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Some Remarks Concerning Semi-$T_{\frac{1}{2}}$ Spaces

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Abstract. In this paper we prove that each subspace of an Alexandroff T_0-space is semi-$T_{\frac{1}{2}}$. In particular, any subspace of the folder X^n, where n is a positive integer and X is either the Khalimsky line $(\mathbb{Z}, \tau_K)$, the Marcus-Wyse plane $(\mathbb{Z}^2, \tau_{MW})$ or any partially ordered set with the upper topology is semi-$T_{\frac{1}{2}}$. Then we study the basic properties of spaces possessing the axiom semi-$T_{\frac{1}{2}}$ such as finite productiveness and monotonicity.

1. Introduction

Recall ([15]) that a set A of a topological space X is called semi-open if there is an open set O such that $O \subset A \subset Cl(O)$. The semi-closed sets are defined as the complements to the semi-open sets. The separation axioms semi-T_i, where $i = 0, \frac{1}{2}$ etc (see [18], [3]), are obtained from the definitions of the usual separation axioms T_i by the replacing of open sets by semi-open ones. For example, a space X satisfies the separation axiom $T_{\frac{1}{2}}$ ([8]) if for each point p of X the set $\{p\}$ is either open or closed, i.e. for each point p of X at least one of the sets $\{p\}$, $X \setminus \{p\}$ is open. Hence, a space X satisfies the separation axiom semi-$T_{\frac{1}{2}}$ if for each point p of X at least one of the sets $\{p\}$, $X \setminus \{p\}$ is semi-open, i.e. for each point p of X the set $\{p\}$ is either semi-open or semi-closed ([5]). Note that the original definition of the $T_{\frac{1}{2}}$ separation axiom was given in [16] via the condition: every set is λ-closed, and the original definition of the semi-$T_{\frac{1}{2}}$ separation axiom was given in [3] via the condition: every semi-generalized closed set is semi-closed. As a rule (cf. [5]) the axiom T_i implies the axiom semi-T_i but the converse does not hold. Moreover, if $i < j$ then the axiom semi-T_j implies the axiom semi-T_i and the converse is not valid.

Recall ([12]) that a topological space X is called Alexandroff if for each point $x \in X$ there is the minimal open set $V(x)$ containing x (hereafter, we will use this notation). In particular, every locally finite space (where each point has an open nbd which is finite) is Alexandroff. It is easy to see that for each point

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$y \in V(x)$ we have $V(y) \subset V(x)$. This implies that if X is a T_0-space and $x, y \in X$ then $V(x) = V(y)$ iff $x = y$. Alexandroff spaces appear by a natural way in studies of topological models of digital images. They are quotient spaces of the Euclidean spaces $\mathbb{R}^n$ defined by special decompositions (see [17]). Some studies of Alexandroff spaces from the general topology point of view can be found for example in [1].

In digital topology simple examples of locally finite $T_{\frac{1}{2}}$-spaces (not T_1) are the Khalimsky line $(\mathbb{Z}, \tau_K)$ ([13]) and the Marcus-Wyse plane $(\mathbb{Z}^2, \tau_{MW})$ ([22], see for the definitions the part 4 of the paper). It is clear that the products $X \times Y$, where X, Y are either $(\mathbb{Z}, \tau_K)$ or $(\mathbb{Z}^2, \tau_{MW})$, are not $T_{\frac{1}{2}}$ (even not $T_{\frac{1}{4}}$, see [2] for the definition). But they are evidently T_0-spaces as well as their subspaces. Since the products $X \times Y$ are also semi-regular (i.e. points and closed sets can be separated by semi-open sets [19]), as well as the spaces $(\mathbb{Z}, \tau_K)$ and $(\mathbb{Z}^2, \tau_{MW})$, the products $X \times Y$ are semi-T_2 ([19]). However, each of the spaces $(\mathbb{Z}, \tau_K)$ and $(\mathbb{Z}^2, \tau_{MW})$ (and so the product $X \times Y$ as well) contains subsets homeomorphic to the space (D, τ), where $D = \{0, 1\}$ and $\tau = \{\emptyset, D, \{1\}\}$, which is evidently not semi-T_1. It is natural to ask if there is a "semi" separation axiom (different from semi-T_0 of course) such that each subspace of the spaces $(\mathbb{Z}, \tau_K)$, $(\mathbb{Z}^2, \tau_{MW})$ and $X \times Y$ satisfies the separation axiom.

In this paper we prove that each Alexandroff T_0-space is semi-$T_{\frac{1}{2}}$. Since Alexandroffness is monotone with respect to any subspace and it is also finitely productive, we get that any subspace of the product $X_1 \times X_2$, where X_1, X_2 are Alexandroff T_0-spaces, is semi-$T_{\frac{1}{2}}$. Note (see the examples in the part 2 of the paper) that in general the axioms T_0 and semi-$T_{\frac{1}{2}}$ are independent, and there is even a space of cardinality 3 such that it is semi-$T_{\frac{1}{2}}$ but not T_0 [11]. Then we study the basic properties of spaces possessing the axiom semi-$T_{\frac{1}{2}}$ such as finite productiveness and monotonicity.

One can refer for the topological notions and notations to [20].

2. The axiom semi-$T_{\frac{1}{2}}$ and Alexandroff spaces.

Theorem 2.1. *Let X be an Alexandroff T_0-space. Then X is semi-$T_{\frac{1}{2}}$.*

Proof. First, let us observe that a singleton S in a space is semi-open iff the set S is open in the space. Assume that X is not semi-$T_{\frac{1}{2}}$. So there is a point x of X such that the set $\{x\}$ is neither open nor semi-closed. Since the set $\{x\}$ is not open, $|V(x)| > 1$. Put $U(x) = \cup\{V(z) : x \notin V(z), z \in X\}$. Since the space X is T_0, for each $y \in V(x) \setminus \{x\}$ we have $V(y) \subset V(x)$ and $x \notin V(y)$. Hence, $U(x) \supset V(x) \setminus \{x\}$. Moreover, $x \in Cl(V(x) \setminus \{x\}) \subset Cl(U(x))$. Note that $Cl(U(x)) = X$. In fact, if $V = X \setminus Cl(U(x)) \neq \emptyset$, then there is $p \in V$ such that $V(p) \subset V$. Note that $x \notin V(p)$. Hence, $V(p) \subset U(x)$. We have a contradiction with the definition of $U(x)$ and V. Let us note that the set $X \setminus \{x\}$ is semi-open. So the set $\{x\}$ is semi-closed. □

Let us observe (cf. [1]) that

(a) if a space X is Alexandroff and $Y \subset X$, then the subspace Y of X is also Alexandroff and for each point $y \in Y$ the set $V(y) \cap Y$ is the minimal open neighborhood of y in Y;
(b) if spaces X and Y are Alexandroff, then the topological product $X \times Y$ is also Alexandroff and for each point $(x, y) \in X \times Y$ the set $V(x) \times V(y)$ is the minimal neighborhood of (x, y) in $X \times Y$.

The following statement is now evident.

Corollary 2.2. *(a) Let X be an Alexandroff T_0-space and $Y \subset X$. Then Y is a semi-$T_{\frac{1}{2}}$ space.*
(b) Let X_1, X_2 be Alexandroff T_0-spaces and $Z \subset X_1 \times X_2$. Then Z is semi-$T_{\frac{1}{2}}$. □

Remark 2.3. *Recall that a space X is locally finite if for each point p of X there is an open set Op containing p such that $|Op| < \infty$. As a rule, spaces considered in digital topology are locally finite. Let us note that there is some interest for an axiomatization of locally finite spaces considered in the digital topology (cf. [14] and [11]). Since each locally finite space is Alexandroff, the statements of Theorem 2.1 and Corollary 2.2 are also valid for locally finite spaces.*

Remark 2.4. *Recall ([7]) that the infinite product $\prod_{\alpha\in\mathcal{A}} X_\alpha$ of Alexandroff spaces X_α, $\alpha \in \mathcal{A}$, endowed with the box topology is also an Alexandroff space. Hence the statement of Corollary 2.2 (b) can be extended to infinite box products of Alexandroff T_0- spaces.*

Example 2.5. *Let X_1 be the set of all real numbers $\mathbb{R}$ and τ_1 be the topology on X defined by the base $\mathcal{B}_1 = \{[x, \infty) : x \in \mathbb{R}\}$. It is easy to see that (X_1, τ_1) is a connected Alexandroff T_0-space which is not locally finite.*

Example 2.6. *Let (X_2, τ_2) be the space (D, τ) from the introduction, i.e. $X_2 = \{0, 1\}$ and $\tau_2 = \{\emptyset, X_2, \{1\}\}$. Note that the space (X_2, τ_2) is $T_{\frac{1}{2}}$ (hence T_0) and locally finite but it is not semi-T_1. Thus Theorem 2.1 (even its analogue for the locally finite spaces) cannot be strengthened to the axiom semi-T_1.*

Example 2.7. *Let $X_3 = \{0, 1, 2\}$ and $\tau_3 = \{\emptyset, X_3, \{2\}\}$. Note that the space (X_3, τ_3) is semi-$T_{\frac{1}{2}}$ (the sets $\{0\}, \{1\}$ are semi-closed and the set $\{2\}$ is open) and locally finite but it is not T_0 (the axiom fails for the pair $0, 1$). Thus the axiom semi-$T_{\frac{1}{2}}$ does not imply the axiom T_0 in the realm of locally finite spaces (and in the realm of Alexandroff spaces as well). Thus the axioms T_0 and semi-$T_{\frac{1}{2}}$ are not equivalent in the realm of locally finite spaces (and in the realm of Alexandroff spaces as well).*

Example 2.8. *Let X_4 be the set of all real numbers and $\tau_4 = \{\emptyset, X_4, (a, \infty) : a \in \mathbb{R}\}$. It is evident that (X_4, τ_4) is a T_0-space. Moreover, each singleton of (X_4, τ_4) is semi-closed. So the space (X_4, τ_4) is semi-T_1 (hence semi-$T_{\frac{1}{2}}$) but it is not semi-T_2 (there are no two disjoint non-empty open sets in the space). Consider the subspaces $Y_1 = (-\infty, 1]$ and $Y_2 = \{1\} \cup \bigcup\limits_{i=1}^{\infty} \left\{\frac{i-1}{i}\right\}$ of the space (X_4, τ_4). Let us note that Y_1 and Y_2 are T_0-spaces but they are not semi-$T_{\frac{1}{2}}$. In fact, the singleton $\{1\}$ in both spaces is neither open nor semi-closed. Moreover, the point 1 in the space Y_1 is the only point which does not have the minimal open neighborhood. Hence, the condition of Alexandroffness cannot be omitted in Theorem 2.1. Let us note that the first example of a T_0-space which is not semi-$T_{\frac{1}{2}}$ was suggested in [6].*

Example 2.9. *Let $X_5 = \{0, 1, 2\}$ and $\tau_5 = \{\emptyset, X_5, \{1, 2\}, \{2\}\}$. Note that the space (X_5, τ_5) is a subspace of the space (X_4, τ_4) (the space (X_5, τ_5) is T_0 and locally finite) but it is not $T_{\frac{1}{2}}$ (the set $\{1\}$ is neither open nor closed). Thus Theorem 2.1 (even its analogue for the locally finite spaces) cannot be strengthened to the axiom $T_{\frac{1}{2}}$. Let us recall ([11]) that for the spaces of cardinality 2 the axioms T_0, semi-$T_{\frac{1}{2}}$ and $T_{\frac{1}{2}}$ coincide. For other examples of T_0 and locally finite spaces which are not $T_{\frac{1}{2}}$ see the part 4 of the paper. Let us also note that the space (X_5, τ_5) is not even $T_{\frac{1}{4}}$.*

Example 2.10. *Let $X_6 = \{0, 1\}$ and $\tau_6 = \{\emptyset, X_6\}$. Observe that the space (X_6, τ_6) is neither T_0 nor semi-$T_{\frac{1}{2}}$ but it is finite and hence it is Alexandroff. So the condition "to be T_0" in Theorem 2.1 cannot be omitted.*

3. Basic properties of semi-$T_{\frac{1}{2}}$ spaces.

Let us recall (cf. [4]) that a singleton $\{p\}$ of a space X is semi-closed iff it is nowhere dense (1) or regular open (2). To say differently, there is an open set U such that $Cl(U) = X$ and $p \notin U$ for (1) or $\{p\} = X \setminus Cl(U)$ for (2). Thus a space X is semi-$T_{\frac{1}{2}}$ iff each singleton of X is either open or nowhere dense (cf. [5]).

Recall ([18]) that the product of two semi-T_i, $i = 0, 1, 2$, spaces is also a semi-T_i space. Each open subset of a semi-T_i, $i = 0, 1, 2$, space is also a semi-T_i space. But in general one cannot omit the openness in the last statement. Here we will show the same for the semi-$T_{\frac{1}{2}}$ spaces.

Proposition 3.1. *If X is a semi-$T_{\frac{1}{2}}$ space and Y is an open subset of X then the subspace Y of X is semi-$T_{\frac{1}{2}}$.*

Proof. Let $p \in Y$. If the set $\{p\}$ is open in X then it is open in Y. If the set $\{p\}$ is nowhere dense in X then it is nowhere dense in Y. Hence, the subspace Y of the space X is semi-$T_{\frac{1}{2}}$. □

Remark 3.2. *Note that in general a closed subset of a semi-$T_{\frac{1}{2}}$ space is not semi-$T_{\frac{1}{2}}$. In fact, consider the space (X_4, τ_4) from Example 2.8 and its subspace Y_1. However, one can easily extend Proposition 3.1 to preopen sets ([23]) (recall that a set S is a preopen set if $S \subset Int(Cl(S))$.*

Proposition 3.3. *Let X_1, X_2 be spaces and $x_2 \in X_2$. Assume that the singleton $\{x_2\}$ is nowhere dense in X_2. Then for each subset Y of X_1 we have that the set $Y \times \{x_2\}$ is semi-closed in the space $X_1 \times X_2$.*

Proof. Let U_2 be an open set of X_2 such that $Cl(U_2) = X_2$ and $x_2 \notin U_2$. Note that the set $U = X_1 \times U_2$ is open in $X = X_1 \times X_2$ and $Cl(U) = X$. Since $Y \times \{x_2\} \subset X \setminus U$ we have that the set $Y \times \{x_2\}$ is semi-closed. □

Corollary 3.4. *Let X_1, X_2 be semi-$T_{\frac{1}{2}}$ spaces. Then the space $X_1 \times X_2$ is also semi-$T_{\frac{1}{2}}$.*

Proof. Let $x_i \in X_i$, $i = 1, 2$. If the singletons $\{x_i\}$, $i = 1, 2$, are open then the singleton $\{(x_1, x_2)\}$ is also open. If one of the sets $\{x_i\}$, $i = 1, 2$, is nowhere dense then by Proposition 3.3 we have that the singleton $\{(x_1, x_2)\}$ is semi-closed. Hence, the space $X_1 \times X_2$ is semi-$T_{\frac{1}{2}}$. □

Remark 3.5. *The statement of Corollary 3.4 can be easily extended to infinite box products.*

4. The axiom semi-$T_{\frac{1}{2}}$ and digital topology.

Let us recall some basic examples of digital topology.

The Khalimsky line ([13]) is the topological space $(\mathbb{Z}, \tau_K)$, where $\mathbb{Z}$ is the set of all integers and τ_K is the topology on $\mathbb{Z}$ generated by the base $\mathcal{B}_K = \{\{2k+1\}, \{2k-1, 2k, 2k+1\} : k \in \mathbb{Z}\}$. One of the interesting properties of the space is the connectedness of $(\mathbb{Z}, \tau_K)$. The folders $(\mathbb{Z}, \tau_K)^n$, where $n \geq 1$, of the Khalimsky line are called the Khalimsky *nD* space.

The Marcus-Wyse plane ([22]) is the topological space $(\mathbb{Z}^2, \tau_{MW})$, where τ_{MW} is the topology on $\mathbb{Z}^2$ generated by the base $\mathcal{B}_{MW} = \{U_p : p \in \mathbb{Z}^2\}$, where for each point $p = (x, y) \in \mathbf{Z}^2$ the set U_p is defined as follows:

$$U_p = \begin{cases} N_4(p) \cup \{p\}, & \text{if } x+y \text{ is even} \\ \{p\}, & \text{if } x+y \text{ is odd} \end{cases}, \quad \text{where } N_4(p) = \{(x-1, y), (x+1, y), (x, y-1), (x, y+1)\}.$$

Here we will discuss the axiomatic properties of the digital topological spaces $(\mathbb{Z}, \tau_K)$, $(\mathbb{Z}^2, \tau_{MW})$ and their products.

It is easy to see that the spaces $(\mathbb{Z}, \tau_K)$, $(\mathbb{Z}^2, \tau_{MW})$ are locally finite and $T_{\frac{1}{2}}$. Hence, they are Alexandroff, T_0 and semi-$T_{\frac{1}{2}}$.

Recall that the s-regularity is finitely productive ([21]), and each T_0 and s-regular space is semi-T_2 ([19]). This implies the following statement.

Proposition 4.1. *Let X_1, X_2 be T_0 and s-regular spaces. Then the product $X = X_1 \times X_2$ is the same. Moreover, X is semi-T_2.*

It is easy to see that the spaces $(\mathbb{Z}, \tau_K)$, $(\mathbb{Z}^2, \tau_{MW})$ are also semi-regular. Hence, by Proposition 4.1, we obtain that any folder $F = X^n$, where $n \geq 2$ and X is $(\mathbb{Z}, \tau_K)$ or $(\mathbb{Z}^2, \tau_{MW})$, is an s-regular T_0-space. In particular, F is also semi-T_2. Let us note that the fact that the folders $(\mathbb{Z}, \tau_K)^n, n \geq 2$, are semi-$T_2$ was first observed in [9]. Since the axiom semi-T_2 implies the axiom semi-T_1, we have that each singleton of F is semi-closed ([18]). But it is easy to see that the spaces $(\mathbb{Z}, \tau_K)$, $(\mathbb{Z}^2, \tau_{MW})$ (and hence the space F) contain subsets homeomorphic to the space (X_2, τ_2) from Example 2.6 which is not semi-T_1.

Furthermore, due to the locally finiteness of the spaces $(\mathbb{Z}, \tau_K)$ and $(\mathbb{Z}^2, \tau_{MW})$ (which implies the Alexandroffness) we have that each subset of F is semi-$T_{\frac{1}{2}}$ by Corollary 2.2. This is an answer to the question posed in the Introduction.

Let us note that each subset of a space possessing the axiom $T_{\frac{1}{2}}$ is also a $T_{\frac{1}{2}}$-space. Hence, the space F is not $T_{\frac{1}{2}}$ (it contains a subset homeomorphic to the space (X_5, τ_5) from Example 2.9 which can be found in the folder $(X_2, \tau_2)^2$). By the same reason the space F is not even $T_{\frac{1}{4}}$.

5. The axiom semi-$T_{\frac{1}{2}}$ and domain theory.

Let us recall [10] that for a poset $(X, \leq)$ an upper set is a subset U of X with the property that, if x is in U and $x \leq y$, then y is in U. As a dual notion of an upper set we say that a lower set of the poset $(X, \leq)$ is a subset L with the property that, if x is in L and $y \leq x$, then y is in L.

For an arbitrary element z of a poset $(X, \leq)$, the smallest upper set containing z is denoted using an up arrow as $\uparrow z = \{x \in X \,|\, z \leq x\}$. For every $z \in X$ take $\uparrow z$. Then, by using the family consisting of X and the sets $\uparrow z$, as a base, we can uniquely establish a topology on X, denoted by τ_{up}. It is well known (cf. [10]) that the the space (X, τ_{up}) is an Alexandroff topological space satisfying the axiom T_0.

Thus Corollary 2.2 implies also the following statement.

Proposition 5.1. *Given a partially ordered set $(X, \leq)$, let (X, τ_{up}) be the upper topological space induced by the given poset $(X, \leq)$. Then each subspace of the folder $(X, \tau_{up})^n$, $n \geq 1$, satisfies the semi-$T_{\frac{1}{2}}$ separation axiom.* □

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