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A note on star-Hurewicz spaces

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Abstract. In this paper, we show the following statements:

(1) There exists a Tychonoff pseudocompact, star-Hurewicz space having a regular-closed subspace which is not star-Hurewicz;

(2) Assuming $\omega_1 < \mathfrak{b}$, there exists a Tychonoff strongly star-Hurewicz (hence star-Hurewicz) space having a regular-closed G_δ-subspace which is not star-Hurewicz (hence not strongly star-Hurewicz);

(3) An open F_σ-subset of a strongly star-Hurewicz space is strongly star-Hurewicz.

1. Introduction

By a space we mean a topological space. We give definitions of terms which are used in this paper. Let $\mathbb{N}$ denote the set of positive integers. Let X be a space and $\mathcal{U}$ a collection of subsets of X. For $A \subseteq X$, let $St(A, \mathcal{U}) = \cup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. As usual, we write $St(x, \mathcal{U})$ instead of $St(\{x\}, \mathcal{U})$.

Let $\mathcal{A}$ and $\mathcal{B}$ be collections of open covers of a space X. Then the symbol $S_1(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(U_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $U_n \in \mathcal{U}_n$ and $(U_n : n \in \mathbb{N})$ is an element of $\mathcal{B}$. The symbol $S_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is an element of $\mathcal{B}$ (see [9, 15]).

Kočinac [10, 11, 12] introduced star selection hypothesis similar to the previous ones. Let $\mathcal{A}$ and $\mathcal{B}$ be collections of open covers of a space X. Then:

(A) The symbol $S^*_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}}\{St(V, \mathcal{U}_n) : V \in \mathcal{V}_n\}$ is an element of $\mathcal{B}$.

(B) The symbol $SS^*_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(K_n : n \in N)$ of finite subsets of X such that $\{St(K_n, \mathcal{U}_n) : n \in \mathbb{N}\} \in \mathcal{B}$.

Let $\mathcal{O}$ denote the collection of all open covers of X.

Definition 1.1. ([10, 11, 12]) A space X is said to be *star-Menger* (*strongly star-Menger*) if it satisfies the selection hypothesis $S^*_{fin}(\mathcal{O}, \mathcal{O})$ ($SS^*_{fin}(\mathcal{O}, \mathcal{O})$, respectively).

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Definition 1.2. ([4, 13]) A space X is said to be *starcompact* (*strongly starcompact*) if for every open cover $\mathcal{U}$ of X there exists a finite $\mathcal{V} \subseteq \mathcal{U}$ ($F \subseteq X$, respectively) such that $St(\cup\mathcal{V}, \mathcal{U}) = X$ ($St(F, \mathcal{U}) = X$, respectively).

In 1925 in [7] (see also [8]), Hurewicz introduced the Hurewicz covering property for a space X in the following way:

H: For each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(\mathcal{V}_n : n \in N)$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $x \in \cup\mathcal{V}_n$ for all but finitely many n.

In [1], two star versions of the Hurewicz property were introduced as follows:

SH: A space X satisfies the *star-Hurewicz property* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(\mathcal{V}_n : n \in N)$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $x \in St(\cup\mathcal{V}_n, \mathcal{U}_n)$ for all but finitely many n.

SSH: A space X satisfies the *strongly star-Hurewicz property* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(A_n : n \in N)$ of finite subsets of X such that for each $x \in X$, $x \in St(A_n, \mathcal{U}_n)$ for all but finitely many n.

Definition 1.3. ([1, 12]) A space X is said to be *star-Hurewicz* (*strongly star-Hurewicz*) if it satisfies the star-Hurewicz property (strongly star-Hurewicz property, respectively).

From the definitions, it is clear that every strongly starcompact space is starcompact and strongly star-Hurewicz, every starcompact space is star-Hurewicz, every strongly star-Hurewicz space is strongly star-Menger, every strongly star-Hurewicz space is star-Hurewicz, every strongly star-Hurewicz space is strongly star-Menger and every star-Hurewicz space is star-Menger.

Bonanzinga et al. in [1] studied star-Hurewicz and related spaces and Kočinac asked the following question in [12]

Question 1.4. *Characterize hereditarily star-Hurewicz (strongly star-Hurewicz) spaces.*

The purpose of this paper is to show the three statements stated in the abstract.

Throughout this paper, let ω denote the first infinite cardinal, ω_1 the first uncountable cardinal and $\mathfrak{c}$ the cardinality of the set of all real numbers. For a cardinal κ, let κ^+ be the smallest cardinal greater than κ. For each pair of ordinals α, β with $\alpha < \beta$, we write $[\alpha, \beta) = \{\gamma : \alpha \leq \gamma < \beta\}$, $(\alpha, \beta) = \{\gamma : \alpha < \gamma < \beta\}$, $(\alpha, \beta] = \{\gamma : \alpha < \gamma \leq \beta\}$ and $[\alpha, \beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$. As usual, a cardinal is an initial ordinal and an ordinal is the set of smaller ordinals. Every cardinal is often viewed as a space with the usual order topology. Other terms and symbols that we do not define follow [6].

2. Main results

In this section, we show the statements stated in the abstract by using the following example from [3, 14]. We make use of two of the cardinals defined in [4]. Define ${}^{\omega}\omega$ as the set of all functions from ω to itself. For all $f, g \in {}^{\omega}\omega$, we say $f \leq^* g$ if and only if $f(n) \leq g(n)$ for all but finitely many n. The *unbounded number*, denoted by $\mathfrak{b}$, is the smallest cardinality of an unbounded subset of $({}^{\omega}\omega, \leq^*)$. The *dominating number*, denoted by $\mathfrak{d}$, is the smallest cardinality of a cofinal subset of $({}^{\omega}\omega, \leq^*)$. It is not difficult to show that $\omega_1 \leq \mathfrak{b} \leq \mathfrak{d} \leq \mathfrak{c}$ and it is known that $\omega_1 < \mathfrak{b} = \mathfrak{c}$, $\omega_1 < \mathfrak{d} = \mathfrak{c}$ and $\omega_1 \leq \mathfrak{b} < \mathfrak{d} = \mathfrak{c}$ are all consistent with the axioms of ZFC (see [4] for details).

Example 2.1. ([3, 14]) *Let $\mathcal{A}$ be an almost disjoint family of infinite subsets of ω (i.e., the intersection of every two distinct elements of $\mathcal{A}$ is finite) and let $X = \omega \cup \mathcal{A}$ be the Isbell-Mrówka space constructed from $\mathcal{A}$ ([4, 6]). Then*

(1) X is strongly star-Hurewicz if and only if $|\mathcal{A}| < \mathfrak{b}$;

(2) If $|\mathcal{A}| = \mathfrak{c}$, then X is not star-Menger.

Example 2.2. *There exists a Tychonoff pseudocompact, star-Hurewicz space having a regular-closed subspace which is not star-Hurewicz.*

Proof. Let $S_1 = \omega \cup \mathcal{A}$ be the same space X as in the construction of Example 2.1, where $\mathcal{A}$ is a maximal almost disjoint family of infinite subsets of ω with $|\mathcal{A}| = \mathfrak{c}$. Clearly S_1 is Tychonoff pseudocompact. But it is not star-Menger by Example 2.1. Thus it is not star-Hurewicz, since every star-Hurewicz space is star-Menger.

Let $D = \{d_\alpha : \alpha < \mathfrak{c}\}$ be a discrete space of cardinality $\mathfrak{c}$ and let $D^* = D \cup \{d^*\}$ be one-point compactification of D, where $d^* \notin D$.

Let

$$S_2 = (D^* \times [0, \mathfrak{c}^+]) \setminus \{\langle d^*, \mathfrak{c}^+\rangle\}$$

be the subspace of the product space $D^* \times [0, \mathfrak{c}^+]$. Then S_2 is pseudocompact Tychonoff, In fact, it has a countably compact, dense subspace $D^* \times [0, \mathfrak{c}^+)$.

First we show that S_2 is star-Hurewicz. We need only show that S_2 is starcompact, since every starcompact space is star-Hurewicz. For this end, let $\mathcal{U}$ be an open cover of S_2. For each $\alpha < \mathfrak{c}$, there exists $U_\alpha \in \mathcal{U}$ such that $\langle d_\alpha, \mathfrak{c}^+\rangle \in U_\alpha$, we can find $\beta_\alpha < \mathfrak{c}^+$ such that $\{d_\alpha\} \times (\beta_\alpha, \mathfrak{c}^+] \subseteq U_\alpha$. Let $\beta = \sup\{\beta_\alpha : \alpha < \mathfrak{c}\}$, then $\beta < \mathfrak{c}^+$. Let $K = D^* \times \{\beta + 1\}$. Then K is compact and $U_\alpha \cap K \neq \emptyset$ for each $\alpha < \mathfrak{c}$. Since $\mathcal{U}$ covers K, there exists a finite subset $\mathcal{U}'$ of $\mathcal{U}$ such that $K \subseteq \cup\mathcal{U}'$. Hence

$$D \times \{\mathfrak{c}^+\} \subseteq St(\cup\mathcal{U}', \mathcal{U}).$$

On the other hand, since $D^* \times [0, \mathfrak{c}^+)$ is countably compact, then it is strongly starcompact(see [4, 13]), we can find a finite subset $\mathcal{U}''$ of $\mathcal{U}$ such that

$$D^* \times [0, \mathfrak{c}^+) \subseteq St(\cup\mathcal{U}'', \mathcal{U}).$$

If we put $\mathcal{V} = \mathcal{U}' \cup \mathcal{U}''$, then $\mathcal{V}$ is a finite subset of $\mathcal{U}$ such that $S_2 = St(\cup\mathcal{V}, \mathcal{U})$, which shows that S_2 is starcompact.

Let $\pi : \mathcal{A} \to D \times \{\mathfrak{c}^+\}$ be a bijection and let X be the quotient image of the disjoint sum $S_1 \oplus S_2$ obtained by identifying A of S_1 with $\pi(A)$ of S_2 for every $A \in \mathcal{A}$. Let $\varphi : S_1 \oplus S_2 \to X$ be the quotient map. Then X is pseudocompact, since S_1 and S_2 are pseudocompact. It is clear that $\varphi(S_1)$ is a regular-closed subspace of X. However $\varphi(S_1)$ is not star-Hurewicz, since it is homeomorphic to S_1.

Finally we show that X is star-Hurewicz. We need only show that S_2 is starcompact. To this end, let $\mathcal{U}$ be an open cover of X. Since $\varphi(S_2)$ is homeomorphic to S_2 and consequently $\varphi(S_2)$ is starcompact. Thus there exists a finite subset $\mathcal{V}'$ of $\mathcal{U}$ such that

$$\varphi(S_2) \subseteq St(\cup\mathcal{V}', \mathcal{U}).$$

On the other hand, since $\varphi(S_1)$ is homeomorphic to S_1, every infinite subset of $\varphi(\omega)$ has an accumulation point in $\varphi(S_1)$. Hence there exists a finite subset F of $\varphi(\omega)$ such that $\varphi(\omega) \subseteq St(F, \mathcal{U})$. In fact, if $\varphi(\omega) \nsubseteq St(F, \mathcal{U})$ for any finite subset $F \subseteq \varphi(\omega)$, then, by induction, we can define a sequence $\{x_n : n \in \omega\}$ in $\varphi(\omega)$ such that $x_n \notin St(\{x_i : i < n\}, \mathcal{U})$ for each $n \in \omega$. By the property of $\varphi(S_1)$ mentioned above, the sequence $\{x_n : n \in \omega\}$ has a limit point x' in $\varphi(S_1)$. Pick $U \in \mathcal{U}$ such that $x' \in U$. Choose $n < m < \omega$ such that $x_n \in U$ and $x_m \in U$. Then $x_m \in St(\{x_i : i < m\}, \mathcal{U})$, which contradicts the definition of the sequence of $\{x_n : n \in \omega\}$. Thus we can find a finite subset $\mathcal{V}''$ of $\mathcal{U}$ such that

$$\varphi(\omega) \subseteq St(\cup\mathcal{V}'', \mathcal{U}).$$

If we put $\mathcal{V} = \mathcal{V}' \cup \mathcal{V}''$, then $\mathcal{V}$ is a finite subset of $\mathcal{U}$ and $X = St(\cup\mathcal{V}, \mathcal{U})$, which completes the proof. □

For the next example, we need the following lemma.

Lemma 2.3. ([2]) *A space X is star-Hurewicz if and only if for every sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exist finite $\mathcal{O}_n \subseteq \mathcal{U}_n$ $(n \in \mathbb{N})$ such that for every $x \in X$, $St(x, \mathcal{U}_n) \cap (\cup\mathcal{O}_n) \neq \emptyset$ for all but finitely many $n \in \mathbb{N}$.*

Example 2.4. *Assuming $\omega_1 < \mathfrak{b}$, there exists a Tychonoff strongly star-Hurewicz (hence star-Hurewicz) space having a regular-closed G_δ-subspace which is not star-Hurewicz (hence not strongly star-Hurewicz).*

Proof. Let $S_1 = \omega \cup \mathcal{A}$ be the same space X as in the construction of Example 2.1 with $|\mathcal{A}| = \omega_1$. Then S_1 is strongly star-Hurewicz by Example 2.1.

Let $D = \{d_\alpha : \alpha < \omega_1\}$ be a discrete space of cardinality ω_1, and let $D^* = D \cup \{d^*\}$ be one-point compactification of D, where $d^* \notin D$.

Let

$$S_2 = (D^* \times [0, \omega_1]) \setminus \{\langle d^*, \omega_1\rangle\}$$

be the subspace of the product space $D^* \times [0, \omega_1]$.

First we show that S_2 is no star-Hurewicz. For each $\alpha < \omega_1$, let

$$U_\alpha = \{d_\alpha\} \times (\alpha, \omega_1] \text{ and } V_\alpha = D^* \times [0, \alpha).$$

Then

$$U_\alpha \cap U_{\alpha'} = \emptyset \text{ if } \alpha \neq \alpha'$$

and

$$U_\alpha \cap V_{\alpha'} = \emptyset \text{ if } \alpha > \alpha'.$$

For each $n \in \mathbb{N}$, let

$$\mathcal{U}_n = \{U_\alpha : \alpha < \omega_1\} \cup \{V_\alpha : \alpha < \omega_1\}.$$

Then $\mathcal{U}_n$ is an open cover of S_2. Let us consider the sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of S_2. It suffices to show that for any finite $\mathcal{V}_n \subseteq \mathcal{U}_n$ $(n \in \mathbb{N})$ there exists $x \in X$, $St(x, \mathcal{U}_n) \cap (\cup\mathcal{V}_n) = \emptyset$ for all $n \in \mathbb{N}$ by Lemma 2.3. Let $(\mathcal{V}_n : n \in \mathbb{N})$ be any sequence such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$. For each $n \in \mathbb{N}$, since $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$, then there exists $\alpha'_n < \omega_1$ such that

$$U_\alpha \notin \mathcal{V}_n \text{ for each } \alpha > \alpha'_n$$

and there exists $\alpha''_n < \omega_1$ such that

$$V_\alpha \notin \mathcal{V}_n \text{ for each } \alpha > \alpha''_n.$$

Let

$$\alpha_0 = \sup(\{\alpha'_n : n \in \mathbb{N}\} \cup \{\alpha''_n : n \in \mathbb{N}\}).$$

Then $\alpha_0 < \omega_1$. If we pick $\alpha' > \alpha_0$, then $U_{\alpha'} \notin \mathcal{V}_n$ and $U_{\alpha'} \cap (\cup\mathcal{V}_n) = \emptyset$ for each $n \in \mathbb{N}$. Since $U_{\alpha'}$ is the only element of $\mathcal{U}_n$ containing $\langle d_{\alpha'}, \omega_1\rangle$ for each $n \in \mathbb{N}$, then $St(\langle d_{\alpha'}, \omega_1\rangle, \mathcal{U}_n) = U_{\alpha'}$ and $U_{\alpha'} \cap (\cup\mathcal{V}_n) = \emptyset$ for each $n \in \mathbb{N}$. Thus we complete the proof.

We assume $S_1 \cap S_2 = \emptyset$. Let $\pi : \mathcal{A} \to D \times \{\omega_1\}$ be a bijection and let X be the quotient image of the disjoint sum $S_1 \oplus S_2$ obtained by identifying A of S_1 with $\pi(A)$ of S_2 for every $A \in \mathcal{A}$. Let $\varphi : S_1 \oplus S_2 \to X$ be the quotient map. Then $\varphi(S_2)$ is a regular-closed subspace of X. For each $n \in \omega$, let $F_n = \{m \in \omega : m \leq n\}$. For each $n \in \omega$, let

$$U_n = X \setminus \varphi(F_n).$$

Then U_n is open in X and $\varphi(S_2) = \bigcap_{n\in\mathbb{N}} U_n$. Thus $\varphi(S_2)$ is a regular-closed G_δ-subspace of X. However $\varphi(S_2)$ is not star-Hurewicz, since it is homeomorphic to S_2.

Finally we show that X is strongly star-Hurewicz. To this end, let $\{\mathcal{U}_n : n \in \mathbb{N}\}$ be a sequence of open covers of X. Since $\varphi(S_1)$ is homeomorphic to S_1, $\varphi(S_1)$ is strongly star-Hurewicz. Thus there exists a sequence $\{F'_n : n \in \mathbb{N}\}$ of finite subsets of $\varphi(S_1)$ such that for each $x \in \varphi(S_1)$, $x \in St(F'_n, \mathcal{U}_n)$ for all but finitely many n. On the other hand, since $\varphi(D^* \times \omega_1)$ is homeomorphic to $D^* \times \omega_1$, then $\varphi(D^* \times \omega_1)$ is countably compact, thus for each $n \in \mathbb{N}$, there exists a finite subset F''_n of $\varphi(D^* \times \omega_1)$ such that

$$\varphi(D^* \times \omega_1) \subseteq St(F''_n, \mathcal{U}_n),$$

since every countably compact space is strongly starcompact (see [4, 13]). For each $n \in \mathbb{N}$, if we put $F_n = F'_n \cup F''_n$. Then the sequence $\{F_n : n \in \mathbb{N}\}$ witnesses for $\{\mathcal{U}_n : n \in \mathbb{N}\}$ that X is strongly star-Hurewicz. Thus we complete the proof. □

Remark 2.5. The authors do not know if there exists an example in ZFC of a regular-closed subspace (or a regular-closed G_δ-subspace) of a Tychonoff strongly star-Hurewicz space that is not strongly star-Hurewicz, and if there exists a Tychonoff star-Hurewicz space having a regular-closed G_δ-subspace which is not star-Hurewicz.

Next we give a positive result.

Theorem 2.6. *An open F_σ-subset of a strongly star-Hurewicz space is strongly star-Hurewicz.*

Proof. Let X be a strongly star-Hurewicz space and let $Y = \cup\{H_n : n \in \mathbb{N}\}$ be an open F_σ-subset of X, where the H_n is closed in X for each $n \in \mathbb{N}$. Without loss of generality, we can assume that $H_n \subseteq H_{n+1}$ for each $n \in \mathbb{N}$. To show that Y is strongly star-Hurewicz. Let $\{\mathcal{U}_n : n \in \mathbb{N}\}$ be a sequence of open covers of Y, we have to find a sequence $\{F_n : n \in \mathbb{N}\}$ of finite subsets of Y such that for each $y \in Y$, $y \in St(F_n, \mathcal{U}_n)$ for all but finitely many n. For each $n \in \mathbb{N}$, let

$$\mathcal{V}_n = \mathcal{U}_n \cup \{X \setminus H_n\}.$$

Then $\{\mathcal{V}_n : n \in \mathbb{N}\}$ is a sequence of open covers of X, there exists a sequence $\{F'_n : n \in \mathbb{N}\}$ of finite subsets of X such that for each $x \in X$, $x \in St(F'_n, \mathcal{V}_n)$ for all but finitely many n, since X is strongly star-Hurewicz. For each $n \in \mathbb{N}$, let $F_n = F'_n \cap Y$. Thus $\{F_n : n \in \mathbb{N}\}$ is a sequence of finite subsets of Y. For each $y \in Y$, there exists $n_0 \in N$ such that $y \in H_n$ and $y \in St(F'_n, \mathcal{V}_n)$ for each $n > n_0$. Hence $y \in St(F_n, \mathcal{U}_n)$ for $n > n_0$, which shows that Y is strongly star-Hurewicz. □

A *cozero-set* in a space X is a set of form $f^{-1}(R \setminus \{0\})$ for some real-valued continuous function f on X. Since a cozero-set is an open F_σ-set, we have the following corollary:

Corollary 2.7. *A cozero-set of a strongly star-Hurewicz space is strongly star-Hure-wicz.*

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