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Fixed Points Results via Simulation Functions

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Abstract. In this paper, we present some fixed point results in the setting of a complete metric spaces by defining a new contractive condition via *admissible mapping* imbedded in *simulation function*. Our results generalize and unify several fixed point theorems in the literature.

Very recently, in [10], Khojasteh *et al.* proposed the notion of *simulation function* to unify the several existing fixed point results in the literature. In this paper, we investigate the existence and uniqueness of fixed points of certain mappings via simulation functions in the context of complete partially ordered metric spaces. We shall also indicate that several results in the literature can be derived from our main results.

Definition 0.1. *(See [10]) A* simulation function *is a mapping* $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$ *satisfying the following conditions:*

(ζ_1) $\zeta(0,0) = 0$;

(ζ_2) $\zeta(t,s) < s - t$ *for all* $t,s > 0$;

(ζ_3) *if* $\{t_n\}, \{s_n\}$ *are sequences in* $(0,\infty)$ *such that* $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, *then*

$$\limsup_{n\to\infty} \zeta(t_n, s_n) < 0. \tag{1}$$

Let $\mathcal{Z}$ *denote the family of all simulation functions* $\zeta : [0,\infty) \times [0,\infty) \to \mathbb{R}$. *Due to the axiom* ($\zeta_2$), *we have*

$$\zeta(t,t) < 0 \text{ for all } t > 0. \tag{2}$$

Example 0.2. *(See e.g.[10, 11, 18]) Let* $\phi_i : [0,\infty) \to [0,\infty)$ *be continuous functions with* $\phi_i(t) = 0$ *if, and only if,* $t = 0$. *For* $i = 1,2,3,4,5,6$, *we define the mappings* $\zeta_i : [0,\infty) \times [0,\infty) \to \mathbb{R}$, *as follows*

(i) $\zeta_1(t,s) = \phi_1(s) - \phi_2(t)$ *for all* $t,s \in [0,\infty)$, *where* $\phi_1(t) < t \le \phi_2(t)$ *for all* $t > 0$.

(ii) $\zeta_2(t,s) = s - \dfrac{f(t,s)}{g(t,s)}t$ *for all* $t,s \in [0,\infty)$, *where* $f,g : [0,\infty)^2 \to (0,\infty)$ *are two continuous functions with respect to each variable such that* $f(t,s) > g(t,s)$ *for all* $t,s > 0$.

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(iii) $\zeta_3(t,s) = s - \phi_3(s) - t$ *for all* $t, s \in [0,\infty)$.

(iv) *If* $\varphi : [0,\infty) \to [0,1)$ *is a function such that* $\limsup_{t\to r^+} \varphi(t) < 1$ *for all* $r > 0$*, and we define*

$$\zeta_4(t,s) = s\varphi(s) - t \qquad \text{for all } s,t \in [0,\infty).$$

(v) *If* $\eta : [0,\infty) \to [0,\infty)$ *is an upper semi-continuous mapping such that* $\eta(t) < t$ *for all* $t > 0$ *and* $\eta(0) = 0$*, and we define*

$$\zeta_5(t,s) = \eta(s) - t \qquad \text{for all } s,t \in [0,\infty).$$

(vi) *If* $\phi : [0,\infty) \to [0,\infty)$ *is a function such that* $\int_0^\varepsilon \phi(u)du$ *exists and* $\int_0^\varepsilon \phi(u)du > \varepsilon$*, for each* $\varepsilon > 0$*, and we define*

$$\zeta_6(t,s) = s - \int_0^t \phi(u)du \qquad \text{for all } s,t \in [0,\infty).$$

It is clear that each function ζ_i $(i = 1,2,3,4,5,6)$ *forms a simulation function.*

One can find more interesting examples of simulation functions in [10, 11, 18].

Suppose (X,d) is a metric space, T is a self-mapping on X and $\zeta \in \mathcal{Z}$. We say that T is a *$\mathcal{Z}$-contraction* with respect to ζ [10], if

$$\zeta(d(Tx,Ty), d(x,y)) \geq 0 \qquad \text{for all } x,y \in X. \tag{3}$$

Again (ζ_2), we have the following inequality

$$q(Tx,Ty) \neq q(x,y) \text{ for all distinct } x,y \in X. \tag{4}$$

Thus, we conclude that T cannot be an isometry whenever T is a $\mathcal{Z}$-contraction. In other words, if a $\mathcal{Z}$-contraction T in a metric space has a fixed point, then it is necessarily unique.

Theorem 0.3. *Every $\mathcal{Z}$-contraction on a complete metric space has a unique fixed point. In fact, every Picard sequence converges to its unique fixed point.*

Let Ψ be the family of functions $\psi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

i) ψ is nondecreasing;

ii) there exist $k_0 \in \mathbb{N}$ and $a \in (0,1)$ and a convergent series of nonnegative terms $\sum_{k=1}^\infty v_k$ such that

$$\psi^{k+1}(t) \leq a\psi^k(t) + v_k,$$

for $k \geq k_0$ and any $t \in \mathbb{R}^+$.

In the literature such functions are called as either Bianchini-Grandolfi gauge functions (see e.g.[20–22]) or (c)-comparison functions (see e.g. [19])

Lemma 0.4. *(See e.g. [19]) If* $\psi \in \Psi$*, then the following hold:*

(i) $(\psi^n(t))_{n\in\mathbb{N}}$ *converges to* 0 *as* $n \to \infty$ *for all* $t \in \mathbb{R}^+$*;*
(ii) $\psi(t) < t$*, for any* $t \in \mathbb{R}^+$*;*
(iii) ψ *is continuous at* 0*;*
(iv) *the series* $\sum_{k=1}^\infty \psi^k(t)$ *converges for any* $t \in \mathbb{R}^+$.

Recently, Samet *et al.* [14] suggested a new contraction type self-mapping to unify several existing results in the literature by auxiliary functions.

Definition 0.5. *Let $\alpha : X \times X \to [0, \infty)$. A self-mapping $T : X \to X$ is called $\alpha-$admissible if the condition*

$$\alpha(x, y) \geq 1 \Longrightarrow \alpha(Tx, Ty) \geq 1,$$

is satisfied for all $x, y \in X$.

Definition 0.6. *Let T be a self-mapping defined on a metric space (X, d). Then, T is called an $\alpha - \psi$ contractive mapping if there exist two auxiliary mappings $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that*

$$\alpha(x, y)d(Tx, Ty) \leq \psi(d(x, y)), \text{ for all } x, y \in X.$$

Clearly, any contractive mapping, that is, a mapping satisfying Banach contraction, is an $\alpha - \psi$ contractive mapping with $\alpha(x, y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$, $k \in (0, 1)$. A number of examples of such type mappings are considered in [14].

The main results in [14] are the following fixed point theorems.

Theorem 0.7. *Let $T : X \to X$ be an $\alpha - \psi$ contractive mapping where (X, d) is a complete metric space. Suppose that*

(i) T is $\alpha-$admissible;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;
(iii) either, T is continuous, or
(iii)′ if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.

Then, there exists $u \in X$ such that $Tu = u$.

Theorem 0.8. *Adding to the hypotheses of Theorem 0.7 the condition: For all $x, y \in X$, there exists $z \in X$ such that $\alpha(x, z) \geq 1$ and $\alpha(y, z) \geq 1$, we obtain uniqueness of the fixed point.*

In this paper, we introduce a new type contractive mapping by using the simulation function together with the admissible mappings in the context of complete metric spaces. We shall also discuss the existing and uniqueness of this new contractive mapping. Moreover, we will list some famous fixed point theorems as consequences of our main result.

1. Main Results

We start with the following definition.

Definition 1.1. *Let T be a self-mapping defined on a metric space (X, d). If there exist $\zeta \in \mathcal{Z}$ and $\alpha : X \times X \to [0, \infty)$ such that*

$$\zeta(\alpha(x, y)d(Tx, Ty), d(x, y)) \geq 0 \qquad \text{for all } x, y \in X, \tag{5}$$

*then we say that T is an α-admissible $\mathcal{Z}$-*contraction *with respect to ζ.*

If $\alpha(x, y) = 1$, then T turns into a *$\mathcal{Z}$-contraction with respect to ζ.*

Remark 1.2. *If T is an α-admissible $\mathcal{Z}$-*contraction *with respect to ζ, then*

$$\alpha(x, y)d(Tx, Ty) < d(x, y) \quad \text{for all } x, y \in X. \tag{6}$$

To prove the assertion, we assume that $x \neq y$. Then $d(x, y) > 0$. If $Tx = Ty$, then $\alpha(x, y)d(Tx, Ty) = 0 < d(x, y)$. Otherwise, $Tx \neq Ty$, then $d(Tx, Ty) > 0$. If $\alpha(x, y) = 0$, then the inequality is satisfied trivially. So assume that $\alpha(x, y) > 0$ and applying (ζ_2) with (5), we derive that

$$0 \leq \zeta(\alpha(x, y)d(Tx, Ty), d(x, y)) < d(x, y) - \alpha(x, y)d(Tx, Ty),$$

so (6) holds.

Popescu [16] proposed the concept of triangular α-orbital admissible as a refinement of the triangular alpha-admissible notion, defined in [15].

Definition 1.3. *[16] Let $T : X \to X$ be a mapping and $\alpha : X \times X \to [0, \infty)$ be a function. We say that T is an α-orbital admissible if*

$$\alpha(x, Tx) \geq 1 \Rightarrow \alpha(Tx, T^2x) \geq 1.$$

Furthermore, T is called a triangular α-orbital admissible if T is α-orbital admissible and

$$\alpha(x, y) \geq 1 \text{ and } \alpha(y, Ty) \geq 1 \Rightarrow \alpha(x, Ty) \geq 1.$$

It is clear that each α-admissible (respectively, triangular α-admissible) mapping is an α-orbital admissible (respectively, triangular α-orbital admissible) mapping. For more details and distinctive examples, see e.g. [16, 17].

We can now state the main result of this paper.

Theorem 1.4. *Let (X, d) be a complete metric space and let $T : X \to X$ be an α-admissible $\mathcal{Z}$-*contraction *with respect to ζ. Suppose that*

(*i*) *T is triangular α-orbital admissible;*
(*ii*) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(*iii*) *T is continuous.*

Then there exists $u \in X$ such that $Tu = u$.

Proof. Due to the condition (*ii*) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Let $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Define an iterative sequence $\{x_n\}$ in X by letting $x_{n+1} = Tx_n$ for all $n \geq 0$. If there exists an n_0 such that $x_{n_0} = x_{n_0+1}$, then $u = x_{n_0}$ becomes a fixed point of T. Consequently, we shall assume that $x_n \neq x_{n+1}$ for all n. So we have

$$d(x_n, x_{n+1}) > 0, \text{ for all } n = 0, 1, \ldots. \tag{7}$$

Regarding that T is $\alpha-$admissible, we derive

$$\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1 \Rightarrow \alpha(Tx_0, Tx_1) = \alpha(x_1, x_2) \geq 1.$$

Recursively, we obtain that

$$\alpha(x_n, x_{n+1}) \geq 1, \text{ for all } n = 0, 1, \ldots. \tag{8}$$

From (5) and (8), it follows that for all $n \geq 1$, we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_n, x_{n-1})d(Tx_n, Tx_{n-1}), d(x_n, x_{n-1})) \\ &= \zeta(\alpha(x_n, x_{n-1})d(x_{n+1}, x_n), d(x_n, x_{n-1})) \\ &< d(x_n, x_{n-1}) - \alpha(x_n, x_{n-1})d(x_{n+1}, x_n). \end{aligned} \tag{9}$$

Consequently, we derive that

$$d(x_n, x_{n+1}) \leq \alpha(x_n, x_{n-1})d(x_n, x_{n+1}) < d(x_n, x_{n-1}) \text{ for all } n = 1, 2, \ldots. \tag{10}$$

Hence, we conclude that the sequence $\{d(x_n, x_{n-1})\}$ is non-decreasing and bounded from below by zero. Consequently, there exists $L \geq 0$ such that $\lim_{n\to\infty} d(x_n, x_{n-1}) = L \geq 0$. We shall prove that

$$\lim_{n\to\infty} d(x_n, x_{n-1}) = 0. \tag{11}$$

Suppose, on the contrary that $L > 0$. Note that from the inequality (10), we derive that

$$\lim_{n\to\infty} \alpha(x_n, x_{n-1})d(x_n, x_{n+1}) = L. \tag{12}$$

Letting $s_n = \alpha(x_n, x_{n-1})d(x_n, x_{n+1})$ and $t_n = d(x_n, x_{n-1})$ and taking (ζ_3) into account, we get that

$$0 \leq \limsup_{n\to\infty} \zeta(\alpha(x_n, x_{n-1})d(x_{n+1}, x_n), d(x_n, x_{n-1})) < 0 \tag{13}$$

which is a contradiction. Thus, we have $L = 0$.

Now, we shall prove that the iterative sequence $\{x_n\}$ is Cauchy. Again we use the method of *Reductio ad absurdum*. Suppose, on the contrary that, $\{x_n\}$ is not a Cauchy sequence. Thus, there exists $\varepsilon > 0$, for all $N \in \mathbb{N}$, there exist $n, m \in \mathbb{N}$ with $n > m > N$ and $d(x_m, x_n) > \varepsilon$. On the other hand, from (11), there exists $n_0 \in \mathbb{N}$ such that

$$d(x_n, x_{n+1}) < \varepsilon \text{ for all } n > n_0. \tag{14}$$

Consider two partial subsequences x_{n_k} and x_{m_k} of x_n such that

$$n_0 \leq n_k < m_k < m_{k+1} \text{ and } d(x_{m_k}, x_{n_k}) > \varepsilon \text{ for all } k. \tag{15}$$

Notice that

$$d(x_{m_{k-1}}, x_{n_k}) \leq \varepsilon \text{ for all } k, \tag{16}$$

where m_k is chosen as a least number $m \in \{n_k, n_{k+1}, n_{k+2}, \ldots\}$ such that (15) is satisfied. We also mention that $n_k + 1 \leq m_k$ for all k.. In fact, the case $n_k + 1 \leq m_k$ is impossible due to (14),(15). Thus, $n_k + 2 \leq m_k$ for all k. It yields that

$$n_k + 1 < m_k < m_k + 1 \text{ for all } k.$$

On account of (15),(16) and the triangle inequality, we derive that

$$\begin{aligned} \varepsilon &< d(x_{m_k}, x_{n_k}) \leq d(x_{m_k}, x_{m_k-1}) + d(x_{m_k-1}, x_{n_k}) \\ &\leq d(x_{m_k}, x_{m_k-1}) + \varepsilon \text{ for all } k. \end{aligned} \tag{17}$$

Due to (11), we deduce that

$$\lim_{k\to\infty} d(x_{m_k}, x_{n_k}) = \varepsilon. \tag{18}$$

Again by the triangle inequality, together with (17) and), we also derive that

$$d(x_{m_k}, x_{n_k}) \leq d(x_{m_k}, x_{m_k+1}) + d(x_{m_k+1}, x_{n_k+1}) + d(x_{n_k+1}, x_{n_k}) \text{ for all } k.$$

Analogously, we have

$$d(x_{m_k+1}, x_{n_k+1}) \leq d(x_{m_k+1}, x_{m_k}) + d(x_{m_k}, x_{n_k}) + d(x_{n_k}, x_{n_k+1}) \text{ for all } k.$$

Combining two inequalities above together with (11), we find that

$$\lim_{k\to\infty} d(x_{m_k+1}, x_{n_k+1}) = \varepsilon. \tag{19}$$

Particularly, there exists $n_1 \in \mathbb{N}$ such that for all $k \geq n_1$ we have

$$d(x_{m_k}, x_{n_k}) > \frac{\varepsilon}{2} > 0 \text{ and } d(x_{m_k+1}, x_{n_k+1}) > \frac{\varepsilon}{2} > 0. \tag{20}$$

Moreover, since T is triangular α-orbital admissible, we have

$$\alpha(x_{m_k}, x_{n_k}) \geq 1. \tag{21}$$

Regarding the fact T is an α-admissible $\mathcal{Z}$*-contraction* with respect to ζ, together with (20) and (21) we get that

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_{m_k}, x_{n_k})d(Tx_{m_k}, Tx_{n_k}), d(x_{m_k}, x_{n_k})) \\ &= \zeta(\alpha(x_{m_k}, x_{n_k})d(x_{m_k+1}, x_{n_k+1}), d(x_{m_k}, x_{n_k})) \\ &< d(x_{m_k}, x_{n_k}) - \alpha(x_{m_k}, x_{n_k})d(x_{m_k+1}, x_{n_k+1}), \end{aligned} \tag{22}$$

for all $k \geq n_1$. Consequently, we have

$$0 < d(x_{m_k+1}, x_{n_k+1}) < \alpha(x_{m_k}, x_{n_k})d(x_{m_k+1}, x_{n_k+1}) < d(x_{m_k}, x_{n_k}),$$

for all $k \geq n_1$. From above inequality, together with (18) and (19), we conclude that $s_n = \alpha(x_{m_k}, x_{n_k})d(x_{m_k+1}, x_{n_k+1}) \to \varepsilon$ as $t_n = d(x_{m_k}, x_{n_k}) \to \varepsilon$. On account of the above observations and regarding the condition (ζ_3), we deduce that

$$0 \leq \limsup_{k\to\infty} \zeta(\alpha(x_{m_k}, x_{n_k})d(x_{m_k+1}, x_{n_k+1}), d(x_{m_k}, x_{n_k})) < 0,$$

which is a contradiction. Hence, $\{x_n\}$ is a Cauchy sequence. Owing to the fact that (X, d) is a complete metric space, there exists $u \in X$ such that

$$\lim_{n\to\infty} d(x_n, u) = 0. \tag{23}$$

Since T is continuous, we derive (23) that

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = \lim_{n\to\infty} d(Tx_n, Tu) = 0. \tag{24}$$

From (23), (24) and the uniqueness of the limit, we conclude that u is a fixed point of T, that is, $Tu = u$. □

Theorem 1.5. *Let* (X, d) *be a complete metric space and let* $T : X \to X$ *be an* α*-admissible* $\mathcal{Z}$-contraction *with respect to* ζ*. Suppose that*

(*i*) *T is triangular α-orbital admissible;*
(*ii*) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(*iii*) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.*

Then there exists $u \in X$ such that $Tu = u$.

Proof. Following the proof of Theorem 1.4, we know that the sequence $\{x_n\}$ defined by $x_{n+1} = Tx_n$ for all $n \geq 0$, converges for some $u \in X$. From (8) and condition (iii), there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, u) \geq 1$ for all k. Applying (5), for all k, we get that

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_{n(k)}, u)d(Tx_{n(k)}, Tu), d(x_{n(k)}, u)) \\ &= \zeta(\alpha(x_{n(k)}, u)d(x_{n(k)+1}, Tu), d(x_{n(k)}, u)) \\ &< d(x_{n(k)}, u) - \alpha(x_{n(k)}, u)d(x_{n(k)+1}, Tu), \end{aligned} \tag{25}$$

which is equivalent to

$$d(x_{n(k)+1}, Tu) = d(Tx_{n(k)}, Tu) \leq \alpha(x_{n(k)}, u)d(Tx_{n(k)}, Tu) \leq d(x_{n(k)}, u). \tag{26}$$

Letting $k \to \infty$ in the above equality, we have $d(u, Tu) = 0$, that is, $u = Tu$. □

For the uniqueness of a fixed point of a α-admissible $\mathcal{Z}$-*contraction* with respect to ζ, we shall suggest the following hypothesis.

(U) For all $x, y \in \mathrm{Fix}(T)$, we have $\alpha(x, y) \geq 1$.

Here, $\mathrm{Fix}(T)$ denotes the set of fixed points of T.

Theorem 1.6. *Adding condition* (U) *to the hypotheses of Theorem 1.4 (resp. Theorem 1.5), we obtain that u is the unique fixed point of T.*

2. Consequences

In this section, we shall illustrate that several existing fixed point results in the literature can be derived from our main results by regarding Example 0.2.

If $\psi \in \Psi$ and we define

$$\zeta_E(t, s) = \psi(s) - t \qquad \text{for all } s, t \in [0, \infty),$$

then ζ_{BW} is a simulation function (cf. Example 0.2 (v)).

We conclude that the main result of Samet *et al.* [14] can be expressed as a corollary of our main result.

Theorem 2.1. *Theorem 0.7 is a consequence of Theorem 1.6.*

Proof. Taking $\zeta_E(t, s) = \psi(s) - t$ for all $s, t \in [0, \infty)$ in Theorem 1.6, we get that

$$\alpha(x, y)d(Tx, Ty) \leq \psi(d(x, y)), \text{ for all } x, y \in X.$$

We skip the details. □

Hence, all consequences, including the famous fixed point theorem of Banach, can be expressed easily from the above theorem as in [14].

We derive that the main result of Khojasteh *et al.* [10] can be expressed as a corollary of our main result.

Theorem 2.2. *Theorem 0.3 is a consequence of Theorem 1.6.*

Proof. It is enough to take $\alpha(x, y) = 1$ for all $x, y \in X$. □

Definitely, all interesting results in [10] can be considered as consequences of our main result.

Conclusion

It is clear that we can list several consequences of our main results by defining the mapping ζ in a proper way like in the Example 0.2. In particular, we are able to get several existing fixed point theorems in the various settings (in the context of *partially ordered set endowed with a metric*, in the setting of *cyclic contraction* etc.) regarding Theorem (and hence Theorem 0.7). We omit the details since they are obvious.

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this article.

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References

[1] S. Banach, Sur les opérations dans les ensembles abstraits et leur application auxéquations intégrales, Fundamenta Mathematicae, 3 (1922), 133-181.

[2] D.W. Boyd and J.S.W. Wong, On Nonlinear Contractions, P. Am. Math. Soc. 20 (2) (1969) 458–464.

[3] N. Mizoguchi, W. Takahashi, Fixed point theorems for multivalued mappings on complete metric spaces, J. Math. Anal. Appl. 141 (1989), 177-188.

[4] B. E. Rhoades, A comparison of various definitions of contractive mappings, transactions of the american mathematical society, 224 (1977), 257-290.

[5] B. E. Rhoades, Some theorems on weakly contractive maps, Nonlinear Anal. (TMA) 47 (2001), 2683-2693.

[6] F. E. Browder, W. V. Petrysyn, The solution by iteration of nonlinear functional equation in Banach spaces, Bull. Amer. Math. Soc. 72 (1966), 571-576.

[7] N. Bilgili, E. Karapınar and B. Samet. Generalized α-ψ-contractive mappings in quasi-metric spaces and related fixed-point theorems Journal of Inequalities and Applications 2014, 2014:36

[8] M.S. Khan, M. Swaleh, and S. Sessa. Fixed point theorems by altering distances between the points. Bull. Aust. Math. Soc. 30 (1) (1984) 1–9.

[9] F. Khojasteh, V. Rakočević, Some new common fixed point results for generalized contractive multi-valued non-self-mappings, Appl. Math. Lett. 25 (2012), 287–293.

[10] F. Khojasteh, S. Shukla, S. Radenović, A new approach to the study of fixed point theorems via simulation functions, FILOMAT, To Appear.

[11] A.F. Roldán-López-de-Hierro, E. Karapınar, C. Roldán-López-de-Hierro, J. Martínez-Moreno, Coincidence point theorems on metric spaces via simulation functions, J. Comput. Appl. Math. 275 (2015) 345–355.

[12] H. Argoubi, B. Samet, C. Vetro, Nonlinear contractions involving simulation functions in a metric space with a partial order, J. Nonlinear Sci. Appl. 8 (2015) 10821094

[13] B. Samet, Best proximity point results in partially ordered metric spaces via simulation functions, Fixed Point Theory and Applications.

[14] B. Samet, C. Vetro, P. Vetro, Fixed point theorem for $\alpha - \psi$ contractive type mappings, Nonlinear Anal. 75 (2012) 2154–2165.

[15] E. Karapınar, P. Kuman, P. Salimi, On $\alpha - \psi$-Meri-Keeler contractive mappings, Fixed Point Theory Appl. (2013), 2013:94.

[16] O. Popescu, Some new fixed point theorems for α-Geraghty contractive type maps in metric spaces, Fixed Point Theory Appl. 2014, 2014:190

[17] E. Karapinar, H.H. Alsulami and M. Noorwali, Some extensions for Geragthy type contractive mappings Journal of Inequalities and Applications 2015, 2015:303 (26 September 2015)

[18] H.H. Alsulami, E. Karapınar, F. Khojasteh, A.F. Roldán-López-de-Hierro, A proposal to the study of contractions in quasi-metric spaces, Discrete Dynamics in Nature and Society 2014, Article ID 269286, 10 pages.

[19] Rus, I. A.: *Generalized contractions and applications*, Cluj University Press, Cluj-Napoca, 2001.

[20] Bianchini R.M., Grandolfi, M.: Transformazioni di tipo contracttivo generalizzato in uno spazio metrico, Atti Acad. Naz. Lincei, VII. Ser., Rend., Cl. Sci. Fis. Mat. Natur. 45 (1968), 212-216.

[21] Proinov, P.D. : A generalization of the Banach contraction principle with high order of convergence of successive approximations Nonlinear Analysis (TMA) 67 (2007), 2361-2369.

[22] Proinov, P.D. : New general convergence theory for iterative processes and its applications to Newton Kantorovich type theorems, J. Complexity 26 (2010), 3-42.