



Academician Mileva Prvanović - the First Doctor of Geometrical Sciences in Serbia

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Abstract. This paper is devoted to academician Mileva Prvanović on the occasion of her 85th birthday. We point out her influence on development of geometrical sciences, as the first doctor of these sciences in Serbia.



1. Introduction

Professor Mileva Prvanović has played a crucial role in enriching the research, growth and development of Differential Geometry over a 60-year period, placing her among the foremost geometers of our day. The long list of her outstanding scientific publications, remarkably cross-referenced, is a proof of her great imagination, creativity and dedication to science. She has been internationally recognized by maintaining

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the highest standards of integrity and excellence for scientific research. Her unique personality, prudence and leadership have enriched this field of Geometry and will continue to guide and encourage all of us in new scientific endeavors.

The 18th Geometrical Seminar, held at Vrnjačka Banja, Serbia, from May 25-28, 2014, was devoted to the anniversary of professor Mileva Prvanović. On the occasion of her 85th birthday, her students, colleagues and friends wish Professor Prvanović a very long, prosperous, healthy and happy life.

We will here mention some remarkable details from the life and career of Professor Prvanović.

2. Life devoted to Geometry

Mileva Prvanović was born on July 16, 1929, at Žilne (Knjaževac, Serbia). Her father was most famous Serbian educator-mathematician, professor of the Teacher Training College in Belgrade.

Mileva Prvanović has studied mathematics at the University of Belgrade from 1947 to 1951, and got doctoral degree in 1955 at the University of Zagreb as the first doctor of Geometry from Serbia. The title of her thesis mentored by Danilo Blanuša was *Parageodesic space and parageodesic curves in a subspace of the Riemannian space*. In the period from 1951 to 1955, she was Teaching Assistant at the Mathematical Institute of the Serbian Academy of Sciences in Belgrade, until the end of 1956, Assistant Professor at the Department of Mathematics of the Faculty of Philosophy (later Faculty of Natural Sciences and Mathematics) in Novi Sad, from 1957 Docent, from 1962 Associate and from 1967 Full Professor for a group of subjects in the field of Geometry. Professor Prvanović retired in 1993. She is the member of Serbian Academy of Sciences and Arts from 1981.

Professor Mileva Prvanović lectured in many subjects related to Geometry at the University and has written several textbooks for these courses.

Geometrical Seminar at Mathematical Institute, SANU Belgrade, and later at the Mathematical faculty was led by Professor Mileva Prvanović. It brought together mathematicians dealing with the Geometry of the various universities throughout Serbia and beyond. Meetings seminars were usually held on Fridays from 11am. At these meetings professor Prvanović usually came from Novi Sad directly from the bus station and first talked her mother by telephone in Belgrade, and then was ready to talk and discuss problems with co-workers (PhD students), which is why they came to the Institute. For each colleague she separated enough time, before or after the joint meeting. Professor Prvanović brought tremendous experience, insight, and expertise to geometry community, and her energy and ideas were infectious.

She pointed to shortcomings, thereby encouraging them to further work. Speaking about the activities that relate to the time when there was no internet, emphasizes that the contacts took place by telephone and via letters. Thus, to the first signed of this text, Professor Prvanović writes: 'Unfortunately all of your results are already known, but from your works, which I have just read, it can be seen that you have not only entered into the problem, but you have perfectly mastered the methods and techniques, I suggest you to consider unsolved problem that belongs to the issues discussed here'. From this relationship towards associates sprang large number of doctoral dissertations from Geometry. It can be said that her contribution to the construction of personnel in this field in Serbia is crucial. She was PhD advisor of:

Judita Cofman, Irena Čomić, Svetislav Minčić, Miroljub Milojević, Milan Janjić (with Prof. Dr Zagorka Šnajder), Jovanka Nikić, Nevena Pušić, Djerdji Nadj, Haizhong Li; Dragoljub Cvetković and Neda Bokan, Srdan Vukmirović at Serbia and Kostadin Trenčevski and Ognjan Jotov at FYR Macedonia. For all these PHD candidates except S. Minčić and M. Janjić she was also their M.Sc advisor and also to Vojislav Petrović, Mihailo Jokić, Jan Djuras and Djordje Lisulov.

The Universities at Niš, Novi Sad, Kragujevac, Skopje, Sarajevo and Belgrade highly appreciated her expertise at the field. Professor Prvanović participated in many international conferences with presentations and lectures on Differential Geometry.

For many years she was Editor-in-Chief of the journal *Publication de L'Institut Mathématique* (Belgrade).

The scientific work of professor Prvanović is very reach and heterogeneous. Some of her papers offered milestones for developing another ones, some of them where initials for PhD theses and for new books.

Taking into account that Professor Neda Bokan at the paper "Prof Dr Mileva Prvanović-her contribution to Differential Geometry, Kragujevac Journal of Mathematics 25(2003), 111-125." has expound and analyzed the work and scientific results of Mileva Prvanović, we will, in this paper, give brief overview of the results of Mileva Prvanović, in the reference list ending with number 79.

In the cited paper of N. Bokan results of scientific work of M. Prvanović are divided in four groups:

a) **Transformations of smooth manifolds**

Separately is pointed out the paper [31], which was a basic for examinations of other authors (for example N. Bokan, N. Pušić, V. Petrović).

b) **Geometry of connections with torsion**

Prolonging the works of E. Brinis, F. Graiff and U.P. Singh, M. Prvanović considers two non-symmetric connections and corresponding curvature tensors. A group of geometers from University of Niš (S. Minčić, M. Stanković, Lj. Velimirović, M. Zlatanović) study various problems related with non-symmetric connections.

c) **Recurrent spaces**

W. Roter and his school have interested in recurrent spaces. M. Prvanović has studied some problems in the sense of this school (see e.g. [54], [55]).

d) **Algebraic structure of curvature**

Algebraic structure of curvature is an area with which M. Prvanović has been interested and cooperated e.g. with N. Blažić and G. Stanilov. The most important results are in the paper [73].

3. Summary of the latest results

In the papers [80]-[87] there were examined differentiable manifolds supplied with the complex or with the product structure. These are, depending on whether the metric is invariant or anti-invariant with respect to the structure, almost Hermitian, almost para-Hermitian, almost product-manifolds, ie. complex manifolds with Norden metric. Applying conformal transformations on the holomorphic and anti-holomorphic curvature tensors, there are determined the corresponding conformal invariant tensors.

Among them is, in the case of Kähler manifold, Bochner's curvature tensor too. As an example, it is determined the holomorphic curvature tensor of locally-conformal Kähler's space, as well as the Riemannian curvature tensor in the case of the space of constant holomorphic sectional curvature.

The holomorphic hypersurfaces of the conformally flat anti Kähler manifolds are examined. The results correspond to well-known theorem of E. Cartan and J. Schouten, according to which the hypersurface of conformally flat Riemannian manifold (when the $\dim \geq 5$) is conformally flat if and only if it is quasi-umbilical. The notion of pseudo-symmetry and Ricci pseudo-symmetry is expanded to the anti-Kähler manifolds and applied to the holomorphic hypersurface of the anti-Kähler's manifolds of constant totally real sectional curvature. It is also proven that anti Kähler manifold of quasi constant totally real sectional curvature satisfies the equation of Roter type.

References

- [1] On Darboux linés in a subspace of a Euclidean space, (*Rev. de la Fac. des Sci. de l'Univ. d'Istanbul*), Vol 19 fas. 1, pp. 13-18, (1954).
- [2] Hyper-Darboux lines on a surface in three dimensional Euclidean space, (*Bull. of the Calcutta Math. Soc.*), Vol 47, pp. 55-60, (1955).
- [3] Lignes de Darboux dans l'espace riemannien, (*Bull. des Sci. Math.*), Vol 78, (1954).
- [4] Hyperlignes de Darboux appartenant a l'espace riemannien, (*Bull. des Sci. Math.*), Vol 78, (1954).
- [5] A field of vectors along a curve of subspace of a Riemannian space, Serbo-Croatian, English summary, (*Zbornik radova Matematičkog instituta SAN*), Vol 4, pp. 135-143 (1955).
- [6] Equation de Gauss d'un sous-éspace plongé dans l'espace riemannien généralisé, (*Bull. de la classe des Sci. d'Acad. de Belgique*), Ser. 3, Vol 41, pp. 615-622, (1955).
- [7] Parageodesic spaces and parageodesic curves of the subspace of a Riemannian space, Serbo-Croatian, French summary, (*Zbornik radova Matematičkog instituta SAN*), Beograd, Vol 5, pp. 117-178, (1956).

- [8] Une généralisation des espaces totalement géodésiques, (Comptes rendus des seances de l'Acad. des Sci.), Vol 242, pp. 2219-2221, (1956).
- [9] Propriétés des espaces paragéodesiques, (Comptes rendus des seances de l'Acad. des Sci.), Vol 242, pp. 2500-2502, (1956).
- [10] A note on the union curvature of the curves of a Riemannian space, (The Mathematical Student), Bombay, Vol 24, No 3-4, pp. 209-215, (1956).
- [11] Some properties of a family of conformal geodesics, Serbo-Croatian, English summary, (Godišnjak Fil. fak.), Novi Sad, Vol 1, pp. 313-318, (1956).
- [12] Système des courbe cyclique d'un sous-espace plongé dans un espace riemannien, (Glasnik matematičko-fizički i astronomski), Zagreb, Vol 12, No 4, pp. 233-243, (1957).
- [13] Sur quelques formule de la géométrie conforme d'un sous-espace, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 11, pp. 53-66, (1957).
- [14] Conformal and projective transformations of the generalized Riemannian space, Serbo-Croatian, French summary, (Godišnjak Fil. fak.), Novi Sad, Vol 3, pp. 265-272, (1958).
- [15] Relative Frenet formulas for curves in a subspace of a Riemannian space, (Tensor), Vol 9, No 3, pp. 190-204, (1959).
- [16] Les vecteurs des courbures cycliques des courbes d'n espace riemannien, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 13, pp. 35-46, (1959).
- [17] Les d'erivées covariantes intrinseques dans l'espace X_n a connexion m'etrique, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 14, pp. 143-156, (1960).
- [18] Congruences of curves of semi-simple group space, Serbo-Croatian, French summary, (Godišnjak Fil. fak.), Novi Sad, Vol 5, pp. 449-456, (1960).
- [19] Harmonic tensoroid fields in vector bundles over Riemannian spaces with boundary), Serbo-Croatian, English summary, (Godišnjak Fil. fak.), Novi Sad, Vol 5, pp. 439-448, (1960).
- [20] Classification of the minimal subspaces of the Riemannian space of constant curvature, (Godišnjak Fil. fak.), Novi Sad, Vol 6, pp. 336-347, (1961).
- [21] Projective and conformal transformations in recurrent and Riccirecurrent Riemannian space, (Tensor), Vol 12, No 3, pp. 219-226, (1962).
- [22] Some theorems on the subspaces with indeterminated lines of curvatures of recurrent Riemannian spaces, Serbo-Croatian, English summary, (Matematički vesnik), Vol 1(16), No 2, pp. 61-87, (1964).
- [23] Some theorems on conformal transformations of a class of compact Riemannian space, Serbo-Croatian, French summary, (Godišnjak Fil. fak.), Novi Sad, Vol 7, pp. 235-242, (1962).
- [24] Rimannian prolongation as a generalized biplanar space, Russian, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 6(20), pp. 9-16, (1966).
- [25] ε -biplanar space of parabolic type, (Matematički vesnik), Vol 3(18), pp. 109-117, (1966).
- [26] A note on recurrent and Ricci-recurrent spaces of affine connexion, Serbo-Croatian, Russian summary, (Godišnjak Fil. fak.), Novi Sad, Vol 9, pp. 349-353, (1966).
- [27] Riemannian spaces with fields of geodesic directions, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 8(22), pp. 76-86, (1968).
- [28] Semidecomposable recurrent Riemannian spaces, Serbo-Croatian, English summary, (Godišnjak Fil. fak.), Novi Sad, Vol 9/2, pp. 717-720, (1968).
- [29] Riemannian spaces with fields of geodesic directions II, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 9(23), pp. 97-106, (1969).
- [30] Une connexion non-symétrique associée a l'espace riemannien, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 10(24), pp. 53-64, (1970).
- [31] Holomorphically projective transformations in a locally product space, (Mathematica balkanica), Vol 1, pp. 195-213, (1971).
- [32] On some semi-Chebyshev decompositions, Serbo-Croatian, English summary, (Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.), Vol 2, pp. 7-21, (1972).
- [33] Some tensors of metric semi-symmetric connexion, (Atti della Acad. della Sci. de Torino), Vol 107, pp. 303-318, (1973).
- [34] On pseudo-metric semi-symmetric connections, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 18(32), pp. 157-165, (1975).
- [35] On two tensors in a locally decomposable Riemannian space, (Univ. u Novom sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.), Vol 6, pp. 49-57, (1976).
- [36] Four curvature tensors corresponding to a connection with torsion, (Sbornik 150 let geometrii Lobačevskogo), Moskva pp. 199-205, (1977).
- [37] $(1^F, 2^F)$ and $(3^F, 4^F)$ -connexion of almost complex and almost product space, (Publication de l'Institut Math. d'Acad. Serbe des Sci.), Vol 22(36), pp. 223-229, (1977).
- [38] Product semi-symmetric connections of the locally decomposable Riemannian spaces, (Bull. de l'Acad. Serb. des Sci. et des Arts, Classe des Sci. math. nature), Vol 64, No 10, pp. 17-27, (1979).
- [39] Holomorphically semi-symmetric connexions, (Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.), Vol 9, pp. 91-99, (1979).
- [40] Conjugate connections on manifolds with complex or product structure, (Colloquia Mathematica Societatis Janos Bolyai), Vol 31, Differential geometry, pp. 563-582, (1979).
- [41] A note on holomorphically projective transformations of the Kähler space, (Tensor), Vol 35, No 1, pp. 99-104, (1981).
- [42] Weyl-Otsuki spaces of the second and third kind (Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.), Vol 11, pp. 161-175, (1981).
- [43] On a special connection of an Otsuki space (Tensor), Vol 37, 237-243, (1982).

- [44] A generalization of the Bochner and contact Bochner curvature tensor, (*Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.*), Vol 12, pp. 349-367, (1982).
- [45] A note on product curvature tensors, (*Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.*), Vol 13, pp. 219-226, (1983).
- [46] Some special product semi-symmetric and some special holomorphically semi-symmetric F-connections, (*Publication de l'Institut Math. d'Acad. Serbe des Sci.*), Vol 33(47), pp. 139-152, (1984).
- [47] Otsuki-Norden space, Russian, (*Izv. VUZ, Matematika*), Vol 7, pp. 59- 63, (1984).
- [48] p-Projective curvature tensors, (*Annales Universitatis Mariae Skłodowska*), Vol 16, pp. 123-133, (1987).
- [49] G. Markov, M. Prvanović, p-holomorphically planer curves and p- holomorphically projective transformation, (*Publications Mathematicae*), Vol 37, No 3-4, pp. 273-284, (1990).
- [50] G. Markov, M. Prvanović, Some spaces whose one of pH-projective curvature tensor vanishes, (*Tensor*), Vol 46, pp. 387-396, (1987).
- [51] Conformally quasi-recurrent manifolds, (*Proc. of the 5th National Seminar on Finsler spaces, February 10-15, 1987, Brasov*), pp. 321-329, (1988).
- [52] Some theorems on conformally quasi-recurrent manifolds, (*Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.*), Vol 19, No 2, pp. 21-31, (1989)
- [53] A note on conformally quasi-recurrent manifolds, (*Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.*), Vol 20, pp. 195-212, (1990).
- [54] On a proper conformally recurrent manifolds, (*Rad Jugoslovenske akademije znanosti i umjetnosti*), Vol 444, (*Matematičke znanosti*), No 8, pp. 137-152, (1989).
- [55] On some hypersurfaces of a recurrent riemannian space, (*Publication de l'Institut Math.*), Vol 47(61), pp. 103-112, (1990).
- [56] On a class of SP-Sasakian manifold, (*Note di Matematica, Lecce*), Vol 10, No 2, pp. 325-334, (1990).
- [57] On SP-Sasakian manifold satisfying some curvature conditions, (*SUT Journal of Mathematics*), Vol 26, No2, pp. 201-206, (1990).
- [58] D-conharmonic change in a special para-Sasakian manifold, (*Mathematica Pannonica*), Vol 2, No 2, pp. 47-57, (1991).
- [59] A quasi-umbilical hypersurface of a conformally recurrent space, (*Colloquia Mathematica Sicietatis János Bolyai*), Vol 56, *Differential Geometry*, pp. 617-628, (1989).
- [60] Product umbilical submanifold of codimension 2 of a Riemannian manifold with symmetric product conformal curvature tensor, (*Sbornik: Pamjaty Lobachevskogo posvyashaetsya*), Vol 1, Kazan University, Kazan, pp. 60-79, (1992).
- [61] Generalized recurrent Riemannian manifold, (*Analele Stiintifice ale Univ. Al. I. Cuza*), Vol 38, s. 1a, *Matematica*, No 4, pp. 423-434, (1992).
- [62] Riemannian manifolds satisfying some curvature conditions of recurrent type, (*Zbornik radova Prirodno-matematičkog fakulteta u Kragujevcu*), Vol 16, pp. 87-95, (1994).
- [63] A note on generalized recurrent Riemannian manifold, *Matematički vesnik*, Vol 45, 57-60, (1993). [63] On weakly symmetric Riemannian manifolds, (*Publ. Math. Debrecen*), Vol 46, No 1-2, pp. 19-25, (1995).
- [64] On some classes of Riemannian manifolds, (*Zbornik In memoriam N.I. Lobatshevskii*), Kazan, Vol 3, No 2, pp. 164-172, (1995).
- [65] Einstein connection of akmost Hermitian manifold, (*Bull. de l'Acad'emie Serbe des Sciences et des Arts, Classe des Sciences math. et naturelles, Sciences mathématiques*), Vol CIX, No 20, pp. 51-59, (1995).
- [66] A note on manifold whose product conformal curvature tensor is semisymmetric, (*Univ. u Novom Sadu, Zb. Rad. Prirod.-mat. Fak., Ser. Mat.*), Vol 25, No 1, pp. 179-199, (1995).
- [67] M. Prvanović, N. Pušić, On manifolds admitting some semi-symmetric metric connection, (*Indian J. Math.*), Vol 37, No 1, pp. 37-67, (1995).
- [68] On warped product manifolds, (*Filomat*), Vol 9, No 2, pp. 169-185, (1995).
- [69] On some classes of semi-symmetric connections in the locally decomposable Riemannian space, (*Facta universitatis (Niš), Ser. Math. Inform.*), Vol 10, pp. 105-116, (1995).
- [70] F. Defever, R. Deszcz and M. Prvanović, On warped product manifolds satisfying some curvature condition of pseudosymmetric type, (*Bulletin of the Greek Math. Soc.*), Vol 36, pp. 43-62, (1994).
- [71] Locally decomposable Riemannian manifold endowed with some semisymmetric F-connection, (*Bull.de l'Acad'emie Serbe des Sciences et des Arts, Classe des Sciences math. et naturelles, Sciences mathématiques*), Vol CXIV, No 22, pp. 45-56, (1997).
- [72] On totoally ombilical submanifolds imbeded in a weakly symmetric Riemannian manifold, Russian, (*Izv. VUZ, Mathematics*), Vol 6, pp. 54-64, (1998).
- [73] On a curvature of Kähler type in an almost Hermitian and almost paraHermitian manifold, (*Mat. vesnik*), Vol 50, pp. 57-64, (1998).
- [74] M. Prvanović, U. C. De and S. Bandhopadlyay, Conformally flat spaces satisfying a certain condition on the Ricci tensor, (*Bull.de l'Acad'emie Serbe des Sciences et des Arts, Classe des Sciences math. et naturelles, Sciences mathématiques*), Vol CXVI, No 23, pp. 15-24, (1998).
- [75] Extended recurrent manifolds, (*Izv. VUZ, Mathematics*), Vol 1, pp. 41- 50, (1999).
- [76] Biholomorphic curvature of an almost Hermitian manifold, (*Novi Sad J. Math.*), Vol 29, No 3, pp. 267-280, (1999).
- [77] On some conformally flat Riemannian manifolds, (*J. Electrotechn. Math.*), Vol 5, No 1, pp. 1-8, (2000).
- [78] Bianchi-type identities on almost Hermitian manifolds, (*Bull.de l'Académie Serbe des Sciences et des Arts, Classe des Sciences math. et naturelles, Sciences mathématiques*), Vol CXXI, No 25, pp. 71-88, (2000).
- [79] N. Blažić and M. Prvanović, Almost Hermitian manifolds and Osserman condition, (*Abh. Math. Sem. Univ. Hamburg*), Vol 71, pp.35-47, (2001).
- [80] R. Deszcz and M. Prvanović, Holomorphic hypersurfaces of a holomorphically conformally flat anti-Kähler manifold, (*Analele Stiintifice ale Univ. Al. I. CuzaDIN IASI (S.N.)MATEMATICA, Tomul LIII, 2007, Suplement*), 123-144.).
- [81] R. Deszcz and M. Prvanović, On Ricci H-pseudosymmetric H-hypersurfaces of some anti-Kähler manifolds, (*Bulletin T.CXXXVII de l'Académie serbe des sciences et des arts - 2007 Classe des Sciences mathématiques et naturelles Sciences mathématiques*), No 33.

- [82] R. Deszcz and M. Prvanović, Roter type equations for a class of anti-Kähler manifolds, (*Annales Univ. Sci. Budapest*, 52 (2009), 103-121.
- [83] M. Prvanović, Some properties of the locally conformal Kähler manifold, (*Bulletin T.CXLI de l'Académie serbe des sciences et des arts - 2007 Classe des Sciences mathématiques et naturelles Sciences mathématiques*, No 35, 9-23.
- [84] M. Prvanović, Conformally invariant tensors of an almost Hermitian manifold associated with the holomorphic curvature tensor, *J. Geom.* 103 (2012), 89-101
- [85] R. M. Prvanović, Conformal change of the metric on almost anti-Hermitian manifolds, (*Bulletin T.CXLIV de l'Académie serbe des sciences et des arts, Classe des Sciences mathématiques et naturelles Sciences mathématiques*, No 37, 43-59.
- [86] Mileva Prvanović The conformal change of the metric of an almost Hermitian manifold applied to the antiholomorphic curvature tensor *Communications in Mathematics*, Vol. 21 (2013), No. 1, 77–90
- [87] Mileva Prvanović and Nevena Pušić, Some conformally invariant tensors on anti-Kähler manifolds and their geometrical properties. *Math. Pannonica*, vol. 24/1 (2013), 15-31