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Spectral Properties and Tensor Products of Quasi-$*$-$A(n)$ Operators

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Abstract. In this paper, we prove that the spectrum is continuous on the class of all quasi-$*$-$A(n)$ operators. And we obtain a sufficient condition for a quasi-$*$-$A(n)$ operator to be normal. Finally, we consider the tensor products of quasi-$*$-$A(n)$ operators, giving a necessary and sufficient condition for $T \otimes S$ to be a quasi-$*$-$A(n)$ operator when T and S are both non-zero operators.

1. Introduction

Let H be an infinite dimensional separable Hilbert space, $B(H)$ denote the C^*-algebra of all bounded linear operators on H. An operator $T \in B(H)$ is said to be hyponormal if $T^*T \geq TT^*$(equivalently, if $\|T^*x\| \leq \|Tx\|$ for all x in H). Though there are many unsolved interesting problems for hyponormal operators (e.g., the invariant subspace problem), one of recent hot topics in operator theory is to study natural extensions of hyponormal operators. In paper [23] authors introduced the class of quasi-$*$-$A(n)$ operators defined as follows:

Definition 1.1. *An operator T is said to be a quasi-$*$-$A(n)$ operator if*

$$T^*|T^{1+n}|^{\frac{2}{1+n}}T \geq T^*|T^*|^2T,$$

where n is a positive integer and $|T| = (T^*T)^{1/2}$.

A quasi-$*$-$A(n)$ operator is a generalization of a hyponormal operator.

Theorem 1.2. *Each hyponormal operator is a quasi-$*$-$A(n)$ operator.*

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Proof. Recall from [1, Theorem 1] that if T is a hyponormal operator, then

$$|T^{1+n}|^{\frac{2}{1+n}} \geq T^*T \geq TT^*,$$

therefore, we have $T^*|T^{1+n}|^{\frac{2}{1+n}}T \geq T^*|T^*|^2T$. $\square$

Example 1.3. *Let T be the unilateral weighted shift operator with weights $\alpha := \{\alpha_n\}_{n\geq 1}$ of positive real numbers, that is,*

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ \alpha_1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \alpha_2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \alpha_3 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \alpha_4 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

It is well known that T is hyponormal if and only if α is monotonically increasing. Simple calculations show that T is a quasi-$*$-$A(n)$ operator if and only if

$$(\alpha_{i+n+1}\alpha_{i+n}\ldots\alpha_{i+2}\alpha_{i+1})^{\frac{1}{n+1}} \geq \alpha_i \ \ (i = 1, 2, 3, \ldots).$$

If $T \in B(H)$, write $N(T)$ for the null space of T; $\sigma(T)$, $\sigma_a(T)$ and $\sigma_p(T)$ for the spectrum, the approximate point spectrum and the point spectrum of T, respectively. In this paper, we study spectral properties and the tensor products of quasi-$*$-$A(n)$ operators.

2. Spectral Properties of Quasi-$*$-$A(n)$ Operators

For every $T \in B(H)$, $\sigma(T)$ is a compact subset of $\mathbb{C}$. The function σ viewed as a function from $B(H)$ into the set of all compact subsets of $\mathbb{C}$, equipped with the Hausdorff metric, is well known to be upper semi-continuous, but fails to be continuous in general. Conway and Morrel [6] have carried out a detailed study of spectral continuity in $B(H)$. Recently, the continuity of spectrum was considered when restricted to certain subsets of the entire manifold of Toeplitz operators in [12, 18]. It has been proved that is continuous in the set of normal operators and hyponormal operators in [14]. And this result has been extended to quasihyponormal operators by Djordjević in [7], to p-hyponormal operators by Hwang and Lee in [19], and to (p, k)-quasihyponormal, M-hyponormal, $*$-paranormal and paranormal operators by Duggal, Jeon and Kim in [10]. In this section we extend this result to quasi-$*$-$A(n)$ operators.

Lemma 2.1. *Let T be a quasi-$*$-$A(n)$ operator. Then the following assertions hold:*
(1) *If T is quasinilpotent, then $T = 0$.*
(2) *For every non-zero $\lambda \in \sigma_p(T)$, the matrix representation of T with respect to the decomposition $H = N(T-\lambda) \oplus (N(T-\lambda))^\perp$ is: $T = \begin{pmatrix} \lambda & 0 \\ 0 & B \end{pmatrix}$ for some operator B satisfying $\lambda \notin \sigma_p(B)$ and $\sigma(T) = \{\lambda\} \cup \sigma(B)$.*

Proof. (1) Suppose T is a quasi-$*$-$A(n)$ operator. T is normaloid by [22], thus $T = 0$.

(2) If $\lambda \neq 0$ and $\lambda \in \sigma_p(T)$, we have that $N(T-\lambda)$ reduces T by [23]. So we have that $T = \begin{pmatrix} \lambda & 0 \\ 0 & B \end{pmatrix}$ on $H = N(T-\lambda) \oplus (N(T-\lambda))^\perp$ for some operator B satisfying $\lambda \notin \sigma_p(B)$ and $\sigma(T) = \{\lambda\} \cup \sigma(B)$. $\square$

Lemma 2.2. *[3] Let H be a complex Hilbert space. Then there exists a Hilbert space K such that $H \subset K$ and a map $\varphi : B(H) \to B(K)$ such that*
(1) φ is a faithful $*$-representation of the algebra $B(H)$ on K;
(2) $\varphi(A) \geq 0$ for any $A \geq 0$ in $B(H)$;
(3) $\sigma_a(T) = \sigma_a(\varphi(T)) = \sigma_p(\varphi(T))$ for any $T \in B(H)$.

Theorem 2.3. *The spectrum σ is continuous on the set of quasi-$*$-$A(n)$ operators.*

Proof. Suppose T is a quasi-$*$-$A(n)$ operator. Let φ: $B(H) \to B(K)$ be Berberian's faithful $*$-representation of Lemma 2.2. In the following, we shall show that $\varphi(T)$ is also a quasi-$*$-$A(n)$ operator. In fact, since T is a quasi-$*$-$A(n)$ operator, we have $T^*(|T^{1+n}|^{\frac{2}{1+n}} - |T^*|^2)T \geq 0$. Hence we have

$$
\begin{aligned}
&(\varphi(T))^*(|(\varphi(T))^{1+n}|^{\frac{2}{1+n}} - |(\varphi(T))^*|^2)\varphi(T) \\
= &\ \varphi(T^*(|T^{1+n}|^{\frac{2}{1+n}} - |T^*|^2)T) \text{ by Lemma 2.2} \\
\geq &\ 0 \text{ by Lemma 2.2,}
\end{aligned}
$$

so $\varphi(T)$ is also a quasi-$*$-$A(n)$ operator. By Lemma 2.1, we have T belongs to the set $C(i)$ (see definition in [10]). So we have that the spectrum σ is continuous on the set of quasi-$*$-$A(n)$ operators by [10, Theorem 1.1]. □

Corollary 2.4. *The spectrum $\sigma(.)$, Weyl spectrum $w(.)$ and Browder spectrum $\sigma_b(.)$ are continuous on quasi-$*$-$A(n)$ operators.*

Proof. Suppose T is a quasi-$*$-$A(n)$ operator. By [22], we have that Weyl's theorem holds for T. So we have that Browder's theorem holds for T. Hence Corollary 2.4 holds by Theorem 2.3 and [8, Theorem 2.2]. □

A complex number λ is said to be in the point spectrum $\sigma_p(T)$ of T if there is a nonzero $x \in H$ such that $(T - \lambda)x = 0$. If in addition, $(T^* - \overline{\lambda})x = 0$, then λ is said to be in the joint point spectrum $\sigma_{jp}(T)$ of T. Analogously, a complex number λ is said to be in the approximate point spectrum $\sigma_a(T)$ of T if there is a sequence $\{x_n\}$ of unit vectors in H such that $(T - \lambda)x_n \to 0$. If in addition, $(T^* - \overline{\lambda})x_n \to 0$, then λ is said to be in the joint approximate point spectrum $\sigma_{ja}(T)$ of T. If an operator is hyponormal, then $N(T) \subseteq N(T^*)$ and $\sigma_{jp}(T) = \sigma_p(T)$. Here we show that if T is a quasi-$*$-$A(n)$ operator, then $\sigma_{jp}(T) \setminus \{0\} = \sigma_p(T) \setminus \{0\}$ and $\sigma_{ja}(T) \setminus \{0\} = \sigma_a(T) \setminus \{0\}$.

Lemma 2.5. *[23] Let T be a quasi-$*$-$A(n)$ operator and $\lambda \neq 0$. Then $Tx = \lambda x$ implies $T^*x = \overline{\lambda}x$.*

The following example provides an operator T which is a quasi-$*$-$A(n)$ operator, however, the relation $N(T) \subseteq N(T^*)$ does not hold.

Example 2.6. *[22] Let* $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ *be operators on* $\mathbb{R}^2$*, and let* $H_n = \mathbb{R}^2$ *for all positive integers* n*. Consider the operator* $T_{A,B}$ *on* $\oplus_{n=1}^{+\infty} H_n$ *defined by*

$$T_{A,B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ A & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & B & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & B & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & B & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & B & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Then $T_{A,B}$ *is a quasi-*$*$*-*$A(n)$ *operator, however for the vector* $x = (0, 0, 1, -2, 0, 0, \cdots)$, $T_{A,B}(x) = 0$*, but* $T^*_{A,B}(x) \neq 0$*. Therefore, the relation* $N(T_{A,B}) \subseteq N(T^*_{A,B})$ *does not always hold.*

Theorem 2.7. *Let* T *be a quasi-*$*$*-*$A(n)$ *operator. Then* $\sigma_{jp}(T)\backslash\{0\} = \sigma_p(T)\backslash\{0\}$*.*

Proof. Clearly by Lemma 2.5. □

Lemma 2.8. *[26] Let* $\varphi : B(H) \to B(K)$ *be Berberian's faithful* $*$*-representation. Then* $\sigma_{ja}(T) = \sigma_{jp}(\varphi(T))$*.*

Theorem 2.9. *Let* T *be a quasi-*$*$*-*$A(n)$ *operator. Then* $\sigma_{ja}(T)\backslash\{0\} = \sigma_a(T)\backslash\{0\}$*.*

Proof. Suppose T is a quasi-$*$-$A(n)$ operator. Let φ: $B(H) \to B(K)$ be Berberian's faithful $*$-representation of Lemma 2.2. Then $\varphi(T)$ is a quasi-$*$-$A(n)$ operator. Hence

$$\begin{aligned} \sigma_a(T)\backslash\{0\} &= \sigma_a(\varphi(T))\backslash\{0\} && \text{by Lemma 2.2} \\ &= \sigma_p(\varphi(T))\backslash\{0\} && \text{by Lemma 2.2} \\ &= \sigma_{jp}(\varphi(T))\backslash\{0\} && \text{by Theorem 2.7} \\ &= \sigma_{ja}(T)\backslash\{0\} && \text{by Lemma 2.8.} \end{aligned}$$

□

In the following, we shall give a sufficient condition for quasi-$*$-$A(n)$ operators to be normal.

Theorem 2.10. *Let* T *be a quasi-*$*$*-*$A(n)$ *operator and Riesz (i.e.,* $\sigma_e(T) = \{0\}$*). Then* T *is compact and normal.*

Proof. Suppose T is a quasi-$*$-$A(n)$ operator. Then we have $T^*(|T^{1+n}|^{\frac{2}{1+n}} - |T^*|^2)T \geq 0$. Let φ: $B(H) \longrightarrow B(K)$ be Berberian's faithful $*$-representation of Lemma 2.2, $\varphi(T)$ is also a quasi-$*$-$A(n)$ operator and hence $\varphi(T)$ is normaloid by [22]. If T is Riesz, then by the West Decomposition Theorem [25], we can write $T = K + Q$, where K is compact and Q is quasinilpotent. Since $\varphi(T) = \varphi(Q)$, and hence $\sigma(\varphi(T)) = \sigma(\varphi(Q)) = \sigma_e(Q) = \{0\}$, we have that $\varphi(T)$ is quasinilpotent. Therefore $\|\varphi(T)\| = r(\varphi(T)) = 0$, and hence $\varphi(T) = 0$. Thus T is compact.

For the normality of T, since T is normaloid, there exists $\lambda \in \sigma(T)$ such that $|\lambda| = \|T\|$. Observe that T is compact, the subspace $N(T - \lambda)$ which is invariant for T and T^* is not equal to $\{0\}$. Consider the family of subspace of the following form

$$N(T - \lambda) \cap N(T - \lambda)^* = N_T(\lambda)$$

and it is easy to see that $N_T(\lambda) \perp N_T(\lambda')$ for $\lambda \neq \lambda'$. Observe that

$$\mathfrak{R} := \sum_{\lambda \in \sigma_p(T)} N_T(\lambda)$$

reduces T. Thus we can write

$$T = \begin{pmatrix} T_1 & 0 \\ 0 & T_2 \end{pmatrix} : \mathfrak{R} \oplus \mathfrak{R}^\perp \to \mathfrak{R} \oplus \mathfrak{R}^\perp.$$

Note that T_1 is normal. If $\mathfrak{R}^\perp = \{0\}$, then evidently T is normal. Hence we assume that $\mathfrak{R}^\perp \neq \{0\}$. We now claim that $T_2 = 0$. Assume to the contrary that $T_2 \neq 0$. Since T_2 is a quasi-$*$-$A(n)$ operator, and hence normaloid, we can find $\mu \in \sigma(T_2)$ such that $\|T_2\| = |\mu|$ and $(T_2 - \mu)y = 0$ for some non-zero element y in $\mathfrak{R}^\perp$. But $(T_2 - \mu)^* y = 0$, then

$$N(T - \mu) \cap N(T - \mu)^* = N_T(\mu)$$

is not equal to $\{0\}$ and is also orthogonal to $\mathfrak{R}$. we obtain a contradiction and $T_2 = 0$, and hence we can conclude that T is normal. □

3. Tensor Products of Quasi-$*$-$A(n)$ Operators

Given non-zero T, $S \in B(H)$, let $T \otimes S$ denote the tensor products on the product space $H \otimes H$. The operation of taking tensor products $T \otimes S$ preserves many properties of $T, S \in B(H)$, but by no means all of them. The normaloid property is invariant under tensor products [21]. $T \otimes S$ is hyponormal if and only if T and S are hyponormal [17]. There exist paranormal operators T and S such that $T \otimes S$ is not paranormal [2]. Duggal [9] showed that for non-zero T, $S \in B(H)$, $T \otimes S$ is a p-hyponormal operator if and only if T, S are p-hyponormal, where an operator T is said to be p-hyponormal if $(T^*T)^p \geq (TT^*)^p$ for $0 < p \leq 1$. This result was extended to $*$-class A [11] and quasi-class A [20], respectively.

In this section we obtain an analogous result for quasi-$*$-$A(n)$ operators. Before we state main theorems, we need several preliminary results.

Lemma 3.1. *[13] Let A be a positive linear operator on a Hilbert space H. Then the following properties* (1) *and* (2) *hold.*

(1) *$(A^\lambda x, x) \geq (Ax, x)^\lambda$ for any $\lambda > 1$ and any unit vector x.*

(2) *$(A^\lambda x, x) \leq (Ax, x)^\lambda$ for any $\lambda \in [0, 1]$ and any unit vector x.*

Lemma 3.2. *[24, Proposition 2.2] Let $A_1, A_2 \in B(H)$, $B_1, B_2 \in B(K)$ be non-negative operators. If A_1 and B_1 are non-zero, then the following assertions are equivalent:*

(1) *$A_1 \otimes B_1 \leq A_2 \otimes B_2$.*

(2) *There exists $c > 0$ for which $A_1 \leq cA_2$ and $B_1 \leq c^{-1}B_2$.*

Theorem 3.3. *Let $T, S \in B(H)$ be non-zero operators. Then $T \otimes S$ is a quasi-$*$-$A(n)$ operator if and only if T and S are quasi-$*$-$A(n)$ operators.*

Proof. Suppose that T and S are quasi-$*$-$A(n)$ operators. Then

$$\begin{aligned}(T\otimes S)^*|(T\otimes S)^{1+n}|^{\frac{2}{1+n}}(T\otimes S) &= T^*|T^{1+n}|^{\frac{2}{1+n}}T\otimes S^*|S^{1+n}|^{\frac{2}{1+n}}S\\ &\geq T^*|T^*|^2T\otimes S^*|S^*|^2S\\ &= (T\otimes S)^*|(T\otimes S)^*|^2(T\otimes S). \end{aligned} \tag{1}$$

Thus $T\otimes S$ is a quasi-$*$-$A(n)$ operator.

Conversely, suppose that $T\otimes S$ is a quasi-$*$-$A(n)$ operator. Without loss of generality, it is enough to show that T is a quasi-$*$-$A(n)$ operator. Since $T\otimes S$ is a quasi-$*$-$A(n)$ operator, we obtain

$$T^*|T^{1+n}|^{\frac{2}{1+n}}T\otimes S^*|S^{1+n}|^{\frac{2}{1+n}}S \geq T^*|T^*|^2T\otimes S^*|S^*|^2S.$$

Therefore, by Lemma 3.2, there exists a positive real number l for which

$$l(T^*|T^{1+n}|^{\frac{2}{1+n}}T) \geq T^*|T^*|^2T$$

and

$$l^{-1}(S^*|S^{1+n}|^{\frac{2}{1+n}}S) \geq S^*|S^*|^2S.$$

Consequently, for arbitrary $x\in H$, it follows from Lemma 3.1 that

$$\begin{aligned}\|T\|^4 = \|T^*T\|^2 &= \sup\{(T^*|T^*|^2Tx,\ x) : \|x\| = 1\}\\ &\leq l\sup\{(T^*|T^{1+n}|^{\frac{2}{1+n}}Tx,\ x) : \|x\| = 1\}\\ &\leq l\sup\{(T^*|T^{1+n}|^2Tx,\ x)^{\frac{1}{1+n}}\|Tx\|^{\frac{2n}{1+n}} : \|x\| = 1\}\\ &\leq l\|T\|^4. \end{aligned} \tag{2}$$

Similarly, $\|S\|^4 \leq l^{-1}\|S\|^4$. Clearly, we must have $l = 1$, and hence T is a quasi-$*$-$A(n)$ operator. □

Let $C_2(H)$ denote the Hilbert-Schmidt class. For each pair of operators $A, B \in B(H)$, there is an operator $\Gamma_{A,B}$ defined on $C_2(H)$ via the formula $\Gamma_{A,B} = AXB$ in [4]. The adjoint of $\Gamma_{A,B}$ is given by the formula $\Gamma^*_{A,B}(X) = A^*XB^*$; see details [4].

In the following, we show that if X is a Hilbert-Schmidt operator, A and $(B^*)^{-1}$ are quasi-$*$-$A(n)$ operators such that $AX = XB$, then $A^*X = XB^*$.

Theorem 3.4. *Let $A, B \in B(H)$. Then $\Gamma_{A,B}$ is a quasi-$*$-$A(n)$ operator on $C_2(H)$ if and only if A and B^* belong to quasi-$*$-$A(n)$ operators.*

Proof. The unitary operator $U : C_2(H) \to H \otimes H$ by a map $(x \otimes y)^* \to x \otimes y$ induces the $*$-isomorphism $\Psi : B(C_2(H)) \to B(H \otimes H)$ by a map $X \to UXU^*$. Then we can obtain $\Psi(\Gamma_{A,B}) = A \otimes B^*$; see details [5]. This completes the proof by Theorem 3.3. $\square$

Theorem 3.5. *Let A and $(B^*)^{-1}$ be quasi-$*$-$A(n)$ operators. If $AX = XB$ for $X \in C_2(H)$, then $A^*X = XB^*$.*

Proof. Let Γ be defined on $C_2(H)$ by $\Gamma Y = AYB^{-1}$. Since A and $(B^{-1})^* = (B^*)^{-1}$ are quasi-$*$-$A(n)$ operators, we have that Γ is a quasi-$*$-$A(n)$ operator on $C_2(H)$ by Theorem 3.4. Moreover, we have $\Gamma X = AXB^{-1} = X$ because of $AX = XB$. Hence X is an eigenvector of Γ. By Lemma 2.5 we have $\Gamma^*X = A^*X(B^{-1})^* = X$, that is, $A^*X = XB^*$. $\square$

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