Filomat 29:9 (2015), 2059–2067
DOI 10.2298/FIL1509059C

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

q-Matrix Polynomials in Several Variables

Bayram Çekim[a]

[a] *Gazi University, Faculty of Science, Department of Mathematics,
Teknik Okullar, TR-06500, Ankara, Turkey*

Abstract. In the present paper, we define $q-$matrix polynomials in several variables which reduces Chan-Chyan-Srivastava and Lagrange-Hermite matrix polynomials in [6]. Then several results involving generating matrix functions for these matrix polynomials are derived.

1. Introduction

In the last twenty years, special matrix functions seem in the studies of applied mathematics [6, 8, 9, 18] and other application areas [3, 4, 10–12]. Furthermore, $q-$calculus has became an active research area in special functions in [1, 5, 13, 15–17, 20, 22] and approximation theory in [14, 19, 21]. Therefore, we use $q-$calculus in the theory of special matrix functions in this paper.

All this paper, for a matrix P in $\mathbb{C}^{N\times N}$, its spectrum $\sigma(P)$ denotes the set of all eigenvalues of P and $\tilde{\mu}(P)$ denotes

$$\tilde{\mu}(P) = \min\{z : z \in \sigma[(P + P^*)/2]\}$$

where P^* denotes the transpose conjugate of P. Let $f(z)$ and $g(z)$ be holomorphic functions in z, which are defined in an open set Ω of the complex plane and A is a matrix in $\mathbb{C}^{N\times N}$ with $\sigma(A) \subset \Omega$, then from the properties of the matrix functional calculus in [7], it follows that:

$$f(A)g(A) = g(A)f(A).$$

The two-norm of A, which will be denoted by $\|A\|$, is defined by

$$\|A\| = \sup_{x\neq 0} \frac{\|Ax\|_2}{\|x\|_2}, \tag{1}$$

where, for a vector $y \in \mathbb{C}^N$, $\|y\|_2 = \left(y^T y\right)^{1/2}$ is the Euclidean norm of y.

In 2012, Salem extended $q-$special functions of complex variable to $q-$special matrix functions. Firstly, he defined

2010 *Mathematics Subject Classification*. Primary 33C45, 33D50; Secondary 33D05,15A15
Keywords. $q-$shifted matrix function, $q-$Chan-Chyan-Srivastava matrix polynomials, $q-$Lagrange-Hermite matrix polynomials.
Received: 06 February 2014; Accepted: 03 July 2014
Communicated by H. M. Srivastava
Email address: bayramcekim@gazi.edu.tr (Bayram Çekim)

$$[A]_q = \frac{I - q^A}{1-q}, \ q \neq 1, \ q^A = e^{A \log q} \tag{2}$$

and $q-$sifted factorial matrix function given by

$$(A;q)_0 = I \ , \ (A;q)_n = \prod_{k=0}^{n-1} (I - Aq^k), \ n = 1,2,... \tag{3}$$

for any complex square matrix A (see [18]). The generalization of (3) is

$$(A;q)_\infty = \prod_{k=0}^{\infty} (I - Aq^k) \ , \ \left|q\right| < 1,$$

converges. Then, he gave some following required some theorems related to q-analysis:

Theorem 1.1. *[18] Let A be a complex square matrix and $\left|q\right| < 1$, then infinite products of matrices*

$$(q^A;q)_\infty = \prod_{k=0}^{\infty} (I - q^{A+kI})$$

converges invertibly if $\tilde{\mu}(A) > 0$ and $q^{-n} \notin \sigma(q^A)$, $n = 0,1,2,...$

Theorem 1.2. *[18] Let A be a complex square matrix, $\left|q\right| < 1$ and $q^{-n} \notin \sigma(A)$, $n = 0,1,2,...,$ then we have*

$$\sum_{n=0}^{\infty} \frac{(a;q)_n}{(q;q)_n} A^n = (A;q)_\infty^{-1} (aA;q)_\infty \ ; \ \|A\| < 1.$$

Theorem 1.3. *[18] For any two matrices $A, B \in \mathbb{C}^{N \times N}$ with $AB = BA$, $q^{-n} \notin \sigma(A)$, $n = 0,1,2,...$ and $\left|q\right| < 1$, we have*

$$\sum_{n=0}^{\infty} \frac{(B;q)_n}{(q;q)_n} A^n = (A;q)_\infty^{-1} (AB;q)_\infty \ ; \ \|A\| < 1. \tag{4}$$

On the other hand, Chan-Chyan-Srivastava polynomials, given in [22], have been studied systematically and comprehensively in the literature. For example, $q-$extension in [1], umbral calculus presentations in [24] and matrix extension in [6] of these multivariable polynomials have been given. Therefore, in the present paper, we construct to $q-$matrix polynomials in several variables and to derive different families of mixed multilateral and multilinear generating matrix functions for these matrix polynomials. Also we define some special cases of our matrix polynomials such as $q-$Chan-Chyan-Srivastava matrix polynomials and $q-$Lagrange-Hermite matrix polynomials and give some results for these matrix polynomials.

2. $q-$Matrix Polynomials in Several Variables

The main object of this section is to present $q-$ matrix polynomials in several variables generated by

$$\prod_{i=1}^{r} \left\{ \frac{(x_i t^{m_i} q^{A_i};q)_\infty}{(x_i t^{m_i};q)_\infty} \right\} = \sum_{n=0}^{\infty} u_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r) t^n$$
$$\left|t\right| < \min \left\{ \left|x_1\right|^{-\frac{1}{m_1}}, ..., \left|x_r\right|^{-\frac{1}{m_r}} \right\} \tag{5}$$

where $0 < |q| < 1$, $A_i \in \mathbb{C}^{N\times N}$ and $m_i \in \mathbb{N}$ for $i = 1,2,...,r$. Note that q must satisfy the condition $|\arg(q)| < \pi$ in the above equation and the rest of paper at the same time. This condition hasn't be rewritten again in the rest of paper.

With the help of (4), take $B \to q^A$, $A \to xt^m I$, we have

$$\sum_{n=0}^{\infty} \frac{(q^A;q)_n}{(q;q)_n} (xt^m)^n = \frac{(xt^m q^A;q)_\infty}{(xt^m;q)_\infty}$$

where $0 < |q| < 1$ and $\|A\| < 1$ in Theorem 1.3 reduces $|t| < |x|^{-\frac{1}{m}}$. Here $q^{-n} \notin \sigma(A)$ for $n = 0,1,2,...$ and $AB = BA$ in Theorem 1.3 are achieved.

Thus, (5) yields the following explicit representation:

$$u_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r) = \sum_{m_1k_1+...+m_rk_r=n} (q^{A_1};q)_{k_1}...(q^{A_r};q)_{k_r} \frac{x_1^{k_1}}{(q;q)_{k_1}} ... \frac{x_r^{k_r}}{(q;q)_{k_r}} \tag{6}$$

where A_i be a matrix in $\mathbb{C}^{N\times N}$ for $i = 1,2,...,r$ and $|t| < \min\left\{|x_1|^{-\frac{1}{m_1}},...,|x_r|^{-\frac{1}{m_r}}\right\}$. For the special case $q \to 1^-$, (5) reduces to

$$\prod_{i=1}^{r}\left\{(1-x_i t^{m_i})^{-A_i}\right\} = \sum_{n=0}^{\infty} u_n^{(A_1,...,A_r)}(x_1,...,x_r)t^n \qquad |t| < \min\left\{|x_1|^{-\frac{1}{m_1}},...,|x_r|^{-\frac{1}{m_r}}\right\} \tag{7}$$

given by Erkus-Duman in [6]. Also, for $m_i = 1$ in (7), we have Chan-Chyan-Srivastava matrix polynomials in [6] and for $m_i = i$ in (7), we get multivariable Lagrange- Hermite matrix polynomials in [6].

We notice that the case $N = 1$, $A_i = \alpha_i$ in (5) reduces to the $q-$extension of the Erkus-Srivastava polynomials in several variables introduced by Erkus-Duman [5]. In this case, it is generated by

$$\prod_{i=1}^{r}\left\{\frac{1}{(x_i t^{m_i};q)_{\alpha_i}}\right\} = \sum_{n=0}^{\infty} u_{n,q}^{(\alpha_1,...,\alpha_r)}(x_1,...,x_r)t^n \qquad |t| < \min\left\{|x_1|^{-\frac{1}{m_1}},...,|x_r|^{-\frac{1}{m_r}}\right\}, \quad \alpha_i \in \mathbb{C}. \tag{8}$$

It is clear that the case $N = 1$, $m_i = 1$, $A_i = \alpha_i$ of the matrix polynomials given by (5) reduces to $q-$Lagrange polynomials in several variables, which are generated by [1]:

$$\prod_{i=1}^{r}\left\{\frac{1}{(x_i t;q)_{\alpha_i}}\right\} = \sum_{n=0}^{\infty} g_{n,q}^{(\alpha_1,...,\alpha_r)}(x_1,...,x_r)t^n \qquad |t| < \min\left\{|x_1|^{-1},...,|x_r|^{-1}\right\}, \ \alpha_i \in \mathbb{C} \tag{9}$$

and also $q \to 1^-$ in (9), it reduces Lagrange polynomials in several variables or Chan-Chyan-Srivastava polynomials are given in [22]. For $N = 1$, if $m_i = i$, $A_i = \alpha_i$ and also $q \to 1^-$ in (5), it also reduces Lagrange-Hermite polynomials, which are given by [2].

3. Bilinear and Bilateral Generating Matrix Functions

In this part, we obtain a number of families of bilateral and bilinear generating matrix functions for $q-$multivariable polynomials which have generating function in (5) and explicit representation in (6) with the help of the similar way as in [23].

We begin the following main theorem.

Theorem 3.1. *Corresponding to an non-vanishing function $\Omega_{\mu}(\mathbf{y})$ of (s complex variables $y_1, ..., y_s$ ($s \in \mathbb{N}$)) with complex order μ, let*

$$\Lambda_{\mu,\nu}(\mathbf{y};w) := \sum_{k=0}^{\infty} a_k \Omega_{\mu+\nu k}(\mathbf{y}) w^k \tag{10}$$

where ($a_k \neq 0$, $\mu, \nu \in \mathbb{C}$) *and*

$$_q\Theta_{n,p}^{\mu,\nu}(\mathbf{x};\mathbf{y};z) := \sum_{k=0}^{[n/p]} a_k u_{n-pk,q}^{(A_1,...,A_r)}(\mathbf{x}) \Omega_{\mu+\nu k}(\mathbf{y}) z^k \tag{11}$$

where $n, p \in \mathbb{N}$; $A_i \in \mathbb{C}^{N\times N}$; $\mathbf{x} = (x_1, ..., x_r)$; $\mathbf{y} = (y_1, ..., y_r)$; $(i = 1, 2, ..., r)$. Then we have

$$\sum_{n=0}^{\infty} {}_q\Theta_{n,p}^{\mu,\nu}\left(\mathbf{x};\mathbf{y};\frac{\eta}{t^p}\right)t^n = \prod_{i=1}^{r}\left\{\frac{(x_i t^{m_i} q^{A_i};q)_\infty}{(x_i t^{m_i};q)_\infty}\right\}\Lambda_{\mu,\nu}(\mathbf{y};\eta) \tag{12}$$

provided that (12) exists for $0 < \left|q\right| < 1$, $|t| < \min\left\{|x_1|^{-\frac{1}{m_1}}, ..., |x_r|^{-\frac{1}{m_r}}\right\}$ and $m_i \in \mathbb{N}$ $(i = 1, 2, ..., r)$.

Proof. The left-hand side of the equality (12) of Theorem 3.1 is denoted by S. Then, upon substituting for the polynomials

$$_q\Theta_{n,p}^{\mu,\nu}\left(\mathbf{x};\mathbf{y};\frac{\eta}{t^p}\right)$$

from the definition of function (11) into the left-hand side of (12), we obtain

$$S = \sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} a_k u_{n-pk,q}^{(A_1,...,A_r)}(\mathbf{x})\Omega_{\mu+\nu k}(\mathbf{y})\eta^k t^{n-pk}. \tag{13}$$

Write $n + pk$ instead of n, we have

$$\begin{aligned} S &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_k u_{n,q}^{(A_1,...,A_r)}(\mathbf{x})\Omega_{\mu+\nu k}(\mathbf{y})\eta^k t^n \\ &= \left(\sum_{n=0}^{\infty} u_{n,q}^{(A_1,...,A_r)}(\mathbf{x}) t^n\right)\left(\sum_{k=0}^{\infty} a_k\Omega_{\mu+\nu k}(\mathbf{y})\eta^k\right) \\ &= \prod_{i=1}^{r}\left\{\frac{(x_i t^{m_i} q^{A_i};q)_\infty}{(x_i t^{m_i};q)_\infty}\right\}\Lambda_{\mu,\nu}(\mathbf{y};\eta), \end{aligned}$$

which is the desired result. □

In a similar way, we can obtain the next lemma.

Lemma 3.2. *For $u_{n,q}^{(A_1,...,A_r)}(x_1, ..., x_r)$, the following formula holds:*

$$\begin{aligned} & u_{n,q}^{(A_1+B_1,...,A_r+B_r)}(x_1, ..., x_r) \\ = & \sum_{k=0}^{n} u_{n-k,q}^{(A_1,...,A_r)}(x_1, ..., x_r) u_{k,q}^{(B_1,...,B_r)}(x_1 q^{A_1}, ..., x_r q^{A_r}) \end{aligned}$$

where A_i and B_i are matrices in $\mathbb{C}^{N\times N}$ satisfying conditions $q^{-n} \notin \sigma(x_i t^{m_i} q^{A_i})$, $n = 0, 1, 2, ...$, $\left\|q^{A_i}\right\| < |x_i t^{m_i}|^{-1}$ for $i = 1, 2, ..., r$, these matrices which commute with one another, $0 < \left|q\right| < 1$, $|t| < \min\left\{|x_1|^{-\frac{1}{m_1}}, ..., |x_r|^{-\frac{1}{m_r}}\right\}$ and $m_i \in \mathbb{N}$ $(i = 1, 2, ..., r)$.

Proof. *It is enough to take $A_i \to A_i + B_i$ in (5) and use Theorem 1.3 for proof.* $\square$

Theorem 3.3. *For a non-vanishing function $\Omega_\mu(\mathbf{y})$ of s complex variables $y_1, ..., y_s$ $(s \in \mathbb{N})$ and for $p \in \mathbb{N}$, $\mu, \nu \in \mathbb{C}$, $\mathbf{y} = (y_1, ..., y_s)$, $A_i, B_i \in \mathbb{C}^{N\times N}$ for $i = 1, 2, ..., r$ satisfy the conditions in Lemma 3.2, let*

$$ {}_q\Xi^{n,p}_{\mu,\nu}(\mathbf{x};\mathbf{y};z) := \sum_{k=0}^{[n/p]} a_k u^{(A_1+B_1,...,A_r+B_r)}_{n-pk,q}(x_1, ..., x_r)\Omega_{\mu+\nu k}(\mathbf{y})z^k \tag{14} $$

where $a_k \neq 0$; $n, k \in \mathbb{N}_0$. Then we derive

$$ \begin{aligned} &\sum_{k=0}^{n}\sum_{l=0}^{[k/p]} a_l u^{(A_1,...,A_r)}_{n-k,q}(x_1, ..., x_r) u^{(B_1,...,B_r)}_{k-pl,q}(x_1 q^{A_1}, ..., x_r q^{A_r})\ \Omega_{\mu+\nu l}(\mathbf{z})w^l \\ = &\ {}_q\Xi^{n,p}_{\mu,\nu}(\mathbf{x};\mathbf{y};z) \end{aligned} \tag{15} $$

provided that (15) exists where all matrices are commutative and $0 < |q| < 1$.

For example, setting

$$ s = 1 \text{ and } \Omega_{\mu+\nu k}(y) = L^{(E,\lambda)}_{\mu+\nu k}(y),\ (\mu, \nu \in \mathbb{N}_0) $$

in Theorem 3.1, where the $n-$th Laguerre matrix polynomials $L^{(E,\lambda)}_n(x)$ are given by [8]

$$ L^{(E,\lambda)}_n(x) = \sum_{k=0}^{n} \frac{(-1)^k \lambda^k}{k!\,(n-k)!} (E+I)_n \left[(E+I)_k\right]^{-1} x^k, $$

where E is a matrix in $\mathbb{C}^{N\times N}$, $E + sI$ is an invertible matrix for every integer $s \geq 0$ and λ is a complex number satisfying $Re(\lambda) > 0$ and they have generating function

$$ \sum_{n=0}^{\infty} L^{(E,\lambda)}_n(x)\eta^n = (1-\eta)^{-(E+I)} \exp\left(\frac{-\lambda x\eta}{1-\eta}\right), \tag{16} $$

$$ |\eta| < 1,\ 0 < x < \infty, $$

then we derive a new kind of bilateral generating matrix functions for $u^{(A_1,...,A_r)}_{n,q}(x_1, ..., x_r)$ and $L^{(E,\lambda)}_n(x)$.

Corollary 3.4. *If $\Lambda_{\mu,\nu}(y;w) := \sum\limits_{k=0}^{\infty} a_k L^{(E,\lambda)}_{\mu+\nu k}(y)w^k$ where $(a_k \neq 0,\ \ \mu, \nu \in \mathbb{N}_0)$; and*

$$ {}_q\Theta^{\mu,\nu}_{n,p}(\mathbf{x};y;z) := \sum_{k=0}^{[n/p]} a_k u^{(A_1,...,A_r)}_{n-pk,q}(\mathbf{x}) L^{(E,\lambda)}_{\mu+\nu k}(y)z^k $$

where $n, p \in \mathbb{N}$, then it satisfies

$$ \sum_{n=0}^{\infty} {}_q\Theta^{\mu,\nu}_{n,p}\left(\mathbf{x};y;\frac{\eta}{t^p}\right)t^n = \prod_{i=1}^{r}\left\{\frac{(x_i t^{m_i} q^{A_i};q)_\infty}{(x_i t^{m_i};q)_\infty}\right\}\Lambda_{\mu,\nu}(y;\eta) \tag{17} $$

provided that (17) exists for $0 < |q| < 1$, $|t| < \min\left\{|x_1|^{-\frac{1}{m_1}}, ..., |x_r|^{-\frac{1}{m_r}}\right\}$ and $m_i \in \mathbb{N}$ $(i = 1, 2, ..., r)$.

Remark 3.5. *For the Laguerre matrix polynomials, by the generating relation (16) and $a_k = 1, \mu = 0, \nu = 1$, we have*

$$\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} u_{n-pk,q}^{(A_1,...,A_r)}(\mathbf{x}) L_k^{(E,\lambda)}(y)\eta^k t^{n-pk} = \prod_{i=1}^{r}\left\{\frac{(x_i t^{m_i} q^{A_i};q)_\infty}{(x_i t^{m_i};q)_\infty}\right\}(1-\eta)^{-(E+I)}\exp\left(\frac{-\lambda y\eta}{1-\eta}\right)$$

where $\left|\eta\right| < 1,\ \ 0 < y < \infty.$

Remark 3.6. *In Theorem 3.1, setting $\Omega_{\mu+\nu k}(z) = u_{n,q}^{(B_1,...,B_r)}(y_1,...,y_r)(B_i \in \mathbb{C}^{N\times N})$ and taking $a_k = 1, \mu = 0, \nu = 1$, we obtain bilinear generating matrix function for $u_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r)$:*

$$\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} u_{n-pk,q}^{(A_1,...,A_r)}(x_1,...,x_r) u_{k,q}^{(B_1,...,B_r)}(y_1,...,y_r)\eta^k t^{n-pk} = \prod_{i=1}^{r}\left\{\frac{(x_i t^{m_i} q^{A_i};q)_\infty}{(x_i t^{m_i};q)_\infty}\right\}\prod_{i=1}^{r}\left\{\frac{(y_i \eta^{m_i} q^{B_i};q)_\infty}{(y_i \eta^{m_i};q)_\infty}\right\}$$

where $0 < \left|q\right| < 1,\ \left|\eta\right| < \min\left\{\left|y_1\right|^{-\frac{1}{m_1}},...,\left|y_r\right|^{-\frac{1}{m_r}}\right\}$ *and* $m_i \in \mathbb{N}\ \ (i = 1,2,...,r)$.

For every appropriate option of the a_k $(k \in \mathbb{N}_0)$, if the multivariable function $\Omega_{\mu+\nu k}(\mathbf{y})$, $\mathbf{y} = (y_1,...,y_s)$, $(s \in \mathbb{N})$, is represented as an appropriate product of a number of simpler functions, the results of Theorem 3.1 can be carried out in order to derive varied families of multilateral and multilinear generating matrix functions for function $u_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r)$ given explicitly by (6).

4. $q-$Chan-Chyan-Srivastava Matrix Polynomials

For $m_i = 1$ and $0 < \left|q\right| < 1$ in (5), we define $q-$Chan-Chyan-Srivastava matrix polynomials as follows:

$$\prod_{i=1}^{r}\left\{\frac{(x_i t q^{A_i};q)_\infty}{(x_i t;q)_\infty}\right\} = \sum_{n=0}^{\infty} g_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r)t^n \qquad |t| < \min\left\{|x_1|^{-1},...,|x_r|^{-1}\right\} \tag{18}$$

where $A_i \in \mathbb{C}^{N\times N}$ $(i = 1,2,...,r)$ or

$$g_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r) = \sum_{k_1+...+k_r=n}(q^{A_1};q)_{k_1}...(q^{A_r};q)_{k_r}\frac{x_1^{k_1}}{(q;q)_{k_1}}...\frac{x_r^{k_r}}{(q;q)_{k_r}} \tag{19}$$

where A_i be a matrix in $\mathbb{C}^{N\times N}$ for $i = 1,2,...,r$ and $|t| < \min\left\{|x_1|^{-1},...,|x_r|^{-1}\right\}$.

Theorem 4.1. *Corresponding to an non-vanishing function $\Omega_\mu(\mathbf{y})$ (s complex variables $y_1,...,y_s$ $(s \in \mathbb{N})$) with complex order μ, let*

$$\Lambda_{\mu,\nu}(\mathbf{y};w) := \sum_{k=0}^{\infty} a_k \Omega_{\mu+\nu k}(\mathbf{y}) w^k \tag{20}$$

where ($a_k \neq 0\,,\ \mu,\nu \in \mathbb{C}$) *and*

$$ {}_{q}\Theta_{n,p}^{\mu,\nu}(\mathbf{x};\mathbf{y};z) := \sum_{k=0}^{[n/p]} a_k g_{n-pk,q}^{(A_1,...,A_r)}(\mathbf{x})\Omega_{\mu+\nu k}(\mathbf{y})z^k \tag{21} $$

where $n,p \in \mathbb{N}$; $A_i \in \mathbb{C}^{N\times N}$; $\mathbf{x} = (x_1,...,x_r)$; $\mathbf{y} = (y_1,...,y_r)$; $(i = 1,2,...,r)$. *Then we obtain*

$$ \sum_{n=0}^{\infty} {}_{q}\Theta_{n,p}^{\mu,\nu}\left(\mathbf{x};\mathbf{y};\frac{\eta}{t^p}\right)t^n = \prod_{i=1}^{r}\left\{\frac{(x_i t q^{A_i};q)_\infty}{(x_i t;q)_\infty}\right\}\Lambda_{\mu,\nu}(\mathbf{y};\eta) \tag{22} $$

provided that (22) exists for $0 < \left|q\right| < 1,\ |t| < \min\left\{|x_1|^{-1},...,|x_r|^{-1}\right\}$.

Lemma 4.2. *For* $g_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r)$, *the following formula holds:*

$$ \begin{aligned} & g_{n,q}^{(A_1+B_1,...,A_r+B_r)}(x_1,...,x_r) \\ = & \sum_{k=0}^{n} g_{n-k,q}^{(A_1,...,A_r)}(x_1,...,x_r)g_{k,q}^{(B_1,...,B_r)}(x_1 q^{A_1},...,x_r q^{A_r}) \end{aligned} $$

where A_i *and* B_i *are matrices in* $\mathbb{C}^{N\times N}$ *satisfying conditions* $q^{-n} \notin \sigma(x_i t q^{A_i})$, $n = 0,1,2,...$, $\left\|q^{A_i}\right\| < |x_i t|^{-1}$ *for* $i = 1,2,...,r$, *these matrices which commute with one another,* $0 < \left|q\right| < 1,\ |t| < \min\left\{|x_1|^{-1},...,|x_r|^{-1}\right\}$.

Theorem 4.3. *For a non-vanishing function* $\Omega_\mu(\mathbf{y})$ *of s complex variables* $y_1,...,y_s$ $(s \in \mathbb{N})$ *and for* $p \in \mathbb{N}$, $\mu,\nu \in \mathbb{C}$, $\mathbf{y} = (y_1,...,y_s)$, $A_i, B_i \in \mathbb{C}^{N\times N}$ *for* $i = 1,2,...,r$ *satisfy the conditions in Lemma 4.2, let*

$$ {}_{q}\Xi_{\mu,\nu}^{n,p}(\mathbf{x};\mathbf{y};z) := \sum_{k=0}^{[n/p]} a_k g_{n-pk,q}^{(A_1+B_1,...,A_r+B_r)}(x_1,...,x_r)\Omega_{\mu+\nu k}(\mathbf{y})z^k \tag{23} $$

where $a_k \neq 0$; $n,k \in \mathbb{N}_0$. *One can get*

$$ \begin{aligned} & \sum_{k=0}^{n}\sum_{l=0}^{[k/p]} a_l g_{n-k,q}^{(A_1,...,A_r)}(x_1,...,x_r)g_{k-pl,q}^{(B_1,...,B_r)}(x_1 q^{A_1},...,x_r q^{A_r})\ \Omega_{\mu+\nu l}(\mathbf{z})w^l \\ = & {}_{q}\Xi_{\mu,\nu}^{n,p}(\mathbf{x};\mathbf{y};z) \end{aligned} \tag{24} $$

provided that (24) exists where all matrices are commutative, $0 < \left|q\right| < 1,\ |t| < \min\left\{|x_1|^{-1},...,|x_r|^{-1}\right\}$.

5. $q-$Lagrange-Hermite matrix polynomials

For $m_i = i$ and $0 < \left|q\right| < 1$ in (5), we define $q-$Lagrange-Hermite matrix polynomials as follows:

$$ \begin{aligned} & \prod_{i=1}^{r}\left\{\frac{(x_i t^i q^{A_i};q)_\infty}{(x_i t^i;q)_\infty}\right\} = \sum_{n=0}^{\infty} h_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r)t^n \\ & \qquad |t| < \min\left\{|x_1|^{-\frac{1}{1}},...,|x_r|^{-\frac{1}{r}}\right\} \end{aligned} \tag{25} $$

where $A_i \in \mathbb{C}^{N\times N}$ for $i = 1,2,...,r$ or

$$ \begin{aligned} & h_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r) \\ = & \sum_{k_1+2k_2+...+rk_r=n} (q^{A_1};q)_{k_1}...(q^{A_r};q)_{k_r}\frac{x_1^{k_1}}{(q;q)_{k_1}}...\frac{x_r^{k_r}}{(q;q)_{k_r}} \end{aligned} \tag{26} $$

where A_i be a matrix in $\mathbb{C}^{N\times N}$ for $i = 1,2,...,r$ and $|t| < \min\left\{|x_1|^{-\frac{1}{1}},...,|x_r|^{-\frac{1}{r}}\right\}$.

Theorem 5.1. *Corresponding to an non-vanishing function $\Omega_\mu(\mathbf{y})$ (s complex variables $y_1,...,y_s$ $(s\in\mathbb{N})$) with complex order μ, let*

$$\Lambda_{\mu,\nu}(\mathbf{y};w):=\sum_{k=0}^{\infty}a_k\Omega_{\mu+\nu k}(\mathbf{y})w^k \tag{27}$$

where $(a_k\neq 0\,,\ \mu,\nu\in\mathbb{C})$; $\mathbf{y}=(y_1,...,y_s)$ and

$$_q\Theta_{n,p}^{\mu,\nu}(\mathbf{x};\mathbf{y};z):=\sum_{k=0}^{[n/p]}a_k h_{n-pk,q}^{(A_1,...,A_r)}(\mathbf{x})\Omega_{\mu+\nu k}(\mathbf{y})z^k \tag{28}$$

where $n,p\in\mathbb{N}$; $A_i\in\mathbb{C}^{N\times N}$; $\mathbf{x}=(x_1,...,x_r)$; $\mathbf{y}=(y_1,...,y_r)$; $(i=1,2,...,r)$. Then we have

$$\sum_{n=0}^{\infty}\ {_q\Theta_{n,p}^{\mu,\nu}}\left(\mathbf{x};\mathbf{y};\frac{\eta}{t^p}\right)t^n=\prod_{i=1}^{r}\left\{\frac{(x_it^iq^{A_i};q)_\infty}{(x_it^i;q)_\infty}\right\}\Lambda_{\mu,\nu}(\mathbf{y};\eta) \tag{29}$$

provided that (29) exists for $0<\left|q\right|<1,\ |t|<\min\left\{|x_1|^{-\frac{1}{1}},...,|x_r|^{-\frac{1}{r}}\right\}$.

Lemma 5.2. *For $h_{n,q}^{(A_1,...,A_r)}(x_1,...,x_r)$, the following formula holds:*

$$h_{n,q}^{(A_1+B_1,...,A_r+B_r)}(x_1,...,x_r) = \sum_{k=0}^{n}h_{n-k,q}^{(A_1,...,A_r)}(x_1,...,x_r)h_{k,q}^{(B_1,...,B_r)}(x_1q^{A_1},...,x_rq^{A_r})$$

where A_i and B_i are matrices in $\mathbb{C}^{N\times N}$ satisfying conditions $q^{-n}\notin\sigma(x_it^iq^{A_i})$, $n=0,1,2,...$, $\left\|q^{A_i}\right\|<\left|x_it^i\right|^{-1}$ for $i=1,2,...,r$, these matrices which commute with one another, $0<\left|q\right|<1,\ |t|<\min\left\{|x_1|^{-\frac{1}{1}},...,|x_r|^{-\frac{1}{r}}\right\}$.

Theorem 5.3. *For a non-vanishing function $\Omega_\mu(\mathbf{y})$ of s complex variables $y_1,...,y_s$ $(s\in\mathbb{N})$ and for $p\in\mathbb{N}$, $\mu,\nu\in\mathbb{C}$, $\mathbf{y}=(y_1,...,y_s)$, $A_i,B_i\in\mathbb{C}^{N\times N}$ for $i=1,2,...,r$ satisfy the condition Lemma 5.2, let*

$$_q\Xi_{\mu,\nu}^{n,p}(\mathbf{x};\mathbf{y};z):=\sum_{k=0}^{[n/p]}a_k h_{n-pk,q}^{(A_1+B_1,...,A_r+B_r)}(x_1,...,x_r)\Omega_{\mu+\nu k}(\mathbf{y})z^k \tag{30}$$

where $a_k\neq 0$; $n,k\in\mathbb{N}_0$. Then it is hold

$$\sum_{k=0}^{n}\sum_{l=0}^{[k/p]}a_l h_{n-k,q}^{(A_1,...,A_r)}(x_1,...,x_r)h_{k-pl,q}^{(B_1,...,B_r)}(x_1q^{A_1},...,x_rq^{A_r})\ \Omega_{\mu+\nu l}(\mathbf{z})w^l = {_q\Xi_{\mu,\nu}^{n,p}}(\mathbf{x};\mathbf{y};z) \tag{31}$$

provided that (31) exists where all matrices are commutative, $0<\left|q\right|<1,\ |t|<\min\left\{|x_1|^{-\frac{1}{1}},...,|x_r|^{-\frac{1}{r}}\right\}$.

Acknowledgement. The author would like to thank Professor H.M. Srivastava for his valuable suggestions which improved this article considerably.

References

[1] A. Altın, E. Erkuş and F. Taşdelen. The $q-$ Lagrange polynomials in several variables, *Taiwanese J. Math.* 10 (5) (2006) 1131-1137.
[2] A. Altın and E. Erkuş. On a multivariable extension of the Lagrange-Hermite polynomials, *Integral Transform Spec. Funct.* 17 (2006) 239-244.

[3] E. Defez, A. Law, J. Villanueva-Oller and R. J. Villanueva. Matrix cubic splines for progressive 3D imaging, *J. Math. Imag. and Vision* 17 (2002) 41-53.

[4] E. Defez, A. Hervás, A. Law, J. Villanueva-Oller and R.J. Villanueva. Progressive transmission of images: PC-based computations, using orthogonal matrix polynomials, *Math. Comput. Modelling* 32 (2000) 1125-1140.

[5] E. Erkuş-Duman, A $q-$ extension of the Erkus-Srivastava polynomials in several variables, *Taiwanese J. Math.* 12 (2) (2008) 539-543.

[6] E. Erkuş-Duman, Matrix extensions of polynomials in several variables, *Util. Math.* 85 (2011) 161-180.

[7] N. Dunford and J. Schwartz, Linear Operators Part I, Addison-Wesley, New York, 1957.

[8] L. Jódar, R. Company and E. Navarro, Laguerre matrix polynomials and system of second-order differential equations, *Appl. Num. Math.* 15 (1994) 53–63.

[9] J. Sastre and E. Defez, On the asymptotics of Laguerre matrix polynomials for large x and n, *Appl. Math. Lett.* 19 (8) (2006) 721–727.

[10] L. Jódar, E. Defez and E. Ponsoda, Matrix quadrature and orthogonal matrix polynomials, *Congressus Numerantium* 106 (1995) 141-153.

[11] J. Sastre, E. Defez and L. Jódar, Application of Laguerre matrix polynomials to the numerical inversion of Laplace transforms of matrix functions, *Appl. Math. Lett.* 24 (9) (2011) 1527–1532.

[12] J. Sastre, J. Ibáñez, E. Defez and P. Ruiz, Efficient orthogonal matrix polynomial based method for computing matrix exponential, *Appl. Math. Comput.* 217 (14)(2011) 6451–6463.

[13] J. Cao and H. M. Srivastava, Some q-generating functions of the Carlitz and Srivastava-Agarwal types associated with the generalized Hahn polynomials and the generalized Rogers-Szegö polynomials, *Appl. Math.Comput*. 219 (2013) 8398-8406.

[14] M. Mursaleen, A. Khan, H. M. Srivastava and K. S. Nisar, Operators constructed by means of q-Lagrange polynomials and A-statistical approximation, *Appl. Math. Comput.* 219 (2013) 6911-6818.

[15] H. M. Srivastava, Some generalizations and basic (or $q-$) extensions of the Bernoulli, Euler and Genocchi polynomials, *Appl. Math. Inform.Sci*. 5 (2011) 390-444.

[16] Q.-M. Luo and H. M. Srivastava, q-Extensions of some relationships between the Bernoulli and Euler polynomials, *Taiwanese J. Math.* 15 (2011) 241-257.

[17] J. Choi, P. J. Anderson and H. M. Srivastava, Carlitz's q-Bernoulli and q-Euler numbers and polynomials and a class of q-Hurwitz zeta functions, *Appl. Math. Comput.* 215 (2009) 1185-1208.

[18] A. Salem. On a $q-$gamma and $q-$beta matrix functions, *Linear and Multilinear Algebra* 60 (6) (2012) 683-696.

[19] E. Erkuş-Duman and O. Duman, Statistical approximation properties of high order operators constructed with the Chan-Chyan-Srivastava polynomials, *Appl. Math. Comput.* 218 (5) (2011) 1927-1933.

[20] S.-J. Liu, C.-J. Chyan, H.-C. Lu and H. M. Srivastava, Bilateral generating functions for the Chan-Chyan-Srivastava polynomials and the generalized Lauricella functions, *Integral Transforms Spec. Funct.* 23 (7)(2012) 539-549.

[21] E. Erkuş-Duman, O. Duman and H. M. Srivastava, Statistical approximation of certain positive linear operators constructed by means of the Chan-Chyan-Srivastava polynomials, *Appl. Math. Comput.* 182 (1) (2006) 213-222.

[22] W.-Ch. C. Chan, Ch.-J. Chyan and H. M. Srivastava, The Lagrange polynomials in several variables, *Integral Transforms Spec. Funct.* 12 (2) (2001) 139-148.

[23] E. Erkus and H. M. Srivastava, A unified presentation of some families of multivariable polynomials, *Integral Transforms Spec. Funct.* 17 (2006) 267-273.

[24] H. M. Srivastava, K. S. Nisar and M. A. Khan, Some umbral calculus presentations of the Chan-Chyan-Srivastava polynomials and the Erkus-Srivastava polynomials, *Proyecciones J. Math.* 33 (2014) 77-90.