Filomat 27:5 (2013), 731–738
DOI 10.2298/FIL1305731E

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Additive–cubic functional equations from additive groups into non–Archimedean Banach spaces

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Abstract. In this paper, we establish the generalized Hyres–Ulam stability of the mixed type additive–cubic functional equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+2f(2x)-4f(x)$$

from additive groups into non–Archimedean Banach spaces.

1. Introduction

We recall that a field $\mathbb{K}$, equipped with a function (non–Archimedean absolute value, valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$, is called a non–Archimedean field if the function $|\cdot|:\mathbb{K}\to[0,\infty)$ satisfies the following conditions:

1. $|r|=0$ if and only if $r=0$;
2. $|rs|=|r||s|$;
3. the strong triangle inequality, namely, $|r+s|\le\max\{|r|,|s|\}$ for all $r,s\in\mathbb{K}$.

Clearly, $|1|=1=|-1|$ and $|n|\le 1$ for all nonzero integer n.

Let Y be a vector space over the non-Archimedean field $\mathbb{K}$ with a non–trivial non–Archimedean valuation $|\cdot|$. A function $\|\cdot\|:Y\to[0,\infty)$ is called a non–Archimedean norm (valuation) if it satisfies the following conditions:

1. $\|x\|=0$ if and only if $x=0$;
2. $\|rx\|=|r|\|x\|$ for all $x\in Y$ and all $r\in\mathbb{K}$;
3. the strong triangle inequality, namely,

$$\|x+y\|\le\max\{\|x\|,\|y\|\}$$

for all $x,y\in Y$.

2010 *Mathematics Subject Classification*. Primary 46S40,54E40; Secondary 39B82
Keywords. Stability; Additive functional equation; Cubic functional equation; Non–Archimedean Banach space
Received: 13 March 2012; Accepted: 27 May 2012
Communicated by Dragan S. Djordjević
Research supported by Semnan University
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In this case, the pair $(Y, \|\cdot\|)$ is called a non–Archimedean space. By a Banach non–Archimedean space we mean one in which every Cauchy sequence is convergent. It follows from the strong triangle inequality that

$$\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| : m \leq j < n-1\}$$

for all $x_n, x_m \in Y$ and all $m, n \in \mathbb{N}$ with $n > m$. Therefore, a sequence $\{x_n\}$ is a Cauchy sequence in non–Archimedean space $(Y, \|\cdot\|)$ if and only if the sequence $\{x_{n+1} - x_n\}$ converges to zero in the space $(Y, \|\cdot\|)$.

In 1940, S. M. Ulam [21] in the University of Wisconsin proposed his famous question about the stability of homomorphisms. In the next year, D. H. Hyers [15] gave a positive answer to the above question for additive groups under the assumption that the groups are Banach spaces. In 1978, Th. M. Rassias [19] proved a generalization of Hyers theorem which allows the Cauchy difference to be unbounded. According to Th. M. Rassias theorem:

Theorem 1.1. *Let $f : E \longrightarrow E'$ be a mapping from a norm vector space E into a Banach space E' subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \longrightarrow E'$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2^p}\|x\|^p$$

for all $x \in E$. If $p < 0$ then inequality (1.1) holds for all $x, y \neq 0$, and (1.2) for $x \neq 0$. Also, if the function $t \mapsto f(tx)$ from $\mathbb{R}$ into E' is continuous for each fixed $x \in E$, then T is linear.

The result of Th. M. Rassias has influenced the development of what is now called the Hyers-Ulam-Rassias stability theory for functional equations. In 1994, a generalization of Rassias theorem was obtained by G¢avruta [12] by replacing the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\phi(x, y)$. Several stability results have been recently obtained for various equations, also for mappings with more general domains and ranges (see [1, 2, 11, 13, 14, 16] and [20]).

K. Jun and H. Kim [17] introduced the following cubic functional equation

$$f(2x+y) + f(2x-y) = 2f(x+y) + 2f(x-y) + 12f(x) \tag{1.1}$$

and they established the general solution and the generalized Hyers–Ulam–Rassias stability problem for the functional equation (1.1). They proved that a function $f : E_1 \to E_2$ satisfies the functional equation (1.1) if and only if there exists a function $B : E_1 \times E_1 \times E_1 \to E_2$ such that $f(x) = B(x, x, x)$ for all $x\ inE_1$, and B is symmetric for each fixed one variable and additive for each fixed two variables. The function is given by

$$B(x, y, z) = \frac{1}{24}[f(x+y+z) + f(x-y-z) - f(x+y-z) - f(x-y+z)]$$

for all $x, y, z \in E_1$.
It is easy to see that the function $f(x) = cx^3$ is a solution of the functional equation (1.1). Thus, it is natural that (1.1) is called a cubic functional equation and every solution of the cubic functional equation (1.1) is said to be a cubic function. K. Jun and H. Kim [17], investigated the generalized Hyers–Ulam–Rassias stability for a mixed type cubic and additive functional equation. For more detailed definitions of mixed type functional equations, we can refer to [6] and [8].

M. S. Moslehian and Th. M. Rassias [18] proved the generalized Hyers–Ulam stability of the Cauchy functional equation and the quadratic functional equation in non–Archimedean spaces.
Recently, M. Eshaghi Gordji and M. Bavand Savadkouhi [5] proved the generalized Hyers–Ulam–Rassias stability of the cubic and quartic functional equations in non–Archimedean spaces (see also [3, 4, 7, 9] and

[10]).
In this paper, we investigate the stability of following functional equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+2f(2x)-4f(x) \tag{1.2}$$

in non–Archimedean space. The function $f(x)=ax+bx^3$ satisfies the functional equation (1.2), which explains why it is called additive–cubic functional equation.

2. Main Results

Throughout this section, we assume that G is an additive group and X is a Banach non–Archimedean space. For a given function $T:G\to X$, we define the difference operator

$$\Delta T(x,y)=T(2x+y)+T(2x-y)-2T(x+y)-2T(x-y)-2T(2x)+4T(x)$$

for all $x,y\in G$.

Theorem 2.1. *Let $\varphi:G\times G\to[0,\infty)$ be a function such that*

$$\lim_{n\to\infty}\frac{\varphi(2^nx,2^ny)}{|2|^n}=0 \tag{2.1}$$

$$\lim_{n\to\infty}\frac{1}{|2^n|}\max\{|2|\varphi(2^nx,2^nx),\max\{|2|\varphi(0,2^nx),\varphi(2^nx,2^{n+1}x)\}\}=0 \tag{2.2}$$

for all $x,y\in G$ and let for each $x\in G$ the limit

$$\lim_{n\to\infty}\max\{\frac{1}{|2^j|}\max\{|2|\varphi(2^jx,2^jx),\max\{|2|\varphi(0,2^jx),\varphi(2^jx,2^{j+1}x)\}\}:\ 0\le j<n\}, \tag{2.3}$$

denoted by $\tilde{\varphi}_A(x)$, exists. Suppose that $f:G\to X$ is a function satisfying

$$\|f(2x+y)+f(2x-y)-2f(x+y)-2f(x-y)-2f(2x)+4f(x)\|\le\varphi(x,y) \tag{2.4}$$

for all $x,y\in G$. Then there exists an additive function $A:G\to X$ such that

$$\|f(2x)-8f(x)-A(x)\|\le\frac{1}{|2|}\tilde{\varphi}_A(x) \tag{2.5}$$

for all $x\in G$. Moreover, if

$$\lim_{i\to\infty}\lim_{n\to\infty}\max\{\frac{1}{|2^j|}\max\{|2|\varphi(2^jx,2^jx),\\ \max\{|2|\varphi(0,2^jx),\varphi(2^jx,2^{j+1}x)\}\}:\ i\le j<n+i\}=0$$

then A is the unique additive function satisfying (2.5).

Proof. Setting $x=0$ in (2.4), we get

$$\|f(y)+f(-y)\|\le\varphi(0,y) \tag{2.6}$$

for all $y\in G$. If we replace y in (2.6) by x, we obtain

$$\|f(x)+f(-x)\|\le\varphi(0,x) \tag{2.7}$$

for all $x\in G$. Replacing y by x and $2x$ in (2.4), respectively, we get the inequalities

$$\|f(3x)-4f(2x)+5f(x)\|\le\varphi(x,x), \tag{2.8}$$

$$\|f(4x) - 2f(3x) - 2f(2x) - 2f(-x) + 4f(x)\| \leq \varphi(x, 2x) \tag{2.9}$$

for all $x \in G$. It follows from (2.7) and (2.9) that

$$\|f(4x) - 2f(3x) - 2f(2x) + 6f(x)\| \leq \max\{|2|\varphi(0, x), \varphi(x, 2x)\} \tag{2.10}$$

for all $x \in G$. combining (2.8) and (2.10) to get

$$\|f(4x) - 10f(2x) + 16f(x)\| \leq \max\{|2|\varphi(x, x), \max\{|2|\varphi(0, x), \varphi(x, 2x)\}\} \tag{2.11}$$

for all $x \in G$. Let $g : G \to X$ be a mapping defined by $g(x) = f(2x) - 8f(x)$ for all $x \in G$. Therefore, we have

$$\|g(2x) - 2g(x)\| \leq \max\{|2|\varphi(x, x), \max\{|2|\varphi(0, x), \varphi(x, 2x)\}\} \tag{2.12}$$

for all $x \in G$. Hence,

$$\|\frac{g(2x)}{2} - g(x)\| \leq \frac{1}{|2|} \max\{|2|\varphi(x, x), \max\{|2|\varphi(0, x), \varphi(x, 2x)\}\} \tag{2.13}$$

Replacing x by $2^{n-1}x$ in (2.13) to get

$$\begin{aligned} &\|\frac{1}{2^n} g(2^n x) - \frac{1}{2^{(n-1)}} g(2^{n-1}x)\| \\ &\leq \frac{1}{|2^n|} \max\{|2|\varphi(2^{n-1}x, 2^{n-1}x), \max\{|2|\varphi(0, 2^{n-1}x), \varphi(2^{n-1}x, 2^n x)\}\} \end{aligned} \tag{2.14}$$

for all $x \in G$. It follows from (2.2) and (2.14) that the sequence $\{\frac{g(2^n x)}{2^n}\}$ is Cauchy. Since X is complete, we conclude that $\{\frac{g(2^n x)}{2^n}\}$ is convergent. Set $A(x) := \lim_{n\to\infty} \frac{g(2^n x)}{2^n}$.
Using induction one can show that

$$\begin{aligned} \|\frac{g(2^n x)}{2^n} - g(x)\| \leq \frac{1}{|2|} \max\{&\frac{1}{|2^i|} \max\{\max\{|2|\varphi(2^i x, 2^i x), \\ &\max\{|2|\varphi(0, 2^i x), \varphi(2^i x, 2^{i+1}x) : \ 0 \leq i < n\} \end{aligned} \tag{2.15}$$

for all $n \in \mathbb{N}$ and all $x \in G$. Letting $n \to \infty$ in (2.15) and using (2.3) one can obtain (2.5). By (2.1) and (2.4), we get

$$\begin{aligned} \|\Delta A(x, y)\| &= \lim_{n\to\infty} \frac{1}{|2^n|} \|\Delta g(2^n x, 2^n y)\| \\ &\leq \lim_{n\to\infty} \max\{\|\frac{\Delta f(2^{n+1}x, 2^{n+1}y)}{2^n}\|, \|\frac{8\Delta f(2^n x, 2^n y)}{2^n}\|\} \\ &\leq \lim_{n\to\infty} \max\{|2|\frac{\varphi(2^{n+1}x, 2^{n+1}y)}{2^{n+1}}, |8|\frac{\varphi(2^n x, 2^n y)}{2^n}\} = 0 \end{aligned}$$

for all $x, y \in G$. Therefore the function $A : G \to X$ satisfies (1.2). If A' is another additive function satisfying (2.5), then

$$\begin{aligned} \|A(x) - A'(x)\| &= \lim_{i\to\infty} |2|^{-i} \|A(2^i x) - A'(2^i x)\| \\ &\leq \lim_{i\to\infty} |2|^{-i} \max\{\ \|A(2^i x) - f(2^i x)\|, \|f(2^i x) - A'(2^i x)\|\ \} \\ &\leq \frac{1}{|2|} \lim_{i\to\infty} \lim_{n\to\infty} \max\{\frac{1}{|2^j|} \max\{|2|\varphi(2^j x, 2^j x), \\ &\max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} : \ i \leq j < n + i\} \\ &= 0. \end{aligned}$$

for all $x \in G$. Therefore $A = A'$. This completes the proof of the uniqueness of A. □

Theorem 2.2. *Let $\varphi : G \times G \to [0, \infty)$ be a function such that*

$$\lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y)}{|2|^{3n}} = 0 \tag{2.16}$$

$$\lim_{n\to\infty} \frac{1}{|2^{3n}|} \max\{|2|\varphi(2^n x, 2^n x), \max\{|2|\varphi(0, 2^n x), \varphi(2^n x, 2^{n+1}x)\}\} = 0 \tag{2.17}$$

for all $x, y \in G$ and let for each $x \in G$ the limit

$$\lim_{n\to\infty} \max\{\frac{1}{|2^{3j}|} \max\{|2|\varphi(2^j x, 2^j x), \max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} : \ 0 \le j < n\}, \tag{2.18}$$

denoted by $\tilde{\varphi}_C(x)$, exists. Suppose that $f : G \to X$ is a function satisfying

$$\|f(2x + y) + f(2x - y) - 2f(x + y) - 2f(x - y) - 2f(2x) + 4f(x)\| \le \varphi(x, y) \tag{2.19}$$

for all $x, y \in G$. Then there exists a cubic function $C : G \to X$ such that

$$\|f(2x) - 2f(x) - C(x)\| \le \frac{1}{|8|}\tilde{\varphi}_C(x) \tag{2.20}$$

for all $x \in G$. If moreover,

$$\lim_{i\to\infty}\lim_{n\to\infty} \max\{\frac{1}{|2^{3j}|} \max\{|2|\varphi(2^j x, 2^j x),$$
$$\max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} : \ i \le j < n + i\} = 0$$

then C is the unique cubic function satisfying (2.20).

Proof. Similar to the proof of Theorem 2.1, we have

$$\|f(4x) - 10f(2x) + 16f(x)\| \le \max\{|2|\varphi(x, x), \max\{|2|\varphi(0, x), \varphi(x, 2x)\}\} \tag{2.21}$$

for all $x \in G$. Let $h : G \to X$ be a mapping defined by $h(x) = f(2x) - 2f(x)$ for all $x \in G$. Therefore, we have

$$\|h(2x) - 8h(x)\| \le \max\{|2|\varphi(x, x), \max\{|2|\varphi(0, x), \varphi(x, 2x)\}\} \tag{2.22}$$

for all $x \in G$. Then we have

$$\|\frac{h(2x)}{8} - h(x)\| \le \frac{1}{|8|} \max\{|2|\varphi(x, x), \max\{|2|\varphi(0, x), \varphi(x, 2x)\}\}. \tag{2.23}$$

Replacing x by $2^{n-1}x$ in (2.23) to obtain

$$\|\frac{1}{2^{3n}}h(2^n x) - \frac{1}{2^{3(n-1)}}h(2^{n-1}x)\|$$
$$\le \frac{1}{|2^{3n}|} \max\{|2|\varphi(2^{n-1}x, 2^{n-1}x), \max\{|2|\varphi(0, 2^{n-1}x), \varphi(2^{n-1}x, 2^n x)\}\} \tag{2.24}$$

for all $x \in G$. It follows from (2.17) and (2.24) that the sequence $\{\frac{h(2^n x)}{2^{3n}}\}$ is Cauchy. Since X is complete, we conclude that $\{\frac{h(2^n x)}{2^{3n}}\}$ is convergent. Set $C(x) := \lim_{n\to\infty} \frac{h(2^n x)}{2^{3n}}$.
Using induction to get

$$\|\frac{h(2^n x)}{2^{3n}} - h(x)\| \le \frac{1}{|8|} \max\{\frac{1}{|2^{3i}|} \max\{|2|\varphi(2^i x, 2^i x),$$
$$\max\{|2|\varphi(0, 2^i x), \varphi(2^i x, 2^{i+1}x)\}\} : \ 0 \le i < n\} \tag{2.25}$$

for all $n \in \mathbb{N}$ and all $x \in G$. By taking n to approach infinity in (2.25) and using (2.18) one can obtain (2.20). By (2.16) and (2.19), we get

$$\begin{aligned}
\|\Delta C(x,y)\| &= \lim_{n\to\infty} \frac{1}{|2^{3n}|}\|\Delta h(2^n x, 2^n y)\| \\
&\leq \lim_{n\to\infty} \max\{\|\frac{\Delta f(2^{n+1}x, 2^{n+1}y)}{2^{3n}}\|, \|\frac{2\Delta f(2^n x, 2^n y)}{2^{3n}}\|\} \\
&\leq \lim_{n\to\infty} \max\{|8|\frac{\varphi(2^{n+1}x, 2^{n+1}y)}{2^{3(n+1)}}, |2|\frac{\varphi(2^n x, 2^n y)}{2^{3n}}\} = 0
\end{aligned}$$

for all $x, y \in G$. Therefore the function $C : G \to X$ satisfies (1.2). If C' is another cubic function satisfying (2.20), then

$$\begin{aligned}
\|C(x) - C'(x)\| &= \lim_{i\to\infty} |2|^{-3i}\|C(2^i x) - C'(2^i x)\| \\
&\leq \lim_{i\to\infty} |2|^{-3i} \max\{\ \|C(2^i x) - f(2^i x)\|, \|f(2^i x) - C'(2^i x)\|\ \} \\
&\leq \frac{1}{|8|} \lim_{i\to\infty}\lim_{n\to\infty} \max\{\frac{1}{|2^{3j}|} \max\{|2|\varphi(2^j x, 2^j x), \\
&\max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} :\ i \leq j < n+i\} \\
&= 0.
\end{aligned}$$

for all $x \in G$. Therefore $C = C'$. □

Theorem 2.3. *Let $\varphi : G \times G \to [0,\infty)$ be a function such that*

$$\lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y)}{|2|^n} = \lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y)}{|2|^{3n}} = 0 \tag{2.26}$$

for all $x, y \in G$ and let for each $x \in G$ the limit

$$\lim_{n\to\infty} \max\{\frac{1}{|2^j|} \max\{|2|\varphi(2^j x, 2^j x), \max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} :\ 0 \leq j < n\},$$

denoted by $\tilde{\varphi}_A(x)$, and

$$\lim_{n\to\infty} \max\{\frac{1}{|2^{3j}|} \max\{|2|\varphi(2^j x, 2^j x), \max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} :\ 0 \leq j < n\},$$

denoted by $\tilde{\varphi}_C(x)$, exist. Suppose that $f : G \to X$ is a function satisfying

$$\|f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 2f(2x) + 4f(x)\| \leq \varphi(x,y) \tag{2.27}$$

for all $x, y \in G$. Then there exist an additive function $A : X \to Y$ and a cubic function $C : G \to X$ such that

$$\|f(x) - A(x) - C(x)\| \leq \frac{1}{|12|} \max\{\tilde{\varphi}_A(x), \frac{1}{|4|}\tilde{\varphi}_C(x)\} \tag{2.28}$$

for all $x \in G$. Moreover, if

$$\begin{aligned}
\lim_{i\to\infty}\lim_{n\to\infty} \max\{&\frac{1}{|2^j|} \max\{|2|\varphi(2^j x, 2^j x), \\
&\max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} :\ i \leq j < n+i\} = 0,
\end{aligned}$$

$$\begin{aligned}
\lim_{i\to\infty}\lim_{n\to\infty} \max\{&\frac{1}{|2^{3j}|} \max\{|2|\varphi(2^j x, 2^j x), \\
&\max\{|2|\varphi(0, 2^j x), \varphi(2^j x, 2^{j+1}x)\}\} :\ i \leq j < n+i\} = 0
\end{aligned}$$

then A is the unique additive function and C is the unique cubic function satisfying (2.28).

Proof. By Theorems 2.1 and 2.2, there exist an additive mapping $A_1 : G \to X$ and a cubic mapping $C_1 : G \to X$ such that

$$\|f(2x) - 8f(x) - A(x)\| \leq \frac{1}{|2|}\tilde{\varphi}_A(x),$$

$$\|f(2x) - 2f(x) - C(x)\| \leq \frac{1}{|8|}\tilde{\varphi}_A(x)$$

for all $x \in G$. So we obtain (2.28) by letting $A(x) = \frac{-1}{6}A_1(x)$ and $C(x) = \frac{1}{6}C_1(x)$ for all $x \in G$. To prove the uniqueness property of A and C, let $A_0, C_0 : G \to X$ be another additive and cubic mappings satisfying (2.28). Let $A' = A - A_0$ and $C' = C - C_0$. So

$$\|f(x) - A(x) - C(x)\| \leq \frac{1}{|12|}\max\{\tilde{\varphi}_A(x), \frac{1}{|4|}\tilde{\varphi}_C(x)\},$$

$$\|f(x) - A_1(x) - C_1(x)\| \leq \frac{1}{|12|}\max\{\tilde{\varphi}_A(x), \frac{1}{|4|}\tilde{\varphi}_C(x)\}$$

thus we get

$$\begin{aligned}\|A'(x) + C'(x)\| &\leq \max\{\|f(x) - A(x) - C(x)\|, \|f(x) - A_1(x) - C_1(x)\|\} \\ &\leq \frac{1}{|12|}\max\{\tilde{\varphi}_A(x), \frac{1}{|4|}\tilde{\varphi}_C(x)\} \qquad (2.29)\end{aligned}$$

for all $x \in G$. Since

$$\lim_{i\to\infty}\lim_{n\to\infty}\max\{\frac{1}{|2^j|}\max\{|2|\varphi(2^jx, 2^jx), \max\{|2|\varphi(0, 2^jx), \varphi(2^jx, 2^{j+1}x)\}\} : i \leq j < n + i\} = 0,$$

$$\lim_{i\to\infty}\lim_{n\to\infty}\max\{\frac{1}{|2^{3j}|}\max\{|2|\varphi(2^jx, 2^jx), \max\{|2|\varphi(0, 2^jx), \varphi(2^jx, 2^{j+1}x)\}\} : i \leq j < n + i\} = 0$$

for all $x \in G$, then (2.29) implies that

$$\lim_{n\to\infty}\frac{1}{|2|^{3n}}\|A'(2^nx) + C'(2^nx)\| = 0$$

for all $x \in G$. So it follows from (2.29) that

$$\|A'(x)\| \leq \frac{1}{|12|}\tilde{\varphi}_A(x)$$

for all $x \in G$. Therefore $A' = 0$. □

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