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Soft Idealization of a Decomposition Theorem

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Abstract. Firstly, we define a new set called soft regular-$\tilde{I}$-closed set in soft ideal topological spaces and obtain some properties of it. Secondly, to obtain a decomposition of continuity in these spaces, we introduce two notions of soft Hayashi-Samuels space and soft $A_{\tilde{I}}$-set. Finally, we give a decomposition of continuity in a domain Hayashi-Samuels space and in a range soft topological space.

1. Introduction

To avoid indefinite terms in mathematics, various concepts such as fuzzy sets [22], vague sets [10], rough sets [18] and interval mathematics [5], have been developed by research from different areas. But in the areas such as economics, engineering, social science, medical science; these concepts do not respond some problems.

It was Molodtsov [15] who first introduced soft set theory in 1999 to solve these problems. To illustrate the efficiency of the soft set theory, Molodtsov et al. [15, 16] applied to various systems such as game theory, Riemann integration, Perron integration, smoothness of functions, operations research, probability, theory of measurement, and so on. In [8], a method named uni-int decision making method is presented by using reintroduced operations in soft set theory.

In 2011, Shabir and Naz [20] used soft set theory on topological spaces and called it soft topological spaces. Also, they introduced the notions of soft interior operator, soft closure operator, soft subspaces, soft separation axioms etc. and investigated some properties of them. In 2012, Aygunoğlu and Aygun [6] defined the notions of soft continuity, soft product topology. One year later, Nazmul and Samatha [17] introduced the concept of neighborhoods on soft topological spaces. At the same year, the notion of soft ideal was given by Sahin and Kucuk [19]. This notion was studied by Kandil et al. [12] and some notions related to it were defined such as soft local functions, soft $*$-topology, soft semi-I-compact spaces and soft connected spaces. These concepts are studied to get new soft topologies from the original one, named soft topological spaces with soft ideal. Akdag and Erol [2] presented the concept of soft I-open sets and studied some properties of soft I-openess. They also studied soft I-open and soft I-continuous functions. Concerning these functions, they obtained some characterizations and several properties of these functions. In 2015, Kandil et al. [13] introduced the notions of I-openness, pre-opennes, α-openness, semi-openness and β-openness in soft topological for soft sets and their continuity in soft topological spaces.

The purpose of this paper is to define a new set called soft regular-$\tilde{I}$-closed set in soft ideal topological spaces and obtain some properties of it.

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2. Preliminaries

Let U be an initial universe set and E be a collection of all probable parameters with respect to U. Here the parameters are characteristics or properties of objects in U. Let $P(U)$ denote the power set of U, and let $A \subseteq E$.

Definition 2.1. ([15]) A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \longrightarrow P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A). The family of all these soft sets denoted by $SS(U)_A$.

Definition 2.2. ([14]) For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if (i) $A \subseteq B$, and (ii) $\forall e \in A$, $F(e) \subseteq G(e)$ are identical approximations. We write $(F, A) \widetilde{\subseteq} (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \widetilde{\supseteq} (G, B)$.

Definition 2.3. ([14]) A soft set (F, A) over U is said to be

a) null soft set denoted by Φ, if $\forall e \in A$, $F(e) = \phi$.

b) absolute soft set denoted by $\widetilde{A}$, if $\forall e \in A$, $F(e) = U$.

Definition 2.4. For two soft sets (F, A) and (G, B) over a common universe U,

a) ([14]) union of two soft sets of (F, A) and (G, B) is the soft set (H, C), where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & , \quad \text{if } e \in A - B \\ G(e) & , \quad \text{if } e \in B - A \\ F(e) \cup G(e) & , \quad \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \widetilde{\cup} (G, B) = (H, C)$.

b) ([9]) intersection of (F, A) and (G, B) is the soft set (H, C), where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A) \widetilde{\cap} (G, B) = (H, C)$.

Let X be an initial universe set and E be the non-empty set of parameters.

Definition 2.5. ([20]) Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ is read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$. $x \notin (F, E)$, if $x \notin F(e)$ for some $e \in E$.

Definition 2.6. ([20]) Let Y be a non-empty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.

Definition 2.7. ([3]) The relative complement of a soft set (F, E) is denoted by $(F, E)'$and is defined by $(F, E)' = (F', E)$ where $F' : E \longrightarrow P(X)$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in E$.

Definition 2.8. ([20]) Let τ be the collection of soft sets over X, then τ is said to be soft topology on X if

1) Φ, $\widetilde{X}$ belong to τ,

2) the union of any number of soft sets in τ belongs to τ,

3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open sets in X.

We will denote all soft open sets(resp. soft closed sets) as $SO(X)$ (resp. $SC(X)$) in X.

Definition 2.9. ([20]) Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ.

Definition 2.10. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then:

a) soft interior [11] of the soft set (F, E) is denoted by $int(F, E)$ and is defined as the union of all soft open sets contained in (F, E). Thus $int(F, E)$ is the largest soft open set contained in (F, E).

b) soft closure [20] of (F, E), denoted by $cl(F, E)$ is the intersection of all soft closed super sets of (F, E). Clearly $cl(F, E)$ is the smallest soft closed set over X which contains (F, E).

Proposition 2.11. ([11]) *Let (X, τ, E) be a soft topological space over X and (F, E) and (G, E) be a soft set over X. Then:*

a) $int\,(int\,(F, E)) = int\,(F, E)$.

b) $(F, E) \mathrel{\widetilde{\subseteq}} (G, E)$ *imples* $int\,(F, E) \mathrel{\widetilde{\subseteq}} int(G, E)$.

c) $cl\,(cl\,(F, E)) = cl\,(F, E)$.

d) $(F, E) \mathrel{\widetilde{\subseteq}} (G, E)$ *imples* $cl\,(F, E) \mathrel{\widetilde{\subseteq}} cl(G, E)$.

Definition 2.12. ([7]) Let (F, E) be a soft set X. The soft set (F, E) is called a soft point, denoted by (x_e, E) *or* x_e, if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E - \{e\}$.

Definition 2.13. ([23]) The *soft point* x_e is said to belong to the soft set (G, E), denoted by $x_e \in (G, E)$, if for the element $e \in E$, $F(e) \mathrel{\widetilde{\subseteq}} G(e)$.

Definition 2.14. ([23]) A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft point x_e if there exists an open soft set (H, E) such that $x_e \in (H, E) \mathrel{\widetilde{\subseteq}} (G, E)$. A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft set (F, E) if there exists an open soft set (H, E) such that $(F, E) \mathrel{\widetilde{\subseteq}} (H, E) \mathrel{\widetilde{\subseteq}} (G, E)$. The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$, is the family of all its neighborhoods.

Definition 2.15. ([1]) Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \longrightarrow Y$ and $p : A \longrightarrow B$ be mappings. Let $f_{pu} : SS(X)_A \longrightarrow SS(Y)_B$ be mapping. Then:

1) If $(F, A) \in SS(X)_A$. Then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b) = \begin{cases} \bigcup_{a \in p^{-1}(b) \cap A} u(F(a)) & , \quad p^{-1}(b) \cap A \neq \phi \\ \phi & , \quad \text{otherwise.} \end{cases}$$

for all $b \in B$.

2) Let $(G, B) \in SS(Y)_B$. The inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))) & , \quad p(a) \in B \\ \phi & , \quad \text{otherwise.} \end{cases}$$

for all $a \in A$.

Definition 2.16. Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is said to be

a) soft α-open [4] if $(F, E) \widetilde{\subseteq} int(cl(int(F, E)))$,

b) soft *pre*-open [4] if $(F, E) \widetilde{\subseteq} int(cl(F, E))$,

c) soft regular closed [21] if $(F, E) = cl(int(F, E))$.

Definition 2.17. ([12]) Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with a fixed set E if

a) $(F, E) \in \tilde{I}$ and $(G, E) \in \tilde{I} \Rightarrow (F, E) \widetilde{\cup} (G, E) \in \tilde{I}$,

b) $(F, E) \in \tilde{I}$ and $(G, E) \widetilde{\subseteq} (F, E) \Rightarrow (G, E) \in \tilde{I}$

i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Definition 2.18. ([12]) Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same of parameters E. Then

$$(F, E)^*(\tilde{I}, \tau)(orF^*_E) = \widetilde{\cup}\{x_e \in \varepsilon : O_{x_e} \widetilde{\cap} (F, E) \notin \tilde{I}, \forall O_{x_e} \in \tau\}$$

is called the soft local function of (F, E) with respect to $\tilde{I}$ and τ, where O_{x_e} is a τ-open soft set containing x_e.

Theorem 2.19. ([12]) *Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same of parameters E. Then the soft closure operator $cl^* : SS(X)_E \to SS(X)_E$ defined by: $cl^*(F, E) = (F, E) \widetilde{\cup} (F, E)^*$ satisfies Kuratowski's axioms.*

Theorem 2.20. ([12]) *Let $\tilde{I}$ and $\tilde{J}$ be any two soft ideals with the same set of parameters E on a soft topological space (X, τ, E). Let (F, E), $(G, E) \in SS(X)_E$. Then:*

1) $(\Phi)^* = \Phi$,

2) $(F, E) \widetilde{\subseteq} (G, E) \Rightarrow (F, E)^* \widetilde{\subseteq} (G, E)^*$,

3) $\tilde{I} \subseteq \tilde{J} \Rightarrow (F, E)^*(\tilde{J}) \widetilde{\subseteq} (F, E)^*(\tilde{I})$,

4) $(F, E)^* \widetilde{\subseteq} cl(F, E)$, *where cl is the soft closure w.r.t.*τ,

5) $(F, E)^*$ *is* τ*-closed soft set*,

6) $((F, E)^*)^* \widetilde{\subseteq} (F, E)^*$,

7) $((F, E) \widetilde{\cup} (G, E))^* = (F, E)^* \widetilde{\cup} (G, E)^*$,

8) $((F, E) \widetilde{\cap} (G, E))^* \widetilde{\subseteq} (F, E)^* \widetilde{\cap} (G, E)^*$,

9) $(H, E) \in \tilde{I} \Rightarrow ((F, E) \widetilde{\cup} (H, E))^* = (F, E)^* = ((F, E) - (H, E))^*$.

Definition 2.21. ([13]) Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is said to be

a) soft semi-$\tilde{I}$-open if $(F, E) \widetilde{\subseteq} cl^*(int(F, E))$,

b) soft α-$\tilde{I}$-open if $(F, E) \widetilde{\subseteq} int(cl^*(int(F, E)))$,

c) soft pre-$\tilde{I}$-open if $(F, E) \widetilde{\subseteq} int(cl^*(F, E))$.

3. Soft Regular-$\tilde{I}$-Closed Sets

Definition 3.1. Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is said to be

a) soft $*$-dense-in-itself if $(F, E) \widetilde{\subseteq} (F, E)^*$,

b) soft τ^*-closed if $(F, E)^* \widetilde{\subseteq} (F, E)$,

c) soft $*$-perfect if $(F, E) = (F, E)^*$,

d) soft α^*-$\tilde{I}$-open if $int(F, E) = int(cl^*(int(F, E)))$,

e) soft $C_{\tilde{I}}$-set if $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where (G, E) is soft open and (H, E) is soft α^*-$\tilde{I}$-open,

f) soft $\tilde{I}$-locally-closed if $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where (G, E) is soft open and (H, E) is soft $*$-perfect,

g) soft A-set if $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where (G, E) is soft open and (H, E) is soft regular closed.

Example 3.2. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, $\tilde{I}_1 = \{\Phi, (G_1, E), (G_2, E), (G_3, E)\}$, $\tilde{I}_2 = \{\Phi\}$ and $\tilde{I}_3 = \{\Phi, (K, E)\}$ where (F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E), (G_3, E), (K, E) are soft sets over X, defined as follows. $F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_2\}$, $F_2(e_1) = \{x_1, x_2\}$, $F_2(e_2) = \{x_2\}$, $F_3(e_1) = \{x_1\}$, $F_3(e_2) = \{x_1, x_2\}$, $G_1(e_1) = \phi$, $G_1(e_2) = \{x_1\}$, $G_2(e_1) = \{x_1\}$, $G_2(e_2) = \phi$, $G_3(e_1) = \{x_1\}$, $G_3(e_2) = \{x_1\}$, $K(e_1) = \phi$, $K(e_2) = \{x_2\}$. Then τ defines a soft topology and $\tilde{I}_1$, $\tilde{I}_2$, $\tilde{I}_3$ define soft ideal on X. Hence $(X, \tau, E, \tilde{I}_i)$ $(i = 1, 2, 3)$ is a soft ideal topological space over X.

a) Let $(H, E) = \{\{x_1, x_2\}, \phi\}$ be a soft set on $(X, \tau, E, \tilde{I}_1)$. Since $(H, E)^* = \{\{x_2\}, \phi\} \widetilde{\subseteq} \{\{x_1, x_2\}, \phi\} = (H, E)$, (H, E) is a soft $*_{\tilde{I}_1}$-dense-in-itself.

b) Let $(H, E) = \{\{x_1, x_2\}, \phi\}$ be a soft set on $(X, \tau, E, \tilde{I}_2)$. Since $(H, E) = \{\{x_1, x_2\}, \phi\} \widetilde{\subseteq} \{\{x_1, x_2\}, \{x_1, x_2\}\} = (H, E)^*$, (H, E) is a soft $\tau^*_{\tilde{I}_2}$-closed.

c) Let $(H, E) = \{\{x_1, x_2\}, \phi\}$ be a soft set on $(X, \tau, E, \tilde{I}_3)$. Since $(H, E) = \{\{x_1, x_2\}, \phi\} = (H, E)^*$, (H, E) is a soft $*_{\tilde{I}_3}$-perfect.

d) Let $(H, E) = \{\{x_2\}, \{x_1\}\}$ be a soft set on $(X, \tau, E, \tilde{I}_1)$. Since $int(H, E) = \Phi$, $(int(H, E))^* = \Phi$ and hence $cl^*(int(H, E)) = int(H, E) \widetilde{\cup} (int(H, E))^* = \Phi$. Thus, we have $int(cl^*(int(H, E))) = \Phi = int(H, E)$ and hence (H, E) is a soft α^*-$\tilde{I}_1$-open set.

e) Let $(L, E) = \{\{x_2\}, \phi\}$, $(F_2, E) = \{\{x_1, x_2\}, \{x_2\}\}$ and $(H, E) = \{\{x_2\}, \{x_1\}\}$ be soft sets on $(X, \tau, E, \tilde{I}_1)$. Since $(L, E) = (F_2, E) \widetilde{\cap} (H, E)$, (L, E) is a soft $C_{\tilde{I}_1}$-set, where (F_2, E) is a soft open set and (H, E) is a soft α^*-$\tilde{I}_1$-open set.

f) Let $(L, E) = \{\{x_1\}, \phi\}$, $(F_3, E) = \{\{x_1\}, \{x_1, x_2\}\}$ and $(H, E) = \{\{x_1, x_2\}, \phi\}$ be soft sets on $(X, \tau, E, \tilde{I}_3)$. Since $(L, E) = (F_3, E) \widetilde{\cap} (H, E)$, (L, E) is a soft $\tilde{I}_3$-locally-closed, where (F_3, E) is a soft open set and (H, E) is a soft $*_{\tilde{I}_3}$-perfect.

g) Since $cl(int(\widetilde{X})) = \widetilde{X}$ and $(F_1, E) = (F_1, E) \widetilde{\cap} \widetilde{X}$, (F_1, E) is a soft A-set, where (F_1, E) is a soft open set and $\widetilde{X}$ is a soft regular closed set.

Definition 3.3. A soft subset (F, E) of soft ideal topological space $(X, \tau, E, \tilde{I})$ is said to be soft regular-$\tilde{I}$-closed if

$(F, E) = (int(F, E))^*$.

We denote by $SR_{\tilde{I}}C(X)$ [resp. $S\alpha\tilde{I}O(X)$, $SP\tilde{I}O(X)$, $SRC(X)$, $S\tilde{I}$-$LC(X)$, $SC_{\tilde{I}}(X)$, $SA(X)$] the family of all soft regular-$\tilde{I}$-closed [soft α-$\tilde{I}$-open, soft *pre*-$\tilde{I}$-open, soft regular closed, soft $\tilde{I}$-locally-closed, soft $C_{\tilde{I}}$-set, soft A-set] subset of (X, τ, E), when there is no chance for confusion with the soft ideal.

Proposition 3.4. *For a soft subset (F, E) of an soft ideal topological space $(X, \tau, E, \tilde{I})$ the following properties hold:*

i) *Every soft regular-$\tilde{I}$-closed set is soft α^*-$\tilde{I}$-open and soft semi-$\tilde{I}$-open,*

ii) *Every soft regular-$\tilde{I}$-closed set is soft $*$-perfect.*

Proof. i) Let (F, E) be a soft regular-$\tilde{I}$-closed set. Then, we have $cl^*(int\,(F, E)) = int(F, E) \tilde{\cup} (int(F, E))^* = int(F, E) \tilde{\cup} (F, E) = (F, E)$. Thus, $int(F, E) = int(cl^*(int(F, E)))$ and $(F, E) \tilde{\subseteq} cl^*(int(F, E))$. Therefore, (F, E) is soft α^*-$\tilde{I}$-open and soft semi-$\tilde{I}$-open.

ii) Let (F, E) be a soft regular-$\tilde{I}$-closed set. Then, we have $(F, E) = (int(F, E))^*$. Since, $int(F, E) \tilde{\subseteq} (F, E)$, $(int(F, E))^* \tilde{\subseteq} (F, E)^*$ by Theorem 2.20. Then we have $(F, E) = (int(F, E))^* \tilde{\subseteq} (F, E)^*$. On the other hand, by Theorem 2.20 it follows from $(F, E) = (int(F, E))^*$ that $(F, E)^* = ((int(F, E))^*)^* \tilde{\subseteq} (int(F, E))^* = (F, E)$. Therefore, we obtain $(F, E) = (F, E)^*$. This shows that (F, E) is soft $*$-perfect. □

Remark 3.5. The converses of Proposition 3.4 need not be true as the following examples show.

Example 3.6. Let consider Example 3.2.

1) In Example 3.2 (*d*), we showed that soft set $(H, E) = \{\{x_2\}, \{x_1\}\}$ is a soft α^*-$\tilde{I}_1$-open set. On the other hand, since $int(H, E) = \Phi \neq \{\{x_2\}, \{x_1\}\} = (H, E)$, (H, E) is not soft regular-$\tilde{I}_1$-closed.

2) Let $(F_2, E) = \{\{x_1, x_2\}, \{x_2\}\}$ be a soft open set. Then (F_2, E) is a soft semi-$\tilde{I}_2$-open set which is not soft regular-$\tilde{I}_2$-closed. For $(F_2, E) = \{\{x_1, x_2\}, \{x_2\}\}$, since $int(F_2, E) = \{\{x_1, x_2\}, \{x_2\}\}$, $(int(F_2, E))^* = \{\{x_1, x_2\}, \{x_1, x_2\}\}$ and hence $cl^*(int(F_2, E)) = int(F_2, E) \tilde{\cup} (int(F_2, E))^* = \{\{x_1, x_2\}, \{x_1, x_2\}\} \tilde{\supseteq} \{\{x_1, x_2\}, \{x_2\}\} = (F_2, E)$. This shows that (F_2, E) is a soft semi-$\tilde{I}_2$-open set. On the other hand, $(int(F_2, E))^* = \{\{x_1, x_2\}, \{x_1, x_2\}\} \neq \{\{x_1, x_2\}, \{x_2\}\} = (F_2, E)$ and hence (F_2, E) is not soft regular-$\tilde{I}_2$-closed.

3) Let $(H, E) = \{\{x_2\}, \{x_1\}\}$ be a soft set. Then (H, E) is a soft $*_{\tilde{I}_3}$-perfect but not soft regular-$\tilde{I}_3$-closed. For $(H, E) = \{\{x_2\}, \{x_1\}\}$, $(H, E)^* = \{\{x_2\}, \{x_1\}\} = (H, E)$ and hence (H, E) is a soft $*_{\tilde{I}_3}$-perfect. On the other hand, since $int(H, E) = \Phi$, $(int(H, E))^* = \Phi \neq \{\{x_2\}, \{x_1\}\} = (H, E)$. This show that (H, E) is not soft regular-$\tilde{I}_3$-closed.

Corollary 3.7. *Every soft regular-$\tilde{I}$-closed set is soft τ^*-closed and soft $*$-dense-in-itself.*

Proof. The proof is obvious from Proposition 3.4. □

Proposition 3.8. *In a soft ideal topological space $(X, \tau, E, \tilde{I})$, every soft regular-$\tilde{I}$-closed set is soft regular closed.*

Proof. Let (F, E) be any soft regular-$\tilde{I}$-closed set. Then, we have $(F, E) = (int(F, E))^*$. Thus, we obtain that $cl(F, E) = cl((int(F, E))^*) = (int(F, E))^* = (F, E)$ by Theorem 2.20. Additionally, by Theorem 2.20, we have $(int\,(F, E))^* \tilde{\subseteq} cl(int(F, E))$ and hence $(F, E) = (int(F, E))^* \tilde{\subseteq} cl(int(F, E)) \tilde{\subseteq} cl(F, E) = (F, E)$. Then we have $(F, E) = cl(int(F, E))$ and hence (F, E) is a soft regular closed set. □

Remark 3.9. The converse of Proposition 3.8 need not be true as the following example shows.

Example 3.10. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, $\tilde{I} = \{\Phi, (G_1, E), (G_2, E), (G_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E), (G_3, E) are soft sets over X, defined as follows. $F_1(e_1) = \{x_1, x_3\}$, $F_1(e_2) = \phi$, $F_2(e_1) = \{x_4\}$, $F_2(e_2) = \{x_4\}$, $F_3(e_1) = \{x_1, x_3, x_4\}$, $F_3(e_2) = \{x_4\}$, $G_1(e_1) = \{x_4\}$, $G_1(e_2) = \phi$, $G_2(e_1) = \phi$, $G_2(e_2) = \{x_4\}$, $G_3(e_1) = \{x_4\}$, $G_3(e_2) = \{x_4\}$. Then τ defines a soft topology and $\tilde{I}$ defines soft ideal on X. Hence $(X, \tau, E, \tilde{I})$ is a soft ideal topological space over X.

Let $(H, E) = \{\{x_2, x_4\}, X\}$ be a soft set. Then (H, E) is a soft regular closed set which is not soft regular-$\tilde{I}$-closed. For $(H, E) = \{\{x_2, x_4\}, X\}$, since $int(H, E) = \{\{x_4\}, \{x_4\}\}$, $cl(int(H, E)) = \{\{x_2, x_4\}, X\} = (H, E)$ and (H, E) is a soft regular closed set. On the other hand, since $int(H, E) = \{\{x_4\}, \{x_4\}\} \in \tilde{I}$, we have $(int(H, E))^* = \Phi \neq (H, E)$ and hence (H, E) is not soft regular-$\tilde{I}$-closed.

Proposition 3.11. *Let $(X, \tau, E, \tilde{I})$ be a soft ideal topological space and $\tilde{I} = \{\Phi\}$ or $\tilde{I} = \tilde{I}_n$, where $\tilde{I}_n$ is the soft ideal of all nowhere dense soft sets in $(X, \tau, E, \tilde{I})$. Then a soft subset (F, E) of X is a soft regular-$\tilde{I}$-closed set if and only if (F, E) is soft regular closed.*

Proof. By Proposition 3.8, every soft regular-$\tilde{I}$-closed set is soft regular closed. If $\tilde{I} = \{\Phi\}$ $\left[resp. \tilde{I} = \tilde{I}_n\right]$, then it is well-known that $(F, E)^* = cl(F, E)$ $[resp. (F, E)^* = cl(int(cl(F, E))]$. Therefore, we obtain $(int(F, E))^* = cl(int(F, E))$ $[resp. (int(F, E))^* = cl(int(cl(int(F, E)))) = cl(int(F, E))]$. Thus, soft regular-$\tilde{I}$-closedness and soft regular closedness are equivalent. □

Remark 3.12. Since every soft open set is soft α-$\tilde{I}$-open, soft regular-$\tilde{I}$-closedness and soft α-$\tilde{I}$-openness (and hence soft openess) are independent of each other as the following example shows.

Example 3.13. In Example 3.10, $(F_1, E) = \{\{x_1, x_3\}, \phi\}$ is a soft open set but not a soft regular-$\tilde{I}$-closed set, since $(int(F_1, E))^* = \{\{x_1, x_2, x_3\}, \{x_1, x_2, x_3\}\} \neq (F_1, E)$. Let X, E, τ be as in Example 3.10 and $\tilde{I}_1 = \{\Phi, (H_1, E), (H_2, E), (H_3, E)\}$ where (H_1, E), (H_2, E), (H_3, E) are soft sets over X, defined as follows. $H_1(e_1) = \{x_3\}$, $H_1(e_2) = \phi$, $H_2(e_1) = \{x_4\}$, $H_2(e_2) = \phi$, $H_3(e_1) = \{x_3, x_4\}$, $H_3(e_2) = \phi$. Then, $\tilde{I}_1$ defines soft ideal on X.

For $(H, E) = \{\{x_2, x_4\}, X\}$, $int(H, E) = \{\{x_4\}, \{x_4\}\}$ and $(int(H, E))^* = \{\{x_2, x_4\}, X\} = (H, E)$. Hence (H, E) is a soft regular-$\tilde{I}_1$-closed set. On the other hand, since $(int(H, E))^* = \{\{x_2, x_4\}, X\}$, we have $cl^*(int(H, E)) = int(H, E) \,\tilde{\cup}\, (int(H, E))^* = \{\{x_2, x_4\}, X\}$ and $int(cl^*(int(H, E))) = \{\{x_4\}, \{x_4\}\} \tilde{\not\supseteq} \{\{x_2, x_4\}, X\} = (H, E)$. Hence (H, E) is not soft α-$\tilde{I}_1$-open.

Remark 3.14. For the relationship related to several soft sets defined above, we have the following diagram:

$$\begin{array}{ccccc} \text{soft } \tau^*\text{-closed} & & \text{soft } \alpha^*\text{-}\tilde{I}\text{-open} & & \text{soft } \alpha\text{-}\tilde{I}\text{-open} \\ \uparrow & & \uparrow & & \downarrow \\ \text{soft } *\text{-perfect} & \longleftarrow & \text{soft regular-}\tilde{I}\text{-closed} & \longrightarrow & \text{soft semi-}\tilde{I}\text{-open} \\ \downarrow & & \downarrow & & \downarrow \\ \text{soft } *\text{-dense-in-itself} & & \text{soft regular closed} & & \text{soft semi-open} \end{array}$$

We can say that soft α^*-$\tilde{I}$-openness and soft τ^*-closedness are independent of each other.

Example 3.15. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, $\tilde{I} = \{\Phi, (G_1, E), (G_2, E), (G_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E), (G_3, E) are soft sets over X, defined as follows. $F_1(e) = \{x_1, x_3\}$, $F_2(e) = \{x_4\}$, $F_3(e) = \{x_1, x_3, x_4\}$, $G_1(e) = \{x_3\}$, $G_2(e) = \{x_4\}$, $G_3(e) = \{x_3, x_4\}$. Then τ defines a soft topology and $\tilde{I}$ defines soft ideal on X. Hence $(X, \tau, E, \tilde{I})$ is a soft ideal topological space over X.

i) For $(F_1, E) = \{x_1, x_3\}$, since $(F_1, E)^* = \{x_1, x_2, x_3\} \tilde{\not\subseteq} \{x_1, x_3\} = (F_1, E)$ and $int(cl^*(int(F_1, E))) = int[int(F_1, E) \,\tilde{\cup}\, (int\,(F_1, E))^*] = \{x_1, x_3\} = int(F_1, E)$, (F_1, E) is a soft α^*-$\tilde{I}$-open set which is not soft τ^*-closed.

ii) For $(H, E) = \{x_2, x_4\}$, since $(H, E) = \{x_2, x_4\} \tilde{\supseteq} \{x_2\} = (H, E)^*$, (H, E) is soft τ^*-closed. Moreover, $cl^*(int(H, E)) = cl^*(\{x_4\}) = \Phi$ and hence $int(cl^*(int\,(H, E))) = \Phi \neq \{x_4\} = int(H, E)$. Therefore, (H, E) is not soft α^*-$\tilde{I}$-open.

Additionally, we can also say that soft regular closed and soft $*$-dense-in-itself are independent notions, as shown the following example.

Example 3.16. Let $(X, \tau, E, \tilde{I})$ be the same soft ideal topological space as in Example 3.15.

i) For $(F_1, E) = \{x_1, x_3\}$, since $(F_1, E)^* = \{x_1, x_2, x_3\} \tilde{\supseteq} \{x_1, x_3\} = (F_1, E)$ and $cl(int(F_1, E)) = cl(F_1, E) = \{x_1, x_2, x_3\} \neq \{x_1, x_3\} = (F_1, E)$, (F_1, E) is a soft $*$-dense-in-itself set which is not soft regular closed.

ii) Moreover, $(H, E) = \{x_2, x_4\}$ is a soft regular closed set which is not soft $*$-dense-in-itself since $(H, E)^* = \{x_2\} \tilde{\not\supseteq} \{x_2, x_4\} = (H, E)$ and $cl(int(H, E)) = cl(\{x_4\}) = \{x_2, x_4\} = (H, E)$.

Finally, we can also say that soft α^*-$\tilde{I}$-openness and soft semi-$\tilde{I}$-openness [resp. soft α-$\tilde{I}$-openness] are independent of each other, as shown the following examples.

Example 3.17. Let $(X, \tau, E, \tilde{I})$ be the same soft ideal topological space as in Example 3.15.

For $(L, E) = \{x_2, x_3, x_4\}$, since $cl^*(int(L, E)) = int(L, E) \,\tilde{\cup}\, (int(L, E))^* = \{x_4\} \tilde{\not\supseteq} \{x_2, x_3, x_4\} = (L, E)$ [resp. $int(cl^*(int(L, E))) = \{x_4\} \tilde{\not\supseteq} \{x_2, x_3, x_4\} = (L, E)$] and $int(cl^*(int(L, E))) = \{x_4\} = int(L, E)$, (L, E) is a soft α^*-$\tilde{I}$-open set which is not soft semi-$\tilde{I}$-open [resp. soft α-$\tilde{I}$-open].

Example 3.18. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, $\tilde{I} = \{\Phi, (G_1, E), (G_2, E), (G_3, E)\}$, where (F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E), (G_3, E) are soft sets over X, defined as follows. $F_1(e_1) = \phi$, $F_1(e_2) = \{x_3\}$, $F_2(e_1) = \{x_1\}$, $F_2(e_2) = \{x_3\}$, $F_3(e_1) = \{x_1, x_2\}$, $F_3(e_2) = \{x_3\}$, $G_1(e_1) = \{x_1\}$, $G_1(e_2) = \phi$, $G_2(e_1) = \{x_3\}$, $G_2(e_2) = \phi$, $G_3(e_1) = \{x_1, x_3\}$, $G_3(e_2) = \phi$. Then τ defines a soft topology and $\tilde{I}$ defines soft ideal on X.

For $(K, E) = \{\{x_1, x_2\}, \{x_2, x_3\}\}$, since $cl^*(int(K, E)) = int(K, E) \,\tilde{\cup}\, (int(K, E))^* = \{\{x_1, x_2\}, \{x_3\}\} \,\tilde{\cup}\, \widetilde{X} = \widetilde{X} \tilde{\supseteq} (K, E)$ [resp. $int(cl^*(int(K, E))) = \widetilde{X} \tilde{\supseteq} (K, E)$] and $int(cl^*(int(K, E)))) = \widetilde{X} \neq int(K, E)$, (K, E) is a soft semi-$\tilde{I}$-open [resp. soft α-$\tilde{I}$-open] which is not soft α^*-$\tilde{I}$-open set.

4. Soft $A_{\tilde{I}}$-Sets

Definition 4.1. A soft ideal topological space $(X, \tau, E, \tilde{I})$ is called soft Hayashi-Samuels space if $\widetilde{X}^* = \widetilde{X}$.

Example 4.2. Let soft ideal topological space $(X, \tau, E, \tilde{I}_1)$ be as in Example 3.2. Since $\widetilde{X}^* = \widetilde{X}$, soft ideal topological space $(X, \tau, E, \tilde{I}_1)$ is a soft Hayashi-Samuels space.

Definition 4.3. A soft subset (F, E) of a soft ideal topological space $(X, \tau, E, \tilde{I})$ is called a soft $A_{\tilde{I}}$-set if $(F, E) = (G, E) \widetilde{\cap} (H, E)$, where $(G, E) \in \tau$ and $(H, E) \in SR_{\tilde{I}}C(X, \tau, E)$.

We denote by $SA_{\tilde{I}}(X)$ the family of all soft $A_{\tilde{I}}$-sets of $(X, \tau, E, \tilde{I})$, when there is no chance for confusion with the soft ideal.

Proposition 4.4. *Let $(X, \tau, E, \tilde{I})$ be a soft ideal topological space and (F, E) a soft subset X. Then the following properties hold:*

a) If (F, E) is soft open set and $(X, \tau, E, \tilde{I})$ is a soft Hayashi-Samuels space, then (F, E) is a soft $A_{\tilde{I}}$-set,

b) If (F, E) is soft regular-$\tilde{I}$-closed set, then (F, E) is a soft $A_{\tilde{I}}$-set.

Proof. Since $\widetilde{X} \in \tau \widetilde{\cap} SR_{\tilde{I}}C(X, \tau, E)$, the proof is obvious. □

Remark 4.5. The converses of Proposition 4.4 need not be true as the following examples show.

Example 4.6. Let $(X, \tau, E, \tilde{I})$ be the same soft ideal topological space as in Example 3.15.

1) Let $(H, E) = \{x_1, x_2, x_3\}$ be a soft set. Then (H, E) is a soft $A_{\tilde{I}}$-set but not soft open. For $(H, E) = \{x_1, x_2, x_3\}$, since $int(H, E) = \{x_1, x_3\}$, $(int(H, E))^* = \{x_1, x_2, x_3\} = (H, E)$ and hence (H, E) is a soft regular-$\tilde{I}$-closed set. Since $(H, E) = \widetilde{X} \widetilde{\cap} (H, E)$ and $\widetilde{X} \in \tau$, (H, E) is a soft $A_{\tilde{I}}$-set. On the other hand, $int(H, E) = \{x_1, x_3\} \neq \{x_1, x_2, x_3\} = (H, E)$ and hence (H, E) is not soft open.

2) Let $(K, E) = \{x_1, x_3\}$ be a soft set. Then (K, E) is a soft $A_{\tilde{I}}$-set but not soft regular-$\tilde{I}$-closed. For $(K, E) = \{x_1, x_3\}$, $(int(K, E))^* = \{x_1, x_2, x_3\} \neq (K, E)$ and hence (K, E) is not a soft regular-$\tilde{I}$-closed set. We showed that $(H, E) = \{x_1, x_2, x_3\}$ is a soft regular-$\tilde{I}$-closed set by (1). Since $(K, E) \in \tau$ and $(K, E) = (K, E) \widetilde{\cap} (H, E)$, (K, E) is a soft $A_{\tilde{I}}$-set.

Proposition 4.7. *Let $(X, \tau, E, \tilde{I})$ be a soft ideal topological space and (F, E) a soft subset of X. Then the following properties hold:*

a) If (F, E) is a soft $A_{\tilde{I}}$-set, then (F, E) is a soft $C_{\tilde{I}}$-set and soft $\tilde{I}$-locally-closed,

b) If (F, E) is a soft $A_{\tilde{I}}$-set, then (F, E) is a soft A-set.

Proof. This is an immediate consequence of Proposition 3.4 and 3.8. □

Remark 4.8. The converses of Proposition 4.7 need not be true as the following examples show.

Example 4.9. Let $(X, \tau, E, \tilde{I})$ be the same soft ideal topological space as in Example 3.15.

1) $(H, E) = \{x_1, x_2\}$ is a soft $C_{\tilde{I}}$-set but not a soft $A_{\tilde{I}}$-set. For $(H, E) = \{x_1, x_2\}$, since $int(cl^*(int(H, E)))) = \Phi = int(H, E)$, (H, E) is a soft α^*-$\tilde{I}$-open set. Also, since $(H, E) = \widetilde{X} \widetilde{\cap} (H, E)$ and $\widetilde{X} \in \tau$, $(H, E) \in SC_{\tilde{I}}(X)$. Moreover, since $(int(H, E))^* = \Phi \neq (H, E)$, (H, E) is not a soft regular -$\tilde{I}$-closed set and $\widetilde{X}$ is the only soft open set which contains (H, E). Hence (H, E) is not a soft $A_{\tilde{I}}$-set. Furthermore, since $(H, E)^* = \{x_1, x_2, x_3\} \neq (H, E)$, (H, E) is not soft $*$-perfect and consequently $(H, E) \notin S\tilde{I}\text{-}LC(X)$.

2) Let $(H, E) = \{x_2\}$. Since, $(H, E)^* = \{x_2\} = (H, E)$ and $(int(H, E))^* = \Phi \neq (H, E)$, (H, E) is a soft $*$-perfect but not a soft regular -$\tilde{I}$-closed. Therefore, (H, E) is a soft $\tilde{I}$-locally-closed and not a soft $A_{\tilde{I}}$-set. Furthermore, since $int(cl^*(int(H, E))) = \Phi = int(H, E)$, we can say that (H, E) is a soft α^*-$\tilde{I}$-open set. Consequently, $(H, E) \in SC_{\tilde{I}}(X)$.

3) Let $(H, E) = \{x_2, x_4\}$. Since, $cl(int(H, E)) = \{x_2, x_4\} = (H, E)$ and $(int(H, E))^* = \Phi \neq (H, E)$, (H, E) is a soft regular closed which is not soft regular -$\tilde{I}$-closed. Therefore (H, E) is a soft A-set which is not a soft $A_{\tilde{I}}$-set.

Proposition 4.10. *For a soft subset (F,E) of soft Hayashi-Samuels space $(X,\tau,E,\tilde{I})$ the following properties are equivalent:*

a) (F,E) *is a soft open set,*

b) (F,E) *is a soft α-$\tilde{I}$-open and a soft $A_{\tilde{I}}$-set,*

c) (F,E) *is a soft pre-$\tilde{I}$-open and a soft $A_{\tilde{I}}$-set.*

Proof. (a)⟶(b) Let (F,E) be a soft open set. Hence $(F,E)\in S\alpha\tilde{I}O(X)$ by [13]. On the other hand, $(F,E)=(F,E)\,\widetilde{\cap}\,\widetilde{X}$, where $(F,E)\in\tau$ and $\widetilde{X}\in SR_{\tilde{I}}C(X)$. Hence $(F,E)\in SA_{\tilde{I}}(X)$.

(b)⟶(c) This is obvious since every soft α-$\tilde{I}$-open set is soft *pre*-$\tilde{I}$-open.

(c)⟶(a) Let (F,E) be a soft *pre*-$\tilde{I}$-open and a soft $A_{\tilde{I}}$-set. Then $(F,E)=(G,E)\,\widetilde{\cap}\,(H,E)$, where $(G,E)\in\tau$ and $(H,E)\in SR_{\tilde{I}}C(X)$. Since $(F,E)\in SP\tilde{I}O(X)$, we have $(F,E)=(G,E)\,\widetilde{\cap}\,(H,E)\,\widetilde{\subseteq}\,int(cl^*((G,E)\,\widetilde{\cap}\,(H,E)))\,\widetilde{\subseteq}\,int(cl^*(G,E)\,\widetilde{\cap}\,cl^*(H,E))$. By Corollary 3.7, (H,E) is soft τ^*-closed and $cl^*(H,E)=(H,E)$. Therefore, we have $int(cl^*(G,E)\,\widetilde{\cap}\,cl^*(H,E))=int(cl^*(G,E)\,\widetilde{\cap}\,(H,E))=int(cl^*(G,E)\,\widetilde{\cap}\,int(H,E))$ and $(G,E)\,\widetilde{\cap}\,(H,E)\,\widetilde{\subseteq}\,(G,E)\,\widetilde{\cap}\,int(cl^*(G,E))\,\widetilde{\cap}\,int(H,E)=int((G,E)\,\widetilde{\cap}\,cl^*(G,E)\,\widetilde{\cap}\,(H,E))=int((G,E)\,\widetilde{\cap}\,(H,E))$. Consequently, we obtain $(G,E)\,\widetilde{\cap}\,(H,E)\,\widetilde{\subseteq}\,int((G,E)\,\widetilde{\cap}\,(H,E))$ and $(F,E)=(G,E)\,\widetilde{\cap}\,(H,E)$ is soft open. □

5. Soft Idealization of a Decomposition Theorem

Definition 5.1. Let $(X_1,\tau_1,A,\tilde{I})$ be a soft ideal topological space and (X_2,τ_2,B) be a soft topological space. Let $u:X_1\longrightarrow X_2$ and $p:A\longrightarrow B$ be mappings. Let $f_{pu}:SS(X_1)_A\longrightarrow SS(X_2)_B$ be a function. Then, the function f_{pu} is called:

a) soft α-$\tilde{I}$-continuous [13] if $f_{pu}^{-1}(G,B)\in S\alpha\tilde{I}O(X_1)$, $\forall(G,B)\in\tau_2$,

b) soft *pre*-$\tilde{I}$-continuous [13] if $f_{pu}^{-1}(G,B)\in SP\tilde{I}O(X_1)$, $\forall(G,B)\in\tau_2$,

c) soft $\tilde{I}$-LC-continuous if $f_{pu}^{-1}(G,B)\in S\tilde{I}\text{-}LC(X_1)$, $\forall(G,B)\in\tau_2$,

d) soft $C_{\tilde{I}}$-continuous if $f_{pu}^{-1}(G,B)\in SC_{\tilde{I}}(X_1)$, $\forall(G,B)\in\tau_2$,

e) soft A-continuous if $f_{pu}^{-1}(G,B)\in SA(X_1)$, $\forall(G,B)\in\tau_2$,

f) soft $A_{\tilde{I}}$-continuous if $f_{pu}^{-1}(G,B)\in SA_{\tilde{I}}(X_1)$, $\forall(G,B)\in\tau_2$.

Example 5.2. i) Let $(X,\tau,E,\tilde{I}_3)$ be the same soft ideal topological space as in Example 3.2 and $Y=\{y_1,y_2\}$, $V=\{v_1,v_2\}$, $\varphi=\{\Phi,\widetilde{Y},(H,B)\}$, $H(v_1)=\{y_2\}$, $H(v_2)=\{y_1\}$. Then (Y,φ,V) is a soft topological space over Y. Let $f_{pu}:SS(X)_E\longrightarrow SS(Y)_V$ be a function defined as follows: $u:X\longrightarrow Y$, $u(x_1)=y_1$, $u(x_2)=y_2$, and $p:E\longrightarrow V$, $p(e_1)=v_1$, $p(e_2)=v_2$. Then, f_{pu} is soft $\tilde{I}$-LC-continuous.

ii) Let $(X,\tau,E,\tilde{I})$ be the same soft ideal topological space as in Example 3.15 and $Y=\{y_1,y_2\}$, $V=\{v\}$, $\varphi=\{\Phi,\widetilde{Y},(H,V)\}$, $H(v)=\{y_1\}$. Then (Y,φ,V) is a soft topological space over Y. Let $f_{pu}:SS(X)_E\longrightarrow SS(Y)_V$ be a function defined as follows: $u:X\longrightarrow Y$, $u(x_1)=u(x_2)=y_1$, $u(x_3)=u(x_4)=y_2$ and $p:E\longrightarrow V$, $p(e)=v$. Then, f_{pu} is soft $C_{\tilde{I}}$-continuous.

iii) Let $(X,\tau,E,\tilde{I})$ be the same soft ideal topological space as in Example 3.15 and $Y=\{y_1,y_2\}$, $V=\{v\}$, $\varphi=\{\Phi,\widetilde{Y},(H,V)\}$, $H(v)=\{y_1\}$. Then (Y,φ,V) is a soft topological space over Y. Let $f_{pu}:SS(X)_E\longrightarrow SS(Y)_V$ be a function defined as follows: $u:X\longrightarrow Y$, $u(x_2)=u(x_4)=y_1$, $u(x_1)=u(x_3)=y_2$ and $p:E\longrightarrow V$, $p(e)=v$. Then, f_{pu} is soft A-continuous.

iv) Let $(X,\tau,E,\tilde{I})$ be the same soft ideal topological space as in Example 3.15 and $Y=\{y_1,y_2\}$, $V=\{v\}$, $\varphi=\{\Phi,\widetilde{Y},(H,V)\}$, $H(v)=\{y_1\}$. Then (Y,φ,V) is a soft topological space over Y. Let $f_{pu}:SS(X)_E\longrightarrow SS(Y)_V$ be a function defined as follows: $u:X\longrightarrow Y$, $u(x_1)=u(x_2)=u(x_3)=y_1$, $u(x_4)=y_2$ and $p:E\longrightarrow V$, $p(e)=v$. Then, f_{pu} is soft $A_{\tilde{I}}$-continuous.

Proposition 5.3. *For a function $f_{pu} : SS(X_1)_A \longrightarrow SS(X_2)_B$, the following properties hold:*

a) If f_{pu} soft $A_{\tilde{I}}$-continuous, then f_{pu} is soft $\tilde{I}$-LC-continuous,

b) If f_{pu} soft $A_{\tilde{I}}$-continuous, then f_{pu} is soft $C_{\tilde{I}}$-continuous,

c) If f_{pu} soft $A_{\tilde{I}}$-continuous, then f_{pu} is soft A-continuous.

Proof. The proof is obvious from Proposition 4.4. □

Remark 5.4. The converses of Proposition 5.3 need not be true as the following examples show.

a) We showed that in Example 5.2 (*i*) f_{pu} is soft $\tilde{I}$-*LC*-continuous. On the other hand, f_{pu} is not soft $A_{\tilde{I}}$-continuous.

b) We showed that in Example 5.2 (*ii*) f_{pu} is soft $C_{\tilde{I}}$-continuous. On the other hand, f_{pu} is not soft $A_{\tilde{I}}$-continuous.

c) We showed that in Example 5.2 (*iii*) f_{pu} is soft A-continuous. On the other hand, f_{pu} is not soft $A_{\tilde{I}}$-continuous.

Theorem 5.5. *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft Hayashi-Samuels space. For a function $f_{pu} : SS(X_1)_A \longrightarrow SS(X_2)_B$, the following properties are equivalent:*

a) f_{pu} is soft continuous,

b) f_{pu} is soft α-$\tilde{I}$-continuous and soft $A_{\tilde{I}}$-continuous,

c) f_{pu} is soft pre-$\tilde{I}$-continuous and soft $A_{\tilde{I}}$-continuous.

Proof. This is an immediate consequence of Proposition 4.10. □

Lemma 5.6. *Let $(X, \tau, E, \tilde{I})$ be a soft ideal topological space and $\tilde{I} = \{\Phi\}$ or $\tilde{I} = \tilde{I}_n$. Then a soft subset (F, E) of $(X, \tau, E, \tilde{I})$ is soft pre-$\tilde{I}$-open if and only if (F, E) is soft pre-open.*

Proof. $\Longrightarrow$ We recall that Kandil et al. [13] showed that every soft *pre-$\tilde{I}$*-open set is soft pre-open.

$\Longleftarrow$ Let (F, E) be a soft pre-open and $\tilde{I} = \{\Phi\}[resp. \tilde{I} = \tilde{I}_n]$, then it is well-known that $(F, E) \tilde{\subseteq} int(cl(F, E))$ and $(F, E)^* = cl(F, E)[resp.(F, E)^* = cl(int(cl(F, E))]$. Therefore, we obtain $int(cl^*(F, E)) = int((F, E) \tilde{\cup} (F, E)^*) \tilde{\supseteq} int(F, E) \tilde{\cup} int((F, E)^*) = int(F, E) \tilde{\cup} int(cl(F, E)) = int(cl(F, E)) \tilde{\supseteq} (F, E)$ $[resp. int(cl^*(F, E)) = int((F, E) \tilde{\cup} (F, E)^*) \tilde{\supseteq} int(F, E) \tilde{\cup} int((F, E)^*) = int(F, E) \tilde{\cup} int(cl(int(cl(F, E)))) = int(cl(F, E)) \tilde{\supseteq} (F, E)]$. Thus, every soft pre-open set is soft *pre-$\tilde{I}$*-open. □

Corollary 5.7. *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft ideal topological space and $\tilde{I} = \{\Phi\}$ or $\tilde{I} = \tilde{I}_n$. For a function $f_{pu} : SS(X_1)_A \longrightarrow SS(X_2)_B$, the following properties are equivalent:*

a) f_{pu} is soft continuous,

b) f_{pu} is soft α-continuous and soft A-continuous,

c) f_{pu} is soft pre-continuous and soft A-continuous.

Proof. 1) Let $\tilde{I} = \{\Phi\}$, we have $(F, A)^* = cl(F, A)$ and $cl^*(F, A) = (F, A) \tilde{\cup} (F, A)^* = cl(F, A)$ for any soft subset (F, A) of X_1. Therefore, we obtain (a) (F, A) is soft α-$\tilde{I}$-open if and only if it is soft α-open and (b) (F, A) is a soft $A_{\tilde{I}}$-set if and only if it is soft A-set. The proof follows Lemma 5.6 and Theorem 5.5 immediately.

2) Let $\tilde{I} = \tilde{I}_n$, then we have $(F, A)^* = cl(int(cl(F, A))$ and $cl^*(F, A) = (F, A) \tilde{\cup} (F, A)^* = (F, A) \tilde{\cup} cl(int(cl(F, A))$ for any soft subset (F, A) of X_1. Therefore, $int(cl^*(int(F, A))) = int[int(F, A) \tilde{\cup} cl(int(cl(int(F, A))))] = int[int(F, A) \tilde{\cup} cl(int(F, A))] = int(cl(int(F, A)))$. We obtain (a) soft α-$\tilde{I}$-open if and only if it is soft α-open and (b) (F, A) is a soft $A_{\tilde{I}}$-set if and only if it is soft A-set. The proof follows Lemma 5.6 and Theorem 5.5 immediately. □

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