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Remarks on strongly star-Hurewicz spaces

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Abstract. A space X is *strongly star-Hurewicz* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(A_n : n \in N)$ of finite subsets of X such that for each $x \in X$, $x \in St(A_n, \mathcal{U}_n)$ for all but finitely many n. In this paper, we continue to investigate topological properties of strongly star-Hurewicz spaces.

1. Introduction

By a space, we mean a topological space. In this section, we give definitions of terms which are used in this paper. Let $\mathbb{N}$ denote the set of positive integers. Let X be a space and $\mathcal{U}$ be a collection of subsets of X. For $A \subseteq X$, let $St(A, \mathcal{U}) = \cup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. As usual, we write $St(x, \mathcal{U})$ instead of $St(\{x\}, \mathcal{U})$.

Let $\mathcal{A}$ and $\mathcal{B}$ be collections of subsets of a space X. Then the symbol $S_1(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(U_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $U_n \in \mathcal{U}_n$ and $\{U_n : n \in \mathbb{N}\}$ is an element of $\mathcal{B}$. The symbol $S_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is an element of $\mathcal{B}$ (see [9,15]).

Kočinac [10, 11, 12] introduced star selection hypothesis similar to the previous ones:

(A) The symbol $S^*_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}}\{St(V, \mathcal{U}_n) : V \in \mathcal{V}_n\}$ is an element of $\mathcal{B}$.

(B) The symbol $SS^*_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(K_n : n \in N)$ of finite subsets of X such that $\{St(K_n, \mathcal{U}_n) : n \in \mathbb{N}\} \in \mathcal{B}$.

Let $\mathcal{O}$ denote the collection of all open covers of X.

Definition 1.1. ([10, 11, 12]) A space X is said to be *star-Menger* (*strongly star-Menger*) if it satisfies the selection hypothesis $S^*_{fin}(\mathcal{O}, \mathcal{O})$ (resp., $SS^*_{fin}(\mathcal{O}, \mathcal{O})$).

In 1925 in [7] (see also [8]), Hurewicz introduced the Hurewicz covering property for a space X in the following way:

H: For each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(\mathcal{V}_n : n \in N)$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $x \in \bigcup \mathcal{V}_n$ for all but finitely many n.

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In [1], two star versions of the Hurewicz property were introduced as follows:

SH: A space X satisfies the *star-Hurewicz property* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(\mathcal{V}_n : n \in N)$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $x \in St(\cup\mathcal{V}_n, \mathcal{U}_n)$ for all but finitely many n.

SSH: A space X satisfies the *strongly star-Hurewicz property* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(A_n : n \in N)$ of finite subsets of X such that for each $x \in X$, $x \in St(A_n, \mathcal{U}_n)$ for all but finitely many n.

Definition 1.2. ([1]) A space X is said to be *strongly star-Hurewicz* (*star-Hurewicz*) if it satisfies the selection hypothesis strongly star-Hurewicz property (resp., star-Hurewicz property).

From the above definitions, we have the following diagram.

$$\begin{array}{ccccc} \textit{compact} & \longrightarrow & \textit{strongly star-Hurewicz} & \longrightarrow & \textit{strongly star-Menger} \\ & & \downarrow & & \downarrow \\ & & \textit{star-Hurewicz} & \longrightarrow & \textit{star-Menger} \end{array}$$

On the study of star-Hurewicz spaces, the readers can see the references [1, 2, 3, 12, 16]. The purpose of this paper is to continue to investigate topological properties of strongly star-Hurewicz spaces.

Throughout this paper, let ω denote the first infinite cardinal, ω_1 the first uncountable cardinal and $\mathfrak{c}$ the cardinality of the set of all real numbers. For each pair of ordinals α, β with $\alpha < \beta$, we write $[\alpha, \beta) = \{\gamma : \alpha \leq \gamma < \beta\}$, $(\alpha, \beta) = \{\gamma : \alpha < \gamma < \beta\}$, $(\alpha, \beta] = \{\gamma : \alpha < \gamma \leq \beta\}$ and $[\alpha, \beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$. As usual, a cardinal is an initial ordinal and an ordinal is the set of smaller ordinals. Every cardinal is often viewed as a space with the usual order topology. Other terms and symbols that we do not define follow [6].

2. Main results

In this section, we study the topological properties of strongly star-Hurewicz spaces.

Theorem 2.1. *A continuous image of a strongly star-Hurewicz space is strongly star-Hurewicz.*

Proof. Let $f : X \to Y$ be a continuous mapping from a strongly star-Hurewicz space X onto a space Y. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of open covers of Y. For each $n \in \mathbb{N}$, let $\mathcal{V}_n = \{f^{-1}(U) : U \in \mathcal{U}_n\}$. Then $(\mathcal{V}_n : n \in \mathbb{N})$ is a sequence of open covers of X. Since X is strongly star-Hurewicz, there exists a sequence $(A_n : n \in N)$ of finite subsets of X such that for each $x \in X$, $x \in St(A_n, \mathcal{U}_n)$ for all but finitely many n. Thus $(f(A_n) : n \in \mathbb{N})$ is a sequence of finite subsets of Y such that for each $y \in Y$, $y \in St(f(A_n), \mathcal{U}_n)$ for all but finitely many n. In fact, let $y \in Y$. Then there is $x \in X$ such that $f(x) = y$. Hence $x \in St(A_n, \mathcal{V}_n)$ for all but finitely many n. Thus $y = f(x) \in St(f(A_n), \{f(U) : U \in \mathcal{V}_n\}) = St(f(A_n), \mathcal{U}_n)$ for all but finitely many n, which shows that Y is strongly star-Hurewicz. □

Next we turn to consider preimages. We shall give a consistent example showing that the preimage of a strongly star-Hurewicz space under a closed 2-to-1 continuous map need not be strongly star-Hurewicz by using the following example from [3]. We make use of one of the cardinals defined in [5]. Define ${}^\omega\omega$ as the set of all functions from ω to itself. For all $f, g \in {}^\omega\omega$, we say $f \leq^* g$ if and only if $f(n) \leq g(n)$ for all but finitely many n. The unbounding number, denoted by $\mathfrak{b}$, is the smallest cardinality of an unbounded subset of $({}^\omega\omega, \leq^*)$. It is not difficult to show that $\omega_1 \leq \mathfrak{b} \leq \mathfrak{c}$. We also use the following example from [3].

Example 2.2. ([3]) *Let $\mathcal{A}$ be an almost disjoint family of infinite subsets of ω (i.e., the intersection of every two distinct elements of $\mathcal{A}$ is finite) and Let $X = \omega \cup \mathcal{A}$ be the Isbell-Mrówka space constructed from $\mathcal{A}$([4],[6]). Then X is strongly star-Hurewicz if and only if $|\mathcal{A}| < \mathfrak{b}$.*

For a space X, recall that the Alexandorff duplicate $A(X)$ of X is constructed in the following way: The underlying set $A(X)$ is $X \times \{0, 1\}$; each point of $X \times \{1\}$ is isolated and a basic neighborhood of $\langle x, 0\rangle \in X \times \{0\}$ is a set of the form $(U \times \{0\}) \cup ((U \times \{1\}) \setminus \{\langle x, 1\rangle\})$, where U is a neighborhood of x in X.

Example 2.3. *Assuming $\mathfrak{b} = \mathfrak{c}$ and $\neg CH$, there exists a closed 2-to-1 continuous map $f : X \to Y$ such that Y is a strongly star-Hurewicz space, but X is not strongly star-Hurewicz.*

Proof. Let $Y = \omega \cup \mathcal{A}$ be the space X of Example 2.2 with $|\mathcal{A}| = \omega_1$. Then Y is strongly star-Hurewicz by Example 2.2.

Let $X = A(Y)$. Then X is not strongly star-Hurewicz. In fact, since $\mathcal{A}$ is a discrete closed subset of Y with $|\mathcal{A}| = \omega_1$, the set $\mathcal{A} \times \{1\}$ is an open and closed subset of $A(Y)$ with $|\mathcal{A} \times \{1\}| = \omega_1$, and each point $\langle a, 1\rangle$ is isolated for each $a \in \mathcal{A}$. Hence X is not strongly star-Hurewicz, since every open and closed subset of a strongly star-Hurewicz space is strongly star-Hurewicz and $\mathcal{A} \times \{1\}$ is not strongly star-Hurewicz.

Let $f : X \to Y$ be the projection. Then f is a closed 2-to-1 continuous map, which completes the proof. □

From the proof of Example 2.3, it is not difficult to show the following result.

Theorem 2.4. *If X is a T_1-space and $A(X)$ is a strongly star-Hurewicz space, then $e(X) < \omega_1$.*

Proof. Suppose that $e(X) \geq \omega_1$. Then there exists a discrete closed subset B of X such that $|B| \geq \omega_1$. Hence $B \times \{1\}$ is a open and closed subset of $A(X)$ and every point of $B \times \{1\}$ is an isolated point. Thus $A(X)$ is not strongly star-Hurewicz, since every open and closed subset of a strongly star-Hurewicz space is strongly star-Hurewicz and $B \times \{1\}$ is not strongly star-Hurewicz. □

Remark 2.5. The author does not know if the Alexandorff duplicate $A(X)$ of a strongly star-Hurewicz space X with $e(X) < \omega_1$ is strongly star-Hurewicz.

Now we give a positive result:

Theorem 2.6. *Let f be an open and closed, finite-to-one continuous map from a space X to a strongly star-Hurewicz space Y. Then X is strongly star-Hurewicz.*

Proof. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of open covers of X and let $y \in Y$. For each $n \in \mathbb{N}$, since $f^{-1}(y)$ is finite, there exists a finite subcollection $\mathcal{U}_{n_y}$ of $\mathcal{U}_n$ such that $f^{-1}(y) \subseteq \bigcup \mathcal{U}_{n_y}$ and $U \cap f^{-1}(y) \neq \emptyset$ for each $U \in \mathcal{U}_{n_y}$. Since f is closed, there exists an open neighborhood $V_{n_y}$of y in Y such that $f^{-1}(V_{n_y}) \subseteq \bigcup\{U : U \in \mathcal{U}_{n_y}\}$. Since f is open, we can assume that

$$V_{n_y} \subseteq \bigcap\{f(U) : U \in \mathcal{U}_{n_y}\}. \tag{1}$$

For each $n \in \mathbb{N}$, taking such open set V_{n_y} for each $y \in Y$, we have an open cover $\mathcal{V}_n = \{V_{n_y} : y \in Y\}$ of Y. Thus $(\mathcal{V}_n : n \in \mathbb{N})$ is a sequence of open covers of Y, so that there exists a sequence $(A_n : n \in \mathbb{N})$ of finite subsets of Y such that for each $y \in Y$, $y \in St(A_n, \mathcal{V}_n)$ for all but finitely many n, since Y is strongly star-Hurewicz. Since f is finite-to-one, the sequence $(f^{-1}(A_n) : n \in N)$ is a sequence of finite subsets of X. We show that for each $x \in X$, $x \in St(f^{-1}(A_n), \mathcal{U}_n)$ for all but finitely many n. Let $x \in X$. Then $f(x) \in St(A_n, \mathcal{V}_n)$ for all but finitely many n. If $f(x) \in St(A_n, \mathcal{V}_n)$, then there exists $y \in Y$ such that $f(x) \in V_{n_y}$ and $V_{n_y} \cap A_n \neq \emptyset$. Since

$$x \in f^{-1}(V_{n_y}) \subseteq \bigcup \mathcal{U}_{n_y},$$

we can choose $U \in \mathcal{U}_{n_y}$ with $x \in U$. Then $V_{n_y} \subseteq f(U)$ by (1). Hence $U \cap f^{-1}(A_n) \neq \emptyset$. Therefore $x \in St(f^{-1}(A_n), \mathcal{U}_n)$. Consequently $x \in St(f^{-1}(A_n), \mathcal{U}_n)$ for all but finitely many n, which shows that X is strongly star-Hurewicz. □

For strongly star-Hurewicz spaces, we give a consistent example showing that the product of a strongly star-Hurewicz space and a compact space need not be strongly star-Hurewicz. For the example, we need the following Lemmas.

Lemma 2.7. ([2]) *A space X is a strongly star-Hurewicz space if and only if for every sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(A_n : n \in \mathbb{N})$ of finite subsets of X such that for every $x \in X$, $St(x, \mathcal{U}_n) \cap A_n \neq \emptyset$ for all but finitely many $n \in \mathbb{N}$.*

Example 2.8. *Assuming $\mathfrak{b} = \mathfrak{c}$ and $\neg CH$, there exist a strongly star-Hurewicz space X and a compact space Y such that $X \times Y$ is not strongly star-Hurewicz.*

Proof. Assuming $\mathfrak{b} = \mathfrak{c}$ and $\neg CH$, let $X = \omega \cup \mathcal{A}$ be the same space as Example 2.2 with $|\mathcal{A}| = \omega_1$. Then X is strongly star-Hurewicz by Example 2.2. Let $D = \{d_\alpha : \alpha < \omega_1\}$ be the discrete space of cardinality ω_1 and let $Y = D \cup \{d^*\}$ be the one-point compactification of D. Then Y is strongly star-Hurewicz, since Y is compact. Let us show that $X \times Y$ is not strongly star-Hurewicz. Since $|\mathcal{A}| = \omega_1$, we can enumerate $\mathcal{A}$ as $\{a_\alpha : \alpha < \omega_1\}$. For each $n \in \mathbb{N}$, let

$$\mathcal{U}_n = \{(\{a_\alpha\} \cup a_\alpha) \times (Y \setminus \{d_\alpha\}) : \alpha < \omega_1\} \cup \{X \times \{d_\alpha\} : \alpha < \omega_1\} \cup \{\omega \times Y\}.$$

Then all the $\mathcal{U}_n$'s are the same and $\mathcal{U}_n$ is an open cover of $X \times Y$ for each $n \in \mathbb{N}$. Let us consider the sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of $X \times Y$. It suffices to show that for any sequence $(A_n : n \in \mathbb{N})$ of finite subsets of $X \times Y$ there exists a point $a \in X \times Y$ such that $St(a, \mathcal{U}_n) \cap A_n \neq \emptyset$ for all $n \in \mathbb{N}$ by Lemma 2.7. Let $(A_n : n \in \mathbb{N})$ be any sequence of finite subsets of $X \times Y$. For each $n \in \mathbb{N}$, since A_n is finite, there exists $\alpha_n < \omega_1$ such that

$$A_n \cap (X \times \{d_\alpha\}) = \emptyset \text{ for each } \alpha > \alpha_n.$$

Let $\beta = \sup\{\alpha_n : n \in \mathbb{N}\}$. Then $\beta < \omega_1$ and

$$(\bigcup_{n \in \mathbb{N}} A_n) \cap (X \times \{d_\alpha\}) = \emptyset \text{ for each } \alpha > \beta.$$

Let $\alpha > \beta$. Since $X \times \{d_\alpha\}$ is the only element of $\mathcal{U}_n$ containing the point $\langle a_\alpha, d_\alpha \rangle$ for each $n \in \mathbb{N}$, $St(\langle a_\alpha, d_\alpha \rangle, \mathcal{U}_n) = X \times \{d_\alpha\}$ for each $n \in \mathbb{N}$. Thus $(\bigcup_{n \in \mathbb{N}} A_n) \cap (X \times \{d_\alpha\}) = \emptyset$, which shows that $X \times Y$ is not strongly star-Hurewicz. □

Remark 2.9. Assuming $\mathfrak{b} = \mathfrak{c}$ and $\neg CH$, Example 2.8 shows that the preimage of a strongly star-Hurewicz space under an open perfect map need not be strongly star-Hurewicz, and also shows that Theorem 2.6 fails to be true if "open and closed, finite-to-one" is replaced by "open perfect". The author does not know if in ZFC, there exist a strongly star-Hurewicz space X and a compact space Y such that $X \times Y$ is not strongly star-Hurewicz.

However, the product of two strongly star-Hurewicz spaces need not be strongly star-Hurewicz. In fact, the following well-known example shows that the product of two countably compact (hence strongly star-Hurewicz) spaces need not be strongly star-Hurewicz. Here we give the proof roughly for the sake of completeness.

Example 2.10. *There exist two Tychonoff countably compact (hence strongly star-Hurewicz) spaces X and Y such that $X \times Y$ is not strongly star-Hurewic.*

Proof. Let D be a discrete space of cardinality $\mathfrak{c}$. We can define $X = \bigcup_{\alpha<\omega_1} E_\alpha$ and $Y = \bigcup_{\alpha<\omega_1} F_\alpha$, where E_α and F_α are the subsets of βD which are defined inductively so as to satisfy the following conditions (1),(2) and (3):

(1) $E_\alpha \cap F_\beta = D$ if $\alpha \neq \beta$;

(2) $|E_\alpha| \leq \mathfrak{c}$ and $|F_\beta| \leq \mathfrak{c}$;

(3) every infinite subset of E_α (resp., F_α) has an accumulation point in $E_{\alpha+1}$ (resp., $F_{\alpha+1}$).

These sets E_α and F_α are well-defined since every infinite closed set in βD has cardinality at least $2^{\mathfrak{c}}$ (see [14]). Then $X \times Y$ is not strongly star-Hurewicz. In fact, the diagonal $\{\langle d, d \rangle : d \in D\}$ is an open and closed subset of $X \times Y$ with cardinality $\mathfrak{c}$ and $\{\langle d, d \rangle\}$ is isolated for each $d \in D$. Thus $X \times Y$ is not strongly star-Hurewicz, since the open and closed subsets of strongly star-Hurewicz spaces are strongly star-Hurewicz and the diagonal $\{\langle d, d \rangle : d \in D\}$ is not strongly star-Hurewicz. □

In [4, Example 3.3.3], van Douwen et al. gave an example showing that there exist a countably compact (and hence strongly star-Hurewicz) space X and a Lindelöf space Y such that $X \times Y$ is not strongly star-Lindelöf. Therefore, this example shows that the product of a strongly star-Hurewicz space X and a Lindelöf space Y need not be strongly star-Hurewicz, since every strongly star-Hurewicz space is strongly star-Lindelöf.

Next we give a condition that implies Lindelöfness. Recall that a space X is *meta-Lindelöf* if every open cover $\mathcal{U}$ of X has a point countable open refinement.

Theorem 2.11. *Every meta-Lindelöf strongly star-Hurewicz space is Lindelöf.*

Proof. Let X be a meta-Lindelöf strongly star-Hurewicz space and $\mathcal{U}$ be an open cover of X. Then there exists a point countable open refinement $\mathcal{V}$ of $\mathcal{U}$. Since X is strongly star-Hurewicz, there exists a sequence $(A_n : n \in N)$ of finite subsets of X such that for each $x \in X$, $x \in St(A_n, \mathcal{V})$ for all but finitely many n.

For each $n \in \mathbb{N}$, let

$$\mathcal{V}_n = \{V \in \mathcal{V} : V \cap A_n \neq \emptyset\}.$$

Then $\mathcal{V}_n$ is a countable subset of $\mathcal{V}$. Let $\mathcal{W} = \bigcup_{n\in\mathbb{N}} \mathcal{V}_n$. Then $\mathcal{W}$ is a countable open cover of X. For each $V \in \mathcal{W}$, choose $U_V \in \mathcal{U}$ such that $V \subseteq U_V$. Then $\{U_V : V \in \mathcal{W}\}$ is a countable subcover of $\mathcal{U}$, which shows that X is Lindelöf. Thus we complete the proof. □

Recall that a space X is *para-Lindelöf* if every open cover $\mathcal{U}$ of X has a locally countable open refinement. Since every para-Lindelöf space is meta-Lindelöf, the following Corollary follows from Theorem 2.11.

Corollary 2.12. *A para-Lindelöf strongly star-Hurewicz space X is Lindelöf.*

Since every Lindelöf space is meta-Lindelöf and para-Lindelöf, the following Corollaries follows from Theorem 2.11.

Corollary 2.13. *Let X be a strongly star-Hurewicz space. Then X is meta-Lindelöf if and only if X is Lindelöf.*

Corollary 2.14. *Let X be a strongly star-Hurewicz space. Then X is para-Lindelöf if and only if X is Lindelöf.*

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