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On nondifferentiable minimax fractional programming involving higher order generalized convexity

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Abstract. In this article, we focus our study on a nondifferentiable minimax fractional programming problem and establish weak, strong and strict converse duality theorems under generalized higher order $(\mathcal{F}, \alpha, \rho, d)$-Type I assumptions. Our results extend and unify some of the known results in the literature.

1. Introduction

Despite optimization problems having seen present in mathematics since very early times, optimization theory has been established as an autonomous field only in relatively recent times. The optimization problems in which the objective function is a ratio of two functions are commonly known as fractional programming problems. In past few years, many authors have shown interest in the field of minimax fractional programming problems due to the fact that it has wide varieties of applications, e.g., in design of electronic circuits, portfolio selection problems, engineering design etc., see [9, 11, 22, 23]. One major context is the zero sum games, where the objective of the first player is to minimize the amount given to the other player, and the objective of the second player is to maximize this amount. Schmittendorf [21] first developed necessary and sufficient optimality conditions for a minimax programming problem. The necessary conditions in [21] were used by Tanimoto [24] to formulate a dual problem and to discuss the duality results, which were extended to fractional analogue of problem considered in [21, 24] by several authors [4, 10, 16–19, 25, 26].

Gupta and Danger [12] considered two different types of second order duals for a nondifferentiable minimax fractional programming problem and established duality theorems under (F, ρ)-convexity. Liu [19] proposed the second order duality theorems for a minimax programming problem under generalized second order B-invex functions. Hu and Jian [13] formulated two types of second order duals in minimax fractional programming by introducing an additional vector r and derived the weak, strong and converse duality theorems under η-bonvexity assumptions. Mishra and Rueda [20], Ahmad [2], Ahmad et al. [6], and Husain et al. [7] discussed the second order duality results for the following nondifferentiable minimax programming problem:

(P) Minimize $\sup_{y \in Y} f(x, y) + (x^T Bx)^{1/2}$,

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subject to $h(x) \leq 0,\ x \in R^n$,

where Y is a compact subset of R^l, $f(.,.) : R^n \times R^l \to R$ and $h(.) : R^n \to R^m$ are twice differentiable functions, B is an $n \times n$ positive semidefinite symmetric matrices. Ahmad et al. [7] formulated a unified higher order dual of (P) and established weak, strong and strict converse duality theorems under higher order (F, α, ρ, d)−Type I assumptions which is an extension of the second order duality results of Mishra and Rueda [20] to a class of higher-order duality. In order to unify the symmetric dual formulations in the literature, Ahmad [3] formulated for a nondifferentiable multiobjective programming problem, where every component of the objective function contains a term involving the support function of a compact convex set. Ahmad [1] studied higher order dual for a nondifferentiable minimax fractional programming problem under generalized higher order η-convexity assumptions and derived weak, strong and strict converse duality theorems. Recently, Jayswal and Stancu-Minasian [15] obtained higher order duality results for (P) in order to relate the efficient solutions of primal and dual problems.

The present paper is constructed on the following lines: In section 2, we discuss the problem and give some definitions needed in the sequel of the paper. In section 3, we formulate a higher order dual for a nondifferentiable minimax fractional programming problem and establish weak, strong and strict converse duality theorems under generalized higher order $(\mathcal{F}, \alpha, \rho, d)$−Type I assumptions followed by conclusion at the last. This paper generalize the several results appeared in the literature [5, 7, 8, 13–15, 19] and references therein.

2. Preliminaries

The problem to be considered in the present analysis is the following nondifferentiable minimax fractional problem:

(NP) Minimize $\sup_{y\in Y} \frac{f(x,y)+(x^TBx)^{1/2}}{g(x,y)-(x^TCx)^{1/2}}$,

subject to $h(x) \leq 0,\ x \in R^n$,

where Y is a compact subset of R^l, $f(.,.),\ g(.,.) : R^n \times R^l \to R$ and $h(.) : R^n \to R^m$ are differentiable functions, B and C are $n \times n$ positive semidefinite symmetric matrices. It is assumed that for each (x, y) in $R^n \times R^m$, $f(x) + (x^TBx)^{1/2} \geq 0$ and $g(x) - (x^TCx)^{1/2} > 0$.

Let $\mathcal{X} = \{x \in R^n : h(x) \leq 0\}$ denote the set of all feasible solutions of (NP). Any point $x \in \mathcal{X}$ is called the feasible point of (NP). For each $(x, y) \in R^n \times R^l$, we define

$$\phi(x, y) = \frac{f(x, y) + (x^TBx)^{1/2}}{g(x, y) - (x^TCx)^{1/2}},$$

such that for each $(x, y) \in R^n \times R^l$, we have

$$f(x, y) + (x^TBx)^{1/2} \geq 0 \text{ and } g(x, y) - (x^TCx)^{1/2} > 0.$$

For each $x \in \mathcal{X}$, we define

$$J(x) = \{j \in J : h_j(x) = 0\},$$

where

$J = \{1, 2, \cdots, m\}$,

$$Y(x) = \left\{ y \in Y : \frac{f(x,y)+(x^TBx)^{1/2}}{g(x,y)-(x^TCx)^{1/2}} = \sup_{z\in Y} \frac{f(x,z)+(x^TBx)^{1/2}}{g(x,z)-(x^TCx)^{1/2}} \right\}.$$

$$S(x) = \Big\{ (s, t, \tilde{y}) \in N \times R^s_+ \times R^{ls} : 1 \leq s \leq n + 1,\ t = (t_1, t_2, \cdots, t_s) \in R^s_+$$
$$\text{with } \sum_{i=1}^{s} t_i = 1,\ \tilde{y} = (\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_s) \text{ with } \bar{y}_i \in Y(x)(i = 1, 2, \cdots, s) \Big\}.$$

Since f and g are continuously differentiable and Y is compact in R^l, it follows that for each $x^* \in \mathcal{X}$, $Y(x^*) \neq \emptyset$, and for any $\bar{y}_i \in Y(x^*)$, we have a positive constant

$$\lambda_\circ = \phi(x^*, \bar{y}_i) = \frac{f(x^*, \bar{y}_i) + (x^{*T}Bx^*)^{1/2}}{g(x^*, \bar{y}_i) - (x^{*T}Cx^*)^{1/2}}.$$

Lemma 2.1. (Generalized Schwartz Inequality). Let A be a positive semidefinite matrix of order n. Then, for all $x,\ w \in R^n$,

$$x^T Aw \leq (x^T Ax)^{1/2}(w^T Aw)^{1/2}. \tag{1}$$

We observe that equality holds if $Ax = \xi Aw$ for some $\xi \geq 0$. Evidently, if $(w^T Aw)^{1/2} \leq 1$, we have

$$x^T Aw \leq (x^T Ax)^{1/2}.$$

Let $\mathcal{F}$ be a sublinear functional and let $d(\cdot,\cdot) : R^n \times R^n \to R$. Let $\rho = (\rho^1, \rho^2)$, where $\rho^1 = (\rho^1_1, \rho^1_2, \ldots, \rho^1_s) \in R^s$ and $\rho^2 = (\rho^2_1, \rho^2_2, \ldots, \rho^2_m) \in R^m$ and let $\alpha = (\alpha^1, \alpha^2) : R^n \times R^n \to R_+ \setminus \{0\}$. Let $\phi(\cdot,\cdot) : R^n \times Y \to R$ and $h(\cdot) : R^n \to R^m$ be continuously differentiable functions at $\bar{x} \in R^n$.

Definition 2.1. A functional $\mathcal{F} : R^n \times R^n \times R^n \mapsto R$ is said to be sublinear in its third argument, if for all $x,\ \bar{x} \in R^n$

(*i*) $\mathcal{F}(x, \bar{x}; a + b) \leq \mathcal{F}(x, \bar{x}; a) + \mathcal{F}(x, \bar{x}; b),\ \forall\ a,\ b \in R^n$;
(*ii*) $\mathcal{F}(x, \bar{x}; \beta a) = \beta \mathcal{F}(x, \bar{x}; a),\ \forall\ \beta \in R,\ \beta \geq 0,$ and $\forall\ a \in R^n$.

From (*ii*), it is clear that $\mathcal{F}(x, \bar{x}; 0) = 0$.

Definition 2.2. [7] For each $j \in J$, (ϕ, h_j) is said to be higher-order $(\mathcal{F}, \alpha, \rho, d)$-pseudoquasi-Type I at $\bar{x} \in R^n$, if for all $x \in \mathcal{X}$, $p \in R^n$ and $\bar{y}_i \in Y(x)$,

$$\phi(x, \bar{y}_i) < \phi(\bar{x}, \bar{y}_i) + K(\bar{x}, \bar{y}_i, p) - p^T \nabla_p K(\bar{x}, \bar{y}_i, p) \ \Rightarrow\ \mathcal{F}(x, \bar{x}; \alpha^1(x, \bar{x})(\nabla_p K(\bar{x}, \bar{y}_i, p))) < -\rho^1_i d^2(x, \bar{x}),\ i = 1, 2, \ldots, s,$$

$$-[h_j(\bar{x}) + H_j(\bar{x}, p) - p^T \nabla_p H_j(\bar{x}, p)] \leq 0 \ \Rightarrow\ \mathcal{F}(x, \bar{x}; \alpha^2(x, \bar{x})(\nabla_p H_j(\bar{x}, p))) \leq -\rho^2_j d^2(x, \bar{x}).$$

In the above definition, if

$$\mathcal{F}(x, \bar{x}; \alpha^1(x, \bar{x})(\nabla_p K(\bar{x}, \bar{y}_i, p))) \geq -\rho^1_i d^2(x, \bar{x}) \ \Rightarrow\ \phi(x, \bar{y}_i) > \phi(\bar{x}, \bar{y}_i) + K(\bar{x}, \bar{y}_i, p) - p^T \nabla_p K(\bar{x}, \bar{y}_i, p),\ i = 1, 2, \ldots, s,$$

then we say that (ϕ, g_j) is higher-order $(\mathcal{F}, \alpha, \rho, d)$-strictly pseudoquasi-Type I at $\bar{x}$.

Assuming the functions f, g and h in problem (NP) continuously differentiable with respect to $x \in R^n$, Lai et al. [19] derived the following necessary conditions for optimality of (NP).

Theorem 2.1 (Necessary Conditions). If x^* is a solution of (NP) satisfying $x^{*T}Bx^* > 0$, $x^{*T}Cx^* > 0$, and $\nabla h_j(x^*)$, $j \in J(x^*)$ are linearly independent, then there exist $(s, t^*, \bar{y}) \in S(x^*)$; $\lambda \in R_+$; $w,\ v \in R^n$ and $\mu^* \in R^m_+$ such that

$$\sum_{i=1}^{s} t^*_i \left\{ \nabla f(x^*, \bar{y}_i) + Bw - \lambda_0 (\nabla g(x^*, \bar{y}_i) - Cv) \right\} + \nabla \sum_{j=1}^{m} \mu^*_j h_j(x^*) = 0,$$

$$f(x^*, \bar{y}_i) + (x^{*T}Bx^*)^{\frac{1}{2}} - \lambda_0 \left(g(x^*, \bar{y}_i) - (x^{*T}Cx^*)^{\frac{1}{2}} \right) = 0,\ i = 1, 2, \cdots, s,$$

$$\sum_{j=1}^{m} \mu^*_j h_j(x^*) = 0,$$

$t_i^* \geq 0 \ (i = 1, 2, \cdots, s), \sum_{i=1}^{s} t_i^* = 1,$

$$\begin{cases} w^T B w \leq 1, \ v^T C v \leq 1, \\ (x^{*T} B x^*)^{1/2} = x^{*T} B w, \ (x^{*T} C x^*)^{1/2} = x^{*T} C v. \end{cases}$$

In the above theorem, both matrices B and C are positive semidefinite at the solution x^*. If either $x^{*T}Bx^*$ or $x^{*T}Cx^*$ is zero, then the functions involved in the objective function of the problem (NP) are not differentiable. To derive these necessary conditions under this situation, for $(s, t^*, \bar{y}) \in S(x^*)$, we define $U_{\bar{y}}(x^*) = \{u \in R^n : u^T \nabla h_j(x^*) \leq 0, j \in J(x^*)\}$ satisfying one of the following conditions:

(i) $x^{*T}Bx^* > 0, \ x^{*T}Cx^* = 0$

$$\Rightarrow u^T \left(\sum_{i=1}^{s} t_i \{ \nabla f(x^*) + \frac{Bx^*}{(x^{*T}Bx^*)^{1/2}} - \lambda_\circ \nabla g(x^*) \} \right) + (u^T(\lambda_\circ^2 C)u)^{1/2} < 0,$$

(ii) $x^{*T}Bx^* = 0, \ x^{*T}Cx^* > 0$

$$\Rightarrow u^T \left(\sum_{i=1}^{s} t_i \{ \nabla f(x^*) - \lambda_\circ (\nabla g(x^*) - \frac{Cx^*}{(x^{*T}Cx^*)^{1/2}}) \} \right) + (u^T B u)^{1/2} < 0,$$

(iii) $x^{*T}Bx^* = 0, \ x^{*T}Cx^* = 0$

$$\Rightarrow u^T \left(\sum_{i=1}^{s} t_i \{ \nabla f(x^*) - \lambda_\circ \nabla g(x^*) \} \right) + (u^T(\lambda_\circ^2 C)u)^{1/2} + (u^T B u)^{1/2} < 0,$$

(iv) $x^{*T}Bx^* > 0, \ x^{*T}Cx^* > 0$

$$\Rightarrow u^T \left(\sum_{i=1}^{s} t_i \{ \nabla f(x^*) - \lambda_\circ \nabla g(x^*) \} \right) + (u^T(\lambda_\circ^2 C)u)^{1/2} + (u^T B u)^{1/2} < 0.$$

If in addition, we insert $U_{\bar{y}}(x^*) = \phi$, then the results of Theorem 2.1 still hold.

3. Higher order nondifferentiable fractional duality

In this section, we consider the following dual of (NP):

$$\text{(ND)} \quad \max_{(s,t,\bar{y}) \in S(z)} \ \sup_{(z,\mu,\lambda,v,w,p) \in L(s,t,\bar{y})} \lambda,$$

where $L(s, t, \bar{y})$ denotes the set of all $(z, \mu, \lambda, v, w, p) \in R^n \times R_+^m \times R_+ \times R^n \times R^n \times R^n$ subject to

$$\sum_{i=1}^{s} t_i [\nabla_p (F(z, \bar{y}_i, p) - \lambda G(z, \bar{y}_i, p)) + Bw + \lambda Cv] + \sum_{j=1}^{m} \mu_j \nabla_p H_j(z, p) = 0, \tag{2}$$

$$\sum_{i=1}^{s} t_i [f(z, \bar{y}_i) + z^T Bw - \lambda(g(z, \bar{y}_i) - z^T Cv)) + F(z, \bar{y}_i, p) - \lambda G(z, \bar{y}_i, p) - p^T \nabla_p \{F(z, \bar{y}_i, p) - \lambda G(z, \bar{y}_i, p)\}] \geq 0, \tag{3}$$

$$\sum_{j=1}^{m} \mu_j [h_j(z) + H_j(z, p) - p^T \nabla_p H_j(z, p)] \geq 0, \tag{4}$$

$$w^T B w \leq 1, \ v^T C v \leq 1, \ (z^T B z)^{1/2} = z^T B w, \ (z^T C z)^{1/2} = z^T C v. \tag{5}$$

If for a triplet $(s,t,\tilde{y}) \in S(z)$, the set $L(s,t,\tilde{y}) = \emptyset$, then we define the supremum over it to be ∞.

Remark 1. Let $F(z,\bar{y}_i,p) = p^T \nabla f(z,\bar{y}_i) + \frac{1}{2} p^T \nabla^2 f(z,\bar{y}_i)p,\ G(z,\bar{y}_i,p) = p^T \nabla g(z,\bar{y}_i) + \frac{1}{2} p^T \nabla^2 g(z,\bar{y}_i)p,\ i = 1,2,\cdots,s$ and $H_j(z,p) = p^T \nabla h_j(z) + \frac{1}{2} p^T \nabla^2 h_j(z)p,\ j = 1,2,\cdots,m$. Then (ND) reduces to the second order dual in [13]. If in addition, $p = 0$, then we get the dual formulated by Ahmad et al. [8].

Theorem 3.1 (Weak duality). Let x and $(z,\mu,\lambda,s,t,v,w,\tilde{y},p)$ be feasible solutions of (NP) and (ND) respectively. Suppose that

$$[\sum_{i=1}^{s} t_i\{f(.,\bar{y}_i) + (.)^T Bw - \lambda(g(.,\bar{y}_i) - (.)^T Cv)\}, \sum_{j=1}^{m} \mu_j h_j(.)]$$

is higher order $(\mathcal{F},\alpha,\rho,d)$-pseudoquasi Type I at z and

$$\frac{\rho_1^1}{\alpha^1(x,z)} + \frac{\rho_1^2}{\alpha^2(x,z)} \geq 0.$$

Then

$$\sup_{y \in Y} \frac{f(x,y) + (x^T Bx)^{1/2}}{g(x,y) - (x^T Cx)^{1/2}} \geq \lambda.$$

Proof. Suppose to the contrary that

$$\sup_{y \in Y} \frac{f(x,y) + (x^T Bx)^{1/2}}{g(x,y) - (x^T Cx)^{1/2}} < \lambda.$$

Then, we have
$f(x,\bar{y}_i) + (x^T Bx)^{1/2} - \lambda(g(x,\bar{y}_i) - (x^T Cx)^{1/2}) < 0$, for all $\bar{y}_i \in Y,\ i = 1,2,\cdots,s$.

It follows from $t_i \geq 0,\ i = 1,2,\ldots,s$, that

$$t_i[f(x,\bar{y}_i) + (x^T Bx)^{1/2} - \lambda(g(x,\bar{y}_i) - (x^T Cx)^{1/2})] \leq 0,\ i = 1,2,\ldots,s,$$

with at least one strict inequality, since $t = (t_1,t_2,\ldots,t_s) \neq 0$. Taking summation over i and using $\sum_{i=1}^{s} t_i = 1$, we have

$$\sum_{i=1}^{s} t_i[f(x,\bar{y}_i) + (x^T Bx)^{1/2} - \lambda(g(x,\bar{y}_i) - (x^T Cx)^{1/2})] < 0.$$

It follows from generalized Schwartz inequality and (5) that

$$\sum_{i=1}^{s} t_i[f(x,\bar{y}_i) + x^T Bw - \lambda(g(x,\bar{y}_i) - x^T Cv)] < 0. \tag{6}$$

From (3) and (6), we have

$$\sum_{i=1}^{s} t_i[f(x,\bar{y}_i) + x^T Bw - \lambda(g(x,\bar{y}_i) - x^T Cv)] < \sum_{i=1}^{s} t_i[f(z,\bar{y}_i) + z^T Bw - \lambda(g(z,\bar{y}_i) - z^T Cv)$$
$$+ F(z,\bar{y}_i,p) - \lambda G(z,\bar{y}_i,p) - p^T \nabla_p \{F(z,\bar{y}_i,p) - \lambda G(z,\bar{y}_i,p)\}]. \tag{7}$$

Also, from (4), we get

$$\sum_{j=1}^{m} \mu_j [h_j(z) + H_j(z,p) - p^T \nabla_p H_j(z,p)] \geq 0. \tag{8}$$

The higher order $(\mathcal{F}, \alpha, \rho, d)$-pseudoquasi Type I assumption on

$$[\sum_{i=1}^{s} t_i \{f(.,\bar{y}_i) + (.)^T Bw - \lambda(g(.,\bar{y}_i) - (.)^T Cv)\}, \sum_{j=1}^{m} \mu_j h_j(.)]$$

at z, with (7) and (8), implies

$$\mathcal{F}(x,z;\alpha^1(x,z)\sum_{i=1}^{s} t_i\{\nabla_p(F(z,\bar{y}_i,p) + Bw - \lambda(G(z,\bar{y}_i,p) - Cv))\}) < -\rho_1^1 d^2(x,z),$$

$$\mathcal{F}(x,z;\alpha^2(x,z)\sum_{j=1}^{m} \mu_j \nabla_p H_j(z,p)) < -\rho_1^2 d^2(x,z).$$

By using $\alpha^1(x,z) > 0$, $\alpha^2(x,z) > 0$, and the sublinearity of $\mathcal{F}$ in the above inequalities, we summarize to get

$$\mathcal{F}(x,z;\sum_{i=1}^{s} t_i\{\nabla_p(F(z,\bar{y}_i,p) + Bw - \lambda(G(z,\bar{y}_i,p) - Cv))\} + \sum_{j=1}^{m} \mu_j \nabla_p H_j(z,p)) < -\left(\frac{\rho_1^1}{\alpha^1(x,z)} + \frac{\rho_1^2}{\alpha^2(x,z)}\right) d^2(x,z).$$

Since $\left(\frac{\rho_1^1}{\alpha^1(x,z)} + \frac{\rho_1^2}{\alpha^2(x,z)}\right) \geq 0$, therefore

$$\mathcal{F}(x,z;\sum_{i=1}^{s} t_i\{\nabla_p(F(z,\bar{y}_i,p) + Bw - \lambda(G(z,\bar{y}_i,p) - Cv))\} + \sum_{j=1}^{m} \mu_j \nabla_p H_j(z,p)) < 0,$$

which contradicts (2), as $\mathcal{F}(x,z;0) = 0$.

Theorem 3.2 (Strong duality). Let x^* be an optimal solution of (NP) and let $\nabla h_j(x^*)$, $j \in J(x^*)$ be linearly independent. Assume that

$$\left.\begin{array}{l} F(x^*,\bar{y}_i^*,0) = 0;\ \nabla_p F(x^*,\bar{y}_i^*,0) = \nabla f(x^*,\bar{y}_i^*),\ i = 1,2,\ldots,s, \\ G(x^*,\bar{y}_i^*,0) = 0;\ \nabla_p G(x^*,\bar{y}_i^*,0) = \nabla g(x^*,\bar{y}_i^*),\ i = 1,2,\ldots,s, \\ H_j(x^*,0) = 0;\ \nabla_p H_j(x^*,0) = \nabla h_j(x^*),\ j \in J. \end{array}\right\} \tag{9}$$

Then there exist $(s^*,t^*,\tilde{y}^*) \in S$ and $(x^*,\mu^*,\lambda^*,v^*,w^*,p^*) \in L(s^*,t^*,\tilde{y}^*)$ such that $(x^*,\mu^*,\lambda^*,v^*,w^*,s^*,t^*,\tilde{y}^*,p^* = 0)$ is a feasible solution of (ND) and the two objectives have the same values. Furthermore, if the assumptions of weak duality (Theorem 3.1) hold for all feasible solutions of (NP) and (ND), then $(x^*,\mu^*,\lambda^*,v^*,w^*,s^*,t^*,\tilde{y}^*,p^* = 0)$ is an optimal solution of (ND).

Proof. Since x^* is an optimal solution of (NP) and $\nabla h_j(x^*)$, $j \in J(x^*)$ are linearly independent, by Theorem 2.1, there exist $(s^*,t^*,\tilde{y}^*) \in S$ and $(x^*,\mu^*,\lambda^*,v^*,w^*,p^*) \in L(s^*,t^*,\tilde{y}^*)$ such that $(x^*,\mu^*,\lambda^*,v^*,w^*,s^*,t^*,\tilde{y}^*,p^* = 0)$ is a feasible solution of (ND) and the problems (NP) and (ND) have the same objectives values and

$$\lambda^* = \frac{f(x^*,\bar{y}_i) + (x^{*T}Bx^*)^{1/2}}{g(x^*,\bar{y}_i) - (x^{*T}Cx^*)^{1/2}}.$$

Theorem 3.3 (Strict converse duality). Let x^* and $(z^*, \mu^*, \lambda^*, s^*, t^*, v^*, w^*, \tilde{y}^*, p^*)$ be the optimal solutions of (NP) and (ND), respectively. Suppose that

$$[\sum_{i=1}^{s} t_i\{f(.,\bar{y}_i) + (.)^T Bw - \lambda(g(.,\bar{y}_i) - (.)^T Cv)\}, \sum_{j=1}^{m} \mu_j h_j(.)]$$

is higher order $(\mathcal{F}, \alpha, \rho, d)$-strictly pseudoquasi Type I at z^* with

$$\frac{\rho_1^1}{\alpha^1(x,z)} + \frac{\rho_1^2}{\alpha^2(x,z)} \geq 0,$$

and that $\nabla h_j(x^*)$, $j \in J(x^*)$ are linearly independent. Then $z^* = x^*$; that is, z^* is an optimal solution of (NP).

Proof. Proof follows on the similar lines of Theorem 3.1.

4. Conclusion

In this paper, a nondifferentiable minimax fractional programming problem is formulated in order to derive weak, strong and strict converse duality theorems under generalized higher order $(\mathcal{F}, \alpha, \rho, d)$-convexity assumptions. The results appeared in this paper can be further generalized to the following related class of nondifferentiable minimax fractional programming problems:

$$\text{(CP)} \qquad \text{Min} \sup_{\nu \in W} \frac{Re[\phi(\xi,\nu) + (z^H Bz)^{1/2}]}{Re[\psi(\xi,\nu) - (z^H Dz)^{1/2}]}$$

$$\text{s. t.} \ -g(\xi) \in S^\circ, \ \xi \in C^{2n},$$

where $\xi = (z, \bar{z})$, $\nu = (\omega, \bar{\omega})$ for $z \in C^n$, $\omega \in C^q$, $\phi(\cdot,\cdot) : C^{2n} \times C^{2q} \to C$ and $\psi(\cdot,\cdot) : C^{2n} \times C^{2q} \to C$ are analytic with respect to ξ, W is a specified compact subset in C^{2q}, S° is a polyhedral cone in C^m and $g : C^{2n} \to C^m$ is analytic. Also B, $D \in C^{n \times n}$ are positive semidefinite Hermitian matrices.

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