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A generalized class of τ^* in ideal spaces

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Abstract. In this paper, a generalized class of τ^* called weakly I_{rg}-open sets in ideal topological spaces is introduced and the notion of weakly I_{rg}-closed sets in ideal topological spaces is studied. The relationships of weakly I_{rg}-closed sets and various properties of weakly I_{rg}-closed sets are investigated.

1. Introduction

In 1999, Dontchev et al. studied the notion of generalized closed sets in ideal topological spaces called I_g-closed sets [2]. In 2008, Navaneethakrishnan and Joseph have studied some characterizations of normal spaces via I_g-open sets [6]. In 2009, Navaneethakrishnan et al. have introduced I_{rg}-open sets to establish some new caharacterizations of mildly normal spaces [7]. The main aim of this paper is to introduce a generalized class of τ^* called weakly I_{rg}-open sets in ideal topological spaces and to study the notion of weakly I_{rg}-closed sets in ideal topological spaces. Moreover, this generalized class of τ^* generalize I_g-open sets and I_{rg}-open sets and pre_I^*-open sets. The relationships of weakly I_{rg}-closed sets and various properties of weakly I_{rg}-closed sets are discussed.

2. Preliminaries

In this paper, (X, τ) represent topological space on which no separation axioms are assumed unless explicitly stated. The closure and the interior of a subset G of a space X will be denoted by $Cl(G)$ and $Int(G)$, respectively. A subset G of a topological space (X, τ) is said to be regular open [9] (resp. regular closed [9]) if $G = Int(Cl(G))$ (resp. $G = Cl(Int(G))$).

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies
(1) $G \in I$ and $H \subset G$ implies $H \in I$ and
(2) $G \in I$ and $H \in I$ implies $G \cup H \in I$ [5].

For a topological space (X, τ) with an ideal I on X, a set operator $(.)^* : P(X) \to P(X)$ where $P(X)$ is the set of all subsets of X, called a local function [5] of G with respect to τ and I is defined as follows: for $G \subset X$, $G^*(I, \tau) = \{x \in X : H \cap G \notin I$ for every $H \in \tau(x)\}$ where $\tau(x) = \{H \in \tau : x \in H\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the $\star$-topology and finer than τ, is defined by $Cl^*(G) = G \cup G^*(I, \tau)$ [4].

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We shall simply write G^* for $G^*(I,\tau)$ and τ^* for $\tau^*(I,\tau)$. For an ideal I on X, (X,τ,I) is said to be an ideal topological space or simply an ideal space. On the other hand, (G,τ_G,I_G) where τ_G is the relative topology on G and $I_G = \{G \cap J : J \in I\}$ is an ideal topological space for an ideal topological space (X,τ,I) and $G \subset X$ [4].

A subset G of an ideal topological space (X,τ,I) is said to be I_g-closed [2] if $G^* \subset H$ whenever $G \subset H$ and H is open in (X,τ,I). A subset G of an ideal topological space (X,τ,I) is said to be I_g-open [2] if $X \backslash G$ is I_g-closed. A subset A of an ideal topological space (X,τ,I) is said to be pre_I^*-open if $A \subset Int^*(Cl(A))$ [3]. A subset A of an ideal topological space (X,τ,I) is said to be pre_I^*-closed if $X \backslash A$ is pre_I^*-open. A subset G of an ideal topological space (X,τ,I) is called I_{rg}-closed [7] if $G^* \subset H$ whenever $G \subset H$ and H is a regular open set in (X,τ,I). Also, G is said to be I_{rg}-open [7] if $X \backslash G$ is an I_{rg}-closed set. A subset A of an ideal topological space (X,τ,I) is said to be I-R closed [1] if $A = Cl^*(Int(A))$.

3. A generalized class of τ^*

Definition 3.1. Let (X,τ,I) be an ideal topological space. A subset G of (X,τ,I) is said to be a weakly I_{rg}-closed set if $(Int(G))^* \subset H$ whenever $G \subset H$ and H is a regular open set in X.

Definition 3.2. Let (X,τ,I) be an ideal topological space and $G \subset X$. Then G is said to be a weakly I_{rg}-open set if $X \backslash G$ is a weakly I_{rg}-closed set.

Remark 3.3. Let (X,τ,I) be an ideal topological space. The following diagram holds for a subset $G \subset X$:

$$\begin{array}{ccccc}
I_g\text{-closed} & \Rightarrow & I_{rg}\text{-closed} & \Rightarrow & \text{weakly } I_{rg}\text{-closed} \\
\Uparrow & & & & \Uparrow \\
\star\text{-closed} & & & & pre_I^*\text{-closed} \\
\Uparrow & & & & \\
I\text{-}R \text{ closed} & & & &
\end{array}$$

These implications are not reversible as shown in the following example and in [1, 2, 7].

Example 3.4. Let $X = \{a,b,c,d\}$, $\tau = \{X, \varnothing, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}\}$ and $I = \{\varnothing, \{b\}\}$. Take $A = \{a,c,d\}$. Then A is I_{rg}-closed but it is not pre_I^*-closed. Take $B = \{a\}$. Then B is pre_I^*-closed and also weakly I_{rg}-closed but it is not I_{rg}-closed.

Theorem 3.5. *Let (X,τ,I) be an ideal topological space and $G \subset X$. The following properties are equivalent:*
(1) G is a weakly I_{rg}-closed set,
(2) $Cl^*(Int(G)) \subset H$ whenever $G \subset H$ and H is a regular open set in X.

Proof. (1) $\Rightarrow$ (2) : Let G be a weakly I_{rg}-closed set in (X,τ,I). Suppose that $G \subset H$ and H is a regular open set in X. We have $(Int(G))^* \subset H$. Since $Int(G) \subset G \subset H$, then

$$(Int(G))^* \cup Int(G) \subset H.$$

This implies that $Cl^*(Int(G)) \subset H$.

(2) $\Rightarrow$ (1) : Let $Cl^*(Int(G)) \subset H$ whenever $G \subset H$ and H is regular open in X. Since $(Int(G))^* \cup Int(G) \subset H$, then $(Int(G))^* \subset H$ whenever $G \subset H$ and H is a regular open set in X. □

Theorem 3.6. *Let (X,τ,I) be an ideal topological space and $G \subset X$. If G is regular open and weakly I_{rg}-closed, then G is $\star$-closed.*

Proof. Let G be a regular open and weakly I_{rg}-closed set in (X,τ,I). Since G is regular open and weakly I_{rg}-closed, $Cl^*(G) = Cl^*(Int(G)) \subset G$. Thus, G is a $\star$-closed set in (X,τ,I). □

Theorem 3.7. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a weakly I_{rg}-closed set, then $(Int(G))^*\backslash G$ contains no any nonempty regular closed set.*

Proof. Let G be a weakly I_{rg}-closed set in (X, τ, I). Suppose that H is a regular closed set such that $H \subset (Int(G))^*\backslash G$. Since G is a weakly I_{rg}-closed set, $X\backslash H$ is regular open and $G \subset X\backslash H$, then $(Int(G))^* \subset X\backslash H$. We have $H \subset X\backslash(Int(G))^*$. Hence,

$$H \subset (Int(G))^* \cap (X\backslash(Int(G))^*) = \emptyset.$$

Thus, $(Int(G))^*\backslash G$ contains no any nonempty regular closed set. □

Theorem 3.8. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a weakly I_{rg}-closed set, then $Cl^*(Int(G))\backslash G$ contains no any nonempty regular closed set.*

Proof. Suppose that H is a regular closed set such that $H \subset Cl^*(Int(G))\backslash G$. By Theorem 3.7, it follows from the fact that $Cl^*(Int(G))\backslash G = ((Int(G))^* \cup Int(G))\backslash G$. □

Remark 3.9. The reverse of Theorem 3.8 is not true in general as shown in the following example.

Example 3.10. Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$ and $I = \{\emptyset, \{b\}\}$. Take $A = \{c\}$. Then $Cl^*(Int(A))\backslash A$ contains no any nonempty regular closed set but A is not weakly I_{rg}-closed.

Theorem 3.11. *Let (X, τ, I) be an ideal topological space. The following properties are equivalent:*
(1) *G is pre_I^*-closed for each weakly I_{rg}-closed set G in (X, τ, I),*
(2) *Each singleton $\{x\}$ of X is a regular closed set or $\{x\}$ is pre_I^*-open.*

Proof. (1) $\Rightarrow$ (2) : Let G be pre_I^*-closed for each weakly I_{rg}-closed set G in (X, τ, I) and $x \in X$. We have $Cl^*(Int(G)) \subset G$ for each weakly I_{rg}-closed set G in (X, τ, I). Assume that $\{x\}$ is not a regular closed set. It follows that X is the only regular open set containing $X\backslash\{x\}$. Then, $X\backslash\{x\}$ is a weakly I_{rg}-closed set in (X, τ, I). Thus, $Cl^*(Int(X\backslash\{x\})) \subset X\backslash\{x\}$ and hence $\{x\} \subset Int^*(Cl(\{x\}))$. Consequently, $\{x\}$ is pre_I^*-open.

(2) $\Rightarrow$ (1) : Let G be a weakly I_{rg}-closed set in (X, τ, I). Let $x \in Cl^*(Int(G))$.

Suppose that $\{x\}$ is pre_I^*-open. We have $\{x\} \subset Int^*(Cl(\{x\}))$. Since $x \in Cl^*(Int(G))$, then $Int^*(Cl(\{x\})) \cap Int(G) \neq \emptyset$. It follows that $Cl(\{x\}) \cap Int(G) \neq \emptyset$. We have $Cl(\{x\} \cap Int(G)) \neq \emptyset$ and then $\{x\} \cap Int(G) \neq \emptyset$. Hence, $x \in Int(G)$. Thus, we have $x \in G$.

Suppose that $\{x\}$ is a regular closed set. By Theorem 3.8, $Cl^*(Int(G))\backslash G$ does not contain $\{x\}$. Since $x \in Cl^*(Int(G))$, then we have $x \in G$.

Consequently, we have $x \in G$. Thus, $Cl^*(Int(G)) \subset G$ and hence G is pre_I^*-closed. □

Theorem 3.12. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If $Cl^*(Int(G))\backslash G$ contains no any nonempty $\star$-closed set, then G is a weakly I_{rg}-closed set.*

Proof. Suppose that $Cl^*(Int(G))\backslash G$ contains no any nonempty $\star$-closed set in (X, τ, I). Let $G \subset H$ and H be a regular open set. Assume that $Cl^*(Int(G))$ is not contained in H. It follows that $Cl^*(Int(G)) \cap (X\backslash H)$ is a nonempty $\star$-closed subset of $Cl^*(Int(G))\backslash G$. This is a contradiction. Hence, G is a weakly I_{rg}-closed set. □

Theorem 3.13. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a weakly I_{rg}-closed set, then $Int(G) = H\backslash K$ where H is I-R closed and K contains no any nonempty regular closed set.*

Proof. Let G be a weakly I_{rg}-closed set in (X, τ, I). Take $K = (Int(G))^*\backslash G$. Then, by Theorem 3.7, K contains no any nonempty regular closed set. Take $H = Cl^*(Int(G))$. Then $H = Cl^*(Int(H))$. Moreover, we have

$$\begin{aligned} H\backslash K &= ((Int(G))^* \cup Int(G))\backslash((Int(G))^*\backslash G) \\ &= ((Int(G))^* \cup Int(G)) \cap (X\backslash(Int(G))^* \cup G) \\ &= Int(G). \end{aligned}$$

□

Theorem 3.14. *Let (X,τ,I) be an ideal topological space and $G \subset X$. Assume that G is a weakly I_{rg}-closed set. The following properties are equivalent:*
(1) *G is pre_I^*-closed,*
(2) *$Cl^*(Int(G))\backslash G$ is a regular closed set,*
(3) *$(Int(G))^*\backslash G$ is a regular closed set.*

Proof. (1) $\Rightarrow$ (2) : Let G be pre_I^*-closed. We have $Cl^*(Int(G)) \subset G$. Then, $Cl^*(Int(G))\backslash G = \emptyset$. Thus, $Cl^*(Int(G))\backslash G$ is a regular closed set.
(2) $\Rightarrow$ (1) : Let $Cl^*(Int(G))\backslash G$ be a regular closed set. Since G is a weakly I_{rg}-closed set in (X,τ,I), then by Theorem 3.8, $Cl^*(Int(G))\backslash G = \emptyset$. Hence, we have $Cl^*(Int(G)) \subset G$. Thus, G is pre_I^*-closed.
(2) $\Leftrightarrow$ (3) : It follows easily from that $Cl^*(Int(G))\backslash G = (Int(G))^*\backslash G$. □

Theorem 3.15. *Let (X,τ,I) be an ideal topological space and $G \subset X$ be a weakly I_{rg}-closed set. Then $G \cup (X\backslash(Int(G))^*)$ is a weakly I_{rg}-closed set in (X,τ,I).*

Proof. Let G be a weakly I_{rg}-closed set in (X,τ,I). Suppose that H is a regular open set such that $G \cup (X\backslash(Int(G))^*) \subset H$. We have

$$\begin{aligned} X\backslash H &\subset X\backslash(G \cup (X\backslash(Int(G))^*)) \\ &= (X\backslash G) \cap (Int(G))^* \\ &= (Int(G))^*\backslash G. \end{aligned}$$

Since $X\backslash H$ is a regular closed set and G is a weakly I_{rg}-closed set, it follows from Theorem 3.7 that $X\backslash H = \emptyset$. Hence, $X = H$. Thus, X is the only regular open set containing $G \cup (X\backslash(Int(G))^*)$. Consequently, $G \cup (X\backslash(Int(G))^*)$ is a weakly I_{rg}-closed set in (X,τ,I). □

Corollary 3.16. *Let (X,τ,I) be an ideal topological space and $G \subset X$ be a weakly I_{rg}-closed set. Then $(Int(G))^*\backslash G$ is a weakly I_{rg}-open set in (X,τ,I).*

Proof. Since $X\backslash((Int(G))^*\backslash G) = G \cup (X\backslash(Int(G))^*)$, it follows from Theorem 3.15 that $(Int(G))^*\backslash G$ is a weakly I_{rg}-open set in (X,τ,I). □

Theorem 3.17. *Let (X,τ,I) be an ideal topological space and $G \subset X$. The following properties are equivalent:*
(1) *G is a $\star$-closed and regular open set,*
(2) *G is I-R closed and a regular open set,*
(3) *G is a weakly I_{rg}-closed and regular open set.*

Proof. (1) $\Rightarrow$ (2) $\Rightarrow$ (3) : Obvious.
(3) $\Rightarrow$ (1) : It follows from Theorem 3.6. □

4. Further properties

Theorem 4.1. *Let (X,τ,I) be an ideal topological space. The following properties are equivalent:*
(1) *Each subset of (X,τ,I) is a weakly I_{rg}-closed set,*
(2) *G is pre_I^*-closed for each regular open set G in X.*

Proof. (1) $\Rightarrow$ (2) : Suppose that each subset of (X,τ,I) is a weakly I_{rg}-closed set. Let G be a regular open set. Since G is weakly I_{rg}-closed, then we have $Cl^*(Int(G)) \subset G$. Thus, G is pre_I^*-closed.
(2) $\Rightarrow$ (1) : Let G be a subset of (X,τ,I) and H be a regular open set such that $G \subset H$. By (2), we have $Cl^*(Int(G)) \subset Cl^*(Int(H)) \subset H$. Thus, G is a weakly I_{rg}-closed set in (X,τ,I). □

Theorem 4.2. *Let (X,τ,I) be an ideal topological space. If G is a weakly I_{rg}-closed set and $G \subset H \subset Cl^*(Int(G))$, then H is a weakly I_{rg}-closed set.*

Proof. Let $H \subset K$ and K be a regular open set in X. Since $G \subset K$ and G is a weakly I_{rg}-closed set, then $Cl^*(Int(G)) \subset K$. Since $H \subset Cl^*(Int(G))$, then

$$Cl^*(Int(H)) \subset Cl^*(Int(G)) \subset K.$$

Thus, $Cl^*(Int(H)) \subset K$ and hence, H is a weakly I_{rg}-closed set. □

Corollary 4.3. *Let (X, τ, I) be an ideal topological space. If G is a weakly I_{rg}-closed and open set, then $Cl^*(G)$ is a weakly I_{rg}-closed set.*

Proof. Let G be a weakly I_{rg}-closed and open set in (X, τ, I). We have $G \subset Cl^*(G) \subset Cl^*(G) = Cl^*(Int(G))$. Hence, by Theorem 4.2, $Cl^*(G)$ is a weakly I_{rg}-closed set in (X, τ, I). □

Theorem 4.4. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a nowhere dense set, then G is a weakly I_{rg}-closed set.*

Proof. Let G be a nowhere dense set in X. Since $Int(G) \subset Int(Cl(G))$, then $Int(G) = \emptyset$. Hence, $Cl^*(Int(G)) = \emptyset$. Thus, G is a weakly I_{rg}-closed set in (X, τ, I). □

Remark 4.5. The reverse of Theorem 4.4 is not true in general as shown in the following example.

Example 4.6. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$ and $I = \{\emptyset, \{b\}\}$. Take $A = \{a, c, d\}$. Then A is weakly I_{rg}-closed but it is not a nowhere dense set.*

Remark 4.7. (1) The intersection of two weakly I_{rg}-closed sets in an ideal topological space need not be a weakly I_{rg}-closed set.

(2) The union of two weakly I_{rg}-closed sets in an ideal topological space need not be a weakly I_{rg}-closed set.

Example 4.8. Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\emptyset, \{a\}, \{d\}, \{a, d\}\}$. Take $A = \{b\}$ and $B = \{c\}$. Then A and B are weakly I_{rg}-closed but $A \cup B$ is not a weakly I_{rg}-closed set.

Example 4.9. Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$ and $I = \{\emptyset, \{b\}\}$. Take $A = \{a, b, c\}$ and $B = \{a, c, d\}$. Then A and B are weakly I_{rg}-closed but $A \cap B$ is not a weakly I_{rg}-closed set.

Lemma 4.10. ([8]) *Let A be an open subset of a topological space (X, τ).*

(1) If G is regular open set in X, then so is $G \cap A$ in the subspace (A, τ_A).

(2) If B $(\subset A)$ is regular open in (A, τ_A), then there exists a regular open set G in (X, τ) such that $B = G \cap A$.

Theorem 4.11. *Let (X, τ, I) be an ideal topological space and $H \subset G \subset X$. If G is an open set in X and H is a weakly I_{rg}-closed in G, then H is a weakly I_{rg}-closed set in X.*

Proof. Let K be a regular open set in X and $H \subset K$. We have $H \subset K \cap G$. By Lemma 4.10, $K \cap G$ is a regular open set in G. Since H is a weakly I_{rg}-closed set in G, then $Cl^*_G(Int_G(H)) \subset K \cap G$. Also, we have

$$Cl^*(Int(H)) \subset Cl^*_G(Int(H)) = Cl^*_G(Int_G(H)) \subset K \cap G \subset K.$$

Hence, $Cl^*(Int(H)) \subset K$. Thus, H is a weakly I_{rg}-closed in (X, τ, I). □

Theorem 4.12. *Let (X, τ, I) be an ideal topological space and $H \subset G \subset X$. If G is a regular open set in X and H is a weakly I_{rg}-closed set in X, then H is a weakly I_{rg}-closed set in G.*

Proof. Let $H \subset K$ and K be a regular open set in G. By Lemma 4.10, there exists a regular open set L in X such that $K = L \cap G$. Since H is a weakly I_{rg}-closed set in X, then $Cl^*(Int(H)) \subset K$. Also, we have

$$Cl^*_G(Int_G(H)) = Cl^*_G(Int(H)) = Cl^*(Int(H)) \cap G \subset K \cap G = K.$$

Thus, $Cl^*_G(Int_G(H)) \subset K$. Hence, H is a weakly I_{rg}-closed set in G. □

Theorem 4.13. *Let (X, τ, I) be an ideal topological space and $G \subset X$. Then G is a weakly I_{rg}-open set if and only if $H \subset Int^*(Cl(G))$ whenever $H \subset G$ and H is a regular closed set.*

Proof. Let H be a regular closed set in X and $H \subset G$. It follows that $X \backslash H$ is a regular open set and $X \backslash G \subset X \backslash H$. Since $X \backslash G$ is a weakly I_{rg}-closed set, then $Cl^*(Int(X \backslash G)) \subset X \backslash H$. We have $X \backslash Int^*(Cl(G)) \subset X \backslash H$. Thus, $H \subset Int^*(Cl(G))$.

Conversely, let K be a regular open set in X and $X \backslash G \subset K$. Since $X \backslash K$ is a regular closed set such that $X \backslash K \subset G$, then $X \backslash K \subset Int^*(Cl(G))$. We have

$$X \backslash Int^*(Cl(G)) = Cl^*(Int(X \backslash G)) \subset K.$$

Thus, $X \backslash G$ is a weakly I_{rg}-closed set. Hence, G is a weakly I_{rg}-open set in (X, τ, I). □

Theorem 4.14. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a weakly I_{rg}-closed set, then $Cl^*(Int(G)) \backslash G$ is a weakly I_{rg}-open set in (X, τ, I).*

Proof. Let G be a weakly I_{rg}-closed set in (X, τ, I). Suppose that H is a regular closed set such that $H \subset Cl^*(Int(G)) \backslash G$. Since G is a weakly I_{rg}-closed set, it follows from Theorem 3.8 that $H = \emptyset$. Thus, we have $H \subset Int^*(Cl(Cl^*(Int(G)) \backslash G))$. It follows from Theorem 4.13 that $Cl^*(Int(G)) \backslash G$ is a weakly I_{rg}-open set in (X, τ, I). □

Theorem 4.15. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a weakly I_{rg}-open set, then $H = X$ whenever H is a regular open set and $Int^*(Cl(G)) \cup (X \backslash G) \subset H$.*

Proof. Let H be a regular open set in X and $Int^*(Cl(G)) \cup (X \backslash G) \subset H$. We have

$$\begin{aligned} X \backslash H &\subset (X \backslash Int^*(Cl(G))) \cap G \\ &= Cl^*(Int(X \backslash G)) \backslash (X \backslash G). \end{aligned}$$

Since $X \backslash H$ is a regular closed set and $X \backslash G$ is a weakly I_{rg}-closed set, it follows from Theorem 3.8 that $X \backslash H = \emptyset$. Thus, we have $H = X$. □

Theorem 4.16. *Let (X, τ, I) be an ideal topological space. If G is a weakly I_{rg}-open set and $Int^*(Cl(G)) \subset H \subset G$, then H is a weakly I_{rg}-open set.*

Proof. Let G be a weakly I_{rg}-open set and $Int^*(Cl(G)) \subset H \subset G$. Since $Int^*(Cl(G)) \subset H \subset G$, then $Int^*(Cl(G)) = Int^*(Cl(H))$. Let K be a regular closed set and $K \subset H$. We have $K \subset G$. Since G is a weakly I_{rg}-open set, it follows from Theorem 4.13 that

$$K \subset Int^*(Cl(G)) = Int^*(Cl(H)).$$

Hence, by Theorem 4.13, H is a weakly I_{rg}-open set in (X, τ, I). □

Corollary 4.17. *Let (X, τ, I) be an ideal topological space and $G \subset X$. If G is a weakly I_{rg}-open and closed set, then $Int^*(G)$ is a weakly I_{rg}-open set.*

Proof. Let G be a weakly I_{rg}-open and closed set in (X, τ, I). Then $Int^*(Cl(G)) = Int^*(G) \subset Int^*(G) \subset G$. Thus, by Theorem 4.16, $Int^*(G)$ is a weakly I_{rg}-open set in (X, τ, I). □

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