$

$$\begin{aligned}
\mathcal{L}V_5(x_t,t) \le\ & 2\Big[x^T(t)U_1Ax(t) + x^T(t)U_1Bx(t-\tau(t)) - x^T(t)U_1Hp(t) + \Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_2^TAx(t) \\
&+\Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_2^TBx(t-\tau(t)) - \Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_2^THp(t) + \Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_3^TAx(t) \\
&+\Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_3^TBx(t-\tau(t)) - \Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_3^THp(t) + \Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_4^TAx(t) \\
&+\Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_4^TBx(t-\tau(t)) - \Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_4^THp(t) + x^T(t)U_2x(t) \\
&-(1-\mu)x^T(t)U_2x(t-\tau(t)) + \Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_5x(t) - (1-\mu)\Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_5x(t-\tau(t)) \\
&+\Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_6^Tx(t) - (1-\mu)\Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_6^Tx(t-\tau(t)) + \Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_7^Tx(t) \\
&-(1-\mu)\Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_7^Tx(t-\tau(t)) + (1-\mu)x^T(t)U_3x(t-\tau(t)) - x^T(t)U_3x(t-h_2) \\
&+(1-\mu)\Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_6x(t-\tau(t)) - \Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_6x(t-h_2) \\
&+(1-\mu)\Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_8x(t-\tau(t)) - \Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_8x(t-h_2) \\
&+(1-\mu)\Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_9^Tx(t-\tau(t)) - \Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_9^Tx(t-h_2) + x^T(t)U_4x(t-h_1) \\
&-(1-\mu)x^T(t)U_4x(t-\tau(t)) + \Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_7x(t-h_1) - (1-\mu)\Big(\int_{t-\tau(t)}^{t} x(s)ds\Big)^T U_7x(t-\tau(t)) \\
&+\Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_9x(t-h_1) - (1-\mu)\Big(\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big)^T U_9x(t-\tau(t)) \\
&+\Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_{10}x(t-h_1) - (1-\mu)\Big(\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big)^T U_{10}x(t-\tau(t))\Big].
\end{aligned} \tag{18}$$

Then by Lemma 3.3 and using $0 \le h_1 \le \tau(t) \le h_2$, we have

$$-\int_{t-\tau(t)}^{t} y^T(s)Q_1y(s)ds \le -\frac{1}{h_2}\Big[\int_{t-\tau(t)}^{t} y(s)ds\Big]^T Q_1\Big[\int_{t-\tau(t)}^{t} y(s)ds\Big], \tag{19}$$

$$-\int_{t-\tau(t)}^{t-h_1} y^T(s)Q_2y(s)ds \le -\frac{1}{h_2-h_1}\Big[\int_{t-\tau(t)}^{t-h_1} y(s)ds\Big]^T Q_2\Big[\int_{t-\tau(t)}^{t-h_1} y(s)ds\Big], \tag{20}$$

$$-\int_{t-h_2}^{t-\tau(t)} y^T(s)(Q_2+Q_1)y(s)ds \leq -\frac{1}{h_2-h_1}\Big[\int_{t-h_2}^{t-\tau(t)} y(s)ds\Big]^T(Q_1+Q_2)\Big[\int_{t-h_2}^{t-\tau(t)} y(s)ds\Big], \tag{21}$$

$$-\int_{t-\tau(t)}^{t} x^T(s)Z_1x(s)ds \leq -\frac{1}{h_2}\Big[\int_{t-\tau(t)}^{t} x(s)ds\Big]^T Z_1\Big[\int_{t-\tau(t)}^{t} x(s)ds\Big], \tag{22}$$

$$-\int_{t-\tau(t)}^{t-h_1} x^T(s)Z_2x(s)ds \leq -\frac{1}{h_2-h_1}\Big[\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big]^T Z_2\Big[\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big], \tag{23}$$

$$-\int_{t-h_2}^{t-\tau(t)} x^T(s)(Z_2+Z_1)x(s)ds \leq -\tfrac{1}{h_2-h_1}\Big[\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big]^T(Z_1+Z_2)\Big[\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big]. \tag{24}$$

Now expression (2) can be written as,

$$\begin{aligned} tr\{g^T(t)g(t)\} &\leq \|G_1x(t)\|^2+\|G_2x(t-\tau(t))\|^2 \\ &= tr\{\xi^T(t)diag(G_1^T,G_2^T,0,0,0,0,0,0)diag(G_1,G_2,0,0,0,0,0,0)\xi(t)\}. \end{aligned}$$

Therefore,

$$\begin{aligned} tr\{g^T(t)Pg(t)\} &\leq tr\{\xi^T(t)diag(G_1^TPG_1,G_2^TPG_2,0,0,0,0,0,0)\xi(t)\} \\ &= x^T(t)G_1^TPG_1x(t)+x^T(t-\tau(t))G_2^TPG_2x(t-\tau(t)). \end{aligned} \tag{25}$$

Since the following inequality is evident from (4) and (6),

$$p^T(t)p(t) \leq q^T(t)q(t),$$

there exist a positive scalar W satisfying

$$\xi^T(t)\Xi_2^TW\Xi_2\xi(t)-p^T(t)Wp(t) \geq 0. \tag{26}$$

Substituting from (14) to (26) into (13) we get,

$$dV(x_t,t) \leq \zeta^T(t)\Pi\zeta(t)+\xi(dw(t)),$$

where Π is defined in Theorem 4.1 with

$$\begin{aligned} \zeta^T(t) &= \Big[\xi^T(t) \quad y^T(t) \quad \int_{t-\tau(t)}^{t} y^T(s)ds \quad \int_{t-h_2}^{t-\tau(t)} y^T(s)ds \quad \int_{t-\tau(t)}^{t-h_1} y^T(s)ds\Big], \\ \xi(dw(t)) &= -2\xi^T(t)N\int_{t-\tau(t)}^{t} g(s)dw(s)-2\xi^T(t)M\int_{t-\tau(t)}^{t-h_1} g(s)dw(s)-2\xi^T(t)S\int_{t-h_2}^{t-\tau(t)} g(s)dw(s) \\ &\quad +2x^T(t)Pg(t)dw(t). \end{aligned}$$

Since $\Pi<0$, there exist a scalar $\alpha>0$ such that

$$\Pi+diag\{\alpha I,0,0,0,0,0,0,0\}<0.$$

Hence we have

$$\frac{\mathcal{E}dV(x_t,t)}{dt} \leq \mathcal{E}(\zeta^T(t)\Pi\zeta(t)) \leq \alpha\mathcal{E}|x(t)|^2.$$

Thus if $\Pi<0$, the equilibrium point of the stochastic system (1) is robustly asymptotically stable in the mean square. The proof is completed. □

Remark 4.2. *Theorem 4.1 provides delay interval-dependent stability criteria for the stochastic system (5). Such stability criteria are derived based on the assumption that the time-varying delay is differentiable and the value of μ is known. The conditions in Theorem 4.1 are formulated in terms of solvability of LMIs [15] and can be easily solved by using MATLAB LMI Control Toolbox. It is worth to note that by applying convex optimization algorithms, we can conclude that the maximum allowable upper bound of the interval time-varying delay, that is, h_2 guarantees the feasibility of the presented LMIs. We can obtain the maximum allowable upper bound h_2 by solving the following optimization problem:*

$$\begin{cases} Max & h_2 \\ s.t. & P > 0, R_l > 0, Q_i > 0, Z_i > 0, U_j > 0, N_k, M_k, S_k, U_m, \Pi < 0, \\ & l = 1,2,3; i = 1,2; j = 1,5,8,10; k = 1,...,8; m = 2,3,4,6,7,9. \end{cases} \tag{27}$$

Remark 4.3. *It is shown in Theorem 4.1 that the addressed stability problem is solvable if a set of LMIs are feasible. The states $x(t)$, $\int_{t-\tau(t)}^{t} x(s)ds$, $\int_{t-h_2}^{t-\tau(t)} x(s)ds$ and $\int_{t-\tau(t)}^{t-h_1} x(s)ds$ are taken as augmentation ones. Thus the feasibility region of delay-dependent stability criterion is improved and leads to less conservative results.*

Remark 4.4. *Theorem 4.1 is delay interval-dependent stable which is generally less conservative than delay-independent stable. Moreover, from the free weighting matrix and the newton-Leibnitz formula, it infers that the time derivative of $\tau(t)$ is no longer required to be less than 1.*

5. Numerical Examples

Example 5.1. *Consider the system (5) with*

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix},$$

$$H = I, \quad T_1 = T_2 = 0.1I, \quad G_1 = G_2 = \sqrt{0.1}I.$$

Recently, remarkable results for stability of uncertain stochastic systems with nonlinear uncertainties were presented in [17] and [8]. In [17], the maximum delay bound for the above system was 1.1812*. By Theorem 4.1 in [8], one can obtain the maximum delay bound as* 2.8987*. But when using our proposed Theorem 4.1 derived in this paper, the obtained result is* 2.9586*. Hence, our proposed stability criterion gives a much less conservative result than those discussed in [17] and [8]. Table 1 shows the different values of h_2 for different μ.*

Methods	$\mu = 0$	$\mu = 0.5$	$\mu = 0.9$	$\mu = 1$
[17]	1.1812	0.8502	0.4606	−−
[18]	2.1491	1.3224	0.9748	−−
[8]	2.8987	−−	−−	−−
Theorem 4.1	2.9586	1.7702	1.2259	1.0566

Table 1: The upper bounds of delay of Example 5.1 for different μ

Example 5.2. *Consider the system (5) with*

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$H = 0.2I, \quad T_1 = T_2 = I, \quad G_1 = G_2 = \sqrt{0.1}I.$$

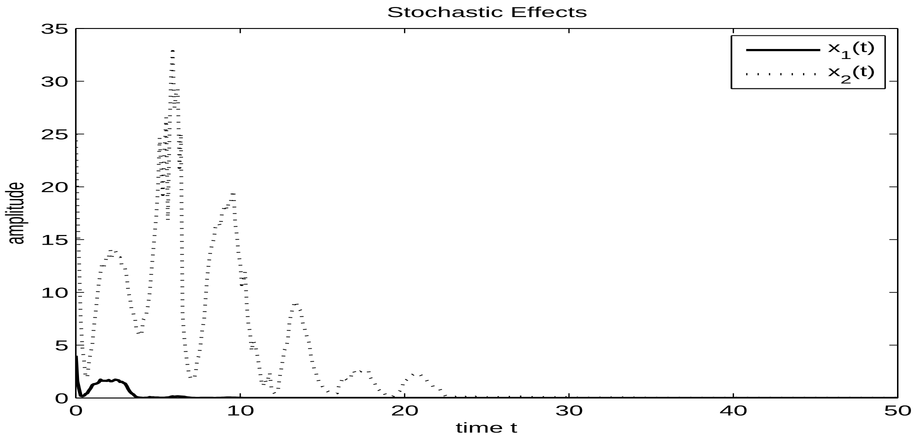

Figure 1: The state trajectories of Example 5.1

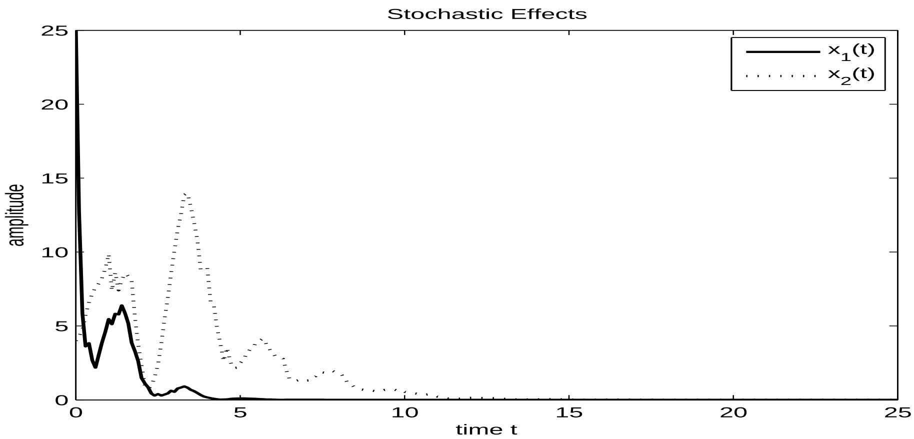

Figure 2: The state trajectories of Example 5.2

It was reported in [19] that the above system is asymptotically stable in the mean square when $0 \leq \tau \leq 1.0660$*. From Theorem 4.1, we conclude that the system (6) is robustly asymptotically stable in the mean square with the maximum allowable upper bound* $h_2 = 2.5220$*. Table.2 shows that the established results in this paper provides larger delay bounds than the existing results in the literature.*

Methods	$\mu = 0$	$\mu = 0.5$	$\mu = 0.9$	$\mu = 1$
[19]	1.0660	0.5252	0.1489	$--$
Theorem 4.1	2.5220	1.4112	1.0714	1.0377

Table 2: The upper bounds of delay of Example 5.2 for different μ

Remark 5.3. *By virtue of Theorem 4.1, we state that our results are computationally efficient as they can be solved efficiently by employing the Matlab LMI toolbox. Besides, Figures 1 and 2 presents an illustrative simulation of the asymptotic stability of system (5) in the mean square for examples 5.1 and 5.2 respectively. It is clear that the proposed stochastic system converges and yield a less conservative result.*

6. Conclusion

This paper has studied the delay-interval dependent stability criterion for uncertain stochastic systems with interval time-varying delays and nonlinear perturbations. Using Lyapunov-Krasovskii functional and stochastic analysis approach, a less conservative stability criterion have been obtained by considering the relationship between the time-varying delays and its lower and upper bounds. Two illustrative examples are given to demonstrate the effectiveness of the obtained results.

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