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Fixed Point Theorems for Two Pairs of Mappings Satisfying a New Type of Common Limit Range Property

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Abstract. The purpose of this paper is to prove a general fixed point theorem for mappings involving almost altering distances and satisfying a new type of common limit range property which generalize the results from Theorem 2.9 [19]. In the last part of the paper, as applications, some fixed point results for mappings satisfying contractive conditions of integral type for almost contractive mappings for ϕ - contractive mappings and (ψ, ϕ) - weak contractive mappings in metric spaces are obtained.

To the memory of Professor Lj. Ćirić (1935–2016)

1. Introduction and Preliminaries

Let (X, d) be a metric space and S, T be two self mappings of X. In [21], Jungck defined S and T to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X, such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t,$$

for some $t \in X$.

This concept has been frequently used to prove the existence theorems in fixed point theory.

Let f, g be self mappings of a nonempty set X. A point $x \in X$ is a coincidence point of f and g if $w = fx = gx$ and w is said to be a point of coincidence of f and g. The set of all coincidence points of f and g is denoted by $C(f, g)$.

In 1994, Pant [31] introduced the notion of pointwise R - weakly commuting mapping. It is proved in [32] that the pointwise R - weakly commuting is equivalent to commutativity at coincidence points.

In [22], Jungck introduced the notion of weakly compatible mappings.

Definition 1.1 ([22]). *Let X be a nonempty set and f, g be self mappings of X. f and g are weakly compatible if $fgu = gfu$ for all $u \in C(f, g)$.*

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Hence, f and g are weakly compatible if and only if f and g are pointwise R - weakly commuting.

The study of common fixed points for noncompatible mappings is also interesting, the work in this regard has been initiated by Pant in [28], [29], [30].

Aamri and El - Moutawakil [1] introduced a generalization of noncompatible mappings.

Definition 1.2 ([1]). *Let S and T be two self mappings of a metric space (X, d). We say that S and T satisfy $(E.A)$ property if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t,$$

for some $t \in X$.

Remark 1.3. *It is clear that two self mappings S and T of a metric space (X, d) will be noncompatible if there exists $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$ but $\lim_{n\to\infty} d(STx_n, TSx_n)$ is non zero or non existent. Therefore, two noncompatible self mappings of a metric space (X, d) satisfy property $(E.A)$.*

It is known [33], [34] that the notions of weakly compatible mappings and mappings satisfying $(E.A)$ property are independent.

In 2005, Liu et al. [26] defined the notion of common property $(E.A)$.

Definition 1.4 ([26]). *Two pairs (A, S) and (B, T) of self mappings defined on a metric space (X, d) are said to satisfy common property $(E.A)$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t,$$

for some $t \in X$.

There exists a vast literature concerning the study of fixed points for mappings satisfying $(E.A)$ property.

In 2011, Sintunawarat and Kumam [46] introduced the notion of common limit range property.

Definition 1.5 ([46]). *A pair (A, S) of self mappings defined on a metric space (X, d) is said to satisfy the common limit range property with respect to S, denoted $CLR_{(S)}$, if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t,$$

for some $t \in S(X)$.

Thus we can infer that a pair (A, S) satisfying the property $(E.A)$ along with the closedness of the subspace $S(X)$ always have $CLR_{(S)}$ - property with respect to S (see Example 2.16, 2.17 [17]).

Recently, Imdad et al. [18] extended the notion of common limit range property to the pairs of self mappings.

Definition 1.6 ([18]). *Two pairs (A, S) and (B, T) of self mappings defined on a metric space (X, d) are said to satisfy common limit range property with respect to S and T, denoted $CLR_{(S,T)}$, if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t,$$

where $t \in S(X) \cap T(X)$.

Some fixed point results for pairs of mappings with $CLR_{(S)}$ and $CLR_{(S,T)}$ property are obtained in [19], [6], [20], [40] and in other papers.

Now, we introduce a new type of limit range property.

Definition 1.7. *Let A, S and T be self mappings of a metric space (X, d). The pair (A, S) is said to satisfy common limit range property with respect to T, denoted $CLR_{(A,S),T}$ if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t,$$

for some $t \in S(X) \cap T(X)$.

Example 1.8. *Let $\mathbb{R}_+$ be the metric space with the usual metric, $Ax = \dfrac{x^2+1}{2}, Sx = \dfrac{x+1}{2}, Tx = x + \dfrac{1}{4}$. Then $S(X) = \left[\dfrac{1}{2}, \infty\right), T(X) = \left[\dfrac{1}{4}, \infty\right), S(X) \cap T(X) = \left[\dfrac{1}{2}, \infty\right)$. Let $\{x_n\}$ be a sequence with $\lim_{n\to\infty} x_n = 0$. Then, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \frac{1}{2} = z$ and $z \in S(X) \cap T(X)$.*

Remark 1.9. *Let A, B, S and T be self mappings of a metric space (X, d). If (A, S) and (B, T) satisfy the common limit range property with respect to S and T, then (A, S) satisfy the common limit range property with respect to T. The converse is not true. In Example 1.8, let $Bx = x^2 + \dfrac{1}{4}$. Let $\{y_n\}$ be a sequence such that $\lim_{n\to\infty} y_n = 0$ which implies $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = \dfrac{1}{4} \neq \dfrac{1}{2}$. Hence (A, S) and (B, T) don't satisfy condition $CLR_{(S,T)}$.*

Definition 1.10 ([24]). *An altering distance is a function $\psi : [0, \infty) \to [0, \infty)$ satisfying:*
(ψ_1) : *ψ is increasing and continuous;*
(ψ_2) : *$\psi(t) = 0$ if and only if $t = 0$.*

Fixed point theorems involving altering distances have been studied in [39], [44], [45] and in other papers.

The notion of almost altering distance is introduced in [40].

Definition 1.11 ([40]). *An almost altering distance is a function $\psi : [0, \infty) \to [0, \infty)$ is a if:*
(ψ_1) : *ψ is continuous;*
(ψ_2) : *$\psi(t) = 0$ if and only if $t = 0$.*

Remark 1.12. *Every altering distance is an almost altering distance, but the converse is not true.*

Example 1.13 ([40]). $\psi(t) = \begin{cases} t, & t \in [0, 1] \\ \dfrac{1}{t}, & t \in (1, \infty). \end{cases}$

2. Implicit Relations

Several fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit function [35], [36] and other papers.

Recently, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, b - metric spaces, ultra - metric spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two and three metric spaces, for single - valued functions, hybrid pairs of mappings and set - valued mappings. The method is used in the study of fixed points for mappings satisfying a contractive/extensive condition of integral type in fuzzy metric spaces, probabilistic metric spaces, intuitionistic metric spaces, partial metric spaces and G - metric spaces.

With this method the proofs of some fixed point theorems are more simple. Also, the method allows the study of local and global properties of fixed point structures.

In 2008, Ali and Imdad [5] introduced a new class of implicit relations. Recently, Imdad and Chauhan [19] employed common limit range property to prove unified metrical common fixed point theorems in metric spaces.

Definition 2.1 ([5]). *Let $\mathfrak{F}$ be the family of lower semi - continuous functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ satisfying the following conditions:*

(F_1) : $F(t, 0, t, 0, 0, t) > 0$, *for all* $t > 0$;
(F_2) : $F(t, 0, 0, t, t, 0) > 0$, *for all* $t > 0$;
(F_3) : $F(t, t, 0, 0, t, t) > 0$, *for all* $t > 0$.

Example 2.2. $F(t_1, ..., t_6) = t_1 - k\max\{t_2, t_3, t_4, t_5, t_6\}$, *where* $k \in [0, 1)$.

Example 2.3. $F(t_1, ..., t_6) = t_1 - at_2 - b\max\{t_3, t_4\} - c\max\{t_5, t_6\}$, *where* $a, b, c \geq 0$ *and* $a + b + c < 1$.

Example 2.4. $F(t_1, ..., t_6) = t_1 - k\max\left\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\right\}$, *where* $k \in [0, 1)$.

Example 2.5. $F(t_1, ..., t_6) = t_1 - k\max\left\{t_2, \frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\right\}$, *where* $k \in [0, 1)$.

Example 2.6. $F(t_1, ..., t_6) = t_1 - \alpha\max\{t_2, t_3, t_4\} - (1 - \alpha)(at_5 + bt_6)$, *where* $\alpha \in (0, 1)$, $a, b \geq 0$ *and* $a + b < 1$.

Example 2.7. $F(t_1, ..., t_6) = t_1 - at_2 - b(t_3 + t_4) - c\min\{t_5, t_6\}$, *where* $a, b, c > 0$ *and* $a + b + c < 1$.

Example 2.8. $F(t_1, ..., t_6) = t_1 - at_2 - \frac{b(t_5 + t_6)}{1 + t_3 + t_4}$, *where* $a, b \geq 0$ *and* $a + 2b < 1$.

Example 2.9. $F(t_1, ..., t_6) = t_1 - \max\{ct_2, ct_3, ct_4, at_5 + bt_6\}$, *where* $c \in (0, 1)$, $a, b \geq 0$ *and* $a + b < 1$.

Other examples are in [9] and [19].

Quite recently, the following theorem is proved in [19].

Theorem 2.10 ([19]). *Let A, B, S and T be self mappings of a metric space (X, d) satisfying the inequality*

$$\begin{aligned} &F(d(Ax, By), d(Sx, Ty), d(Sx, Ax), \\ &d(Ty, By), d(Sx, By), d(Ty, Ax)) < 0, \end{aligned} \tag{1}$$

for all $x, y \in X$, where $F \in \mathfrak{F}$. If the pairs (A, S) and (B, T) share the $CLR_{(S,T)}$ - property, then

1) $C(A, S) \neq \emptyset$,
2) $C(B, T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point.

The purpose of this paper is to prove a general common fixed point theorem for two pairs of mappings involving almost altering distances and satisfying a new common limit range property for the mappings by Definition 1.7, which generalizes the result from Theorem 2.10.

In the last part of the paper, as applications, some fixed point results for mappings satisfying contractive conditions of integral type, for almost contractive mappings, for φ - contractive mappings and (ψ, ϕ) - weak contractive mappings in metric spaces and for a sequence of mappings are obtained.

3. Main Results

Lemma 3.1 ([2]). *Let f and g be weakly compatible self mappings of a nonempty set X. If f and g have a unique point of coincidence $w = fx = gx$ for some $x \in X$, then w is the unique common fixed point of f and g.*

Theorem 3.2. *Let (X, d) be a metric space and A, B, S and T be self mappings of X satisfying the inequality*

$$F(\psi(d(Ax, By)), \psi(d(Sx, Ty)), \psi(d(Sx, Ax)), \psi(d(Ty, By)), \psi(d(Sx, By)), \psi(d(Ty, Ax))) \leq 0, \tag{2}$$

for all $x, y \in X$, F satisfying (F_3) and ψ is an almost altering distance.

If there exist $u, v \in X$ such that $Au = Su$ and $Bv = Tv$, then there exists $t \in X$ such that t is the unique point of coincidence of A and S, as well the unique point of coincidence of B and T.

Proof. First we prove that $Su = Tv$. Suppose that $Su \neq Tv$. Then by (2) we get

$$F(\psi(d(Au,Bv)), \psi(d(Su,Tv)), \psi(d(Su,Au)), \psi(d(Tv,Bv)), \psi(d(Su,Bv)), \psi(d(Tv,Au))) \leq 0,$$

$$F(\psi(d(Su,Tv)), \psi(d(Su,Tv)), 0, 0, \psi(d(Su,Tv)), \psi(d(Su,Tv))) \leq 0,$$

a contradiction of (F_3). Hence $\psi(d(Su,Tv)) = 0$, which implies $Su = Tv = Au = Bv = t$ for some $t \in X$.

Suppose that there exists $w \neq u$ such that $Aw = Sw$. Then, by (2) we obtain

$$F(\psi(d(Aw,Bv)), \psi(d(Sw,Tv)), \psi(d(Sw,Aw)), \psi(d(Tv,Bv)), \psi(d(Sw,Bv)), \psi(d(Tv,Aw))) \leq 0,$$

$$F(\psi(d(Sw,Tv)), \psi(d(Sw,Tv)), 0, 0, \psi(d(Sw,Tv)), \psi(d(Sw,Tv))) \leq 0,$$

a contradiction of (F_3) if $\psi(d(Sw,Tv)) > 0$. Hence, $\psi(d(Sw,Tv)) = 0$, which implies $Sw = Tv = Bv = Au = Su = t$. Hence t is the unique point of coincidence of A and S.

Similarly, t is the unique point of coincidence of B and T. □

Theorem 3.3. *Let (X,d) be a metric space and A, B, S and T be self mappings of X satisfying the inequality (2) for all $x, y \in X$, $F \in \mathfrak{F}$ and ψ is an altering distance. If A, S, T satisfy $CLR_{(A,S),T}$ - property, then*

i) $C(A,S) \neq \varnothing$,

ii) $C(B,T) \neq \varnothing$.

Moreover, if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point.

Proof. Since A, S, T satisfy $CLR_{(A,S),T}$ - property, there exists a sequences $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, where $z \in S(X) \cap T(X)$.

Hence $z \in T(X)$, which implies $z = Tu$ for some $u \in X$.

By (2) we have

$$F(\psi(d(Ax_n,Bu)), \psi(d(Sx_n,Tu)), \psi(d(Sx_n,Ax_n)), \psi(d(Tu,Bu)), \psi(d(Sx_n,Bu)), \psi(d(Tu,Ax_n))) \leq 0.$$

Letting n tends to infinity we obtain

$$F(\psi(d(z,Bu)), 0, 0, \psi(d(z,Bu)), \psi(d(z,Bu)), 0) \leq 0,$$

a contradiction of (F_2) if $\psi(d(z,Bu)) > 0$. Hence, $\psi(d(z,Bu)) = 0$, which implies $z = Bu = Tu$ and $C(B,T) \neq \varnothing$.

On the other hand $z \in S(X)$, which implies $z = Sv$ for $v \in X$. Again, by (2) we obtain

$$F(\psi(d(Av,Bu)), \psi(d(Sv,Tu)), \psi(d(Sv,Av)), \psi(d(Tu,Bu)), \psi(d(Sv,Bu)), \psi(d(Tu,Av))) \leq 0,$$

$$F(\psi(d(Av,z)), 0, \psi(d(Av,z)), 0, 0, \psi(d(z,Av))) \leq 0,$$

a contradiction of (F_1) if $\psi(d(Av,z)) > 0$. Hence, $\psi(d(Av,z)) = 0$, which implies $z = Av = Sv$ and $C(A,S) \neq \varnothing$.

By Theorem 3.2, z is the unique point of coincidence of (A,S) and (B,T).

Moreover, if (A,S) and (B,T) are weakly compatible, then by Lemma 3.1, z is the unique fixed point of (A,S) and (B,T). Hence, z is the unique common fixed point of A, B, S and T. □

If $\psi(t) = t$, then by Theorem 3.3 we obtain

Theorem 3.4. *Let (X,d) be a metric space and A, B, S and T be self mappings of X satisfying the inequality*

$$F(d(Ax,By), d(Sx,Ty), d(Sx,Ax), d(Ty,By), d(Sx,By), d(Ty,Ax)) \leq 0, \tag{3}$$

for all $x, y \in X$ and $F \in \mathfrak{F}$. If A, S, T satisfy $CLR_{(A,S),T}$ - property, then

i) $C(A,S) \neq \varnothing$,

ii) $C(B,T) \neq \varnothing$.

Moreover, if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point.

Remark 3.5. *By Remark 1.9 it follows that Theorem 3.3 is a generalization of Theorem 2.10.*

For a function $f : (X, d) \to (X, d)$ we denote

$$Fix\,(f) = \{x \in X : x = fx\}.$$

Theorem 3.6. *Let A, B, S and T be self mappings of (X, d). If the inequality (2) holds for all $x \in X$ and $F \in \mathfrak{F}$, then*

$$[Fix\,(S) \cap Fix\,(T)] \cap Fix\,(A) = [Fix\,(S) \cap Fix\,(T)] \cap Fix\,(B)\,.$$

Proof. Let $x \in [Fix\,(S) \cap Fix\,(T)] \cap Fix\,(A)$. Then by (2) we have

$$F(\psi\,(d(Ax, Bx))\,, \psi\,(d(Sx, Tx))\,, \psi\,(d(Sx, Ax))\,, \psi\,(d(Tx, Bx))\,, \psi\,(d(Sx, Bx))\,, \psi\,(d(Tx, Ax))) \le 0,$$

$$F(\psi\,(d(x, Bx))\,, 0, 0, \psi\,(d(x, Bx))\,, \psi\,(d(x, Bx))\,, 0) \le 0,$$

a contradiction of (F_2) if $\psi\,(d(x, Bx)) > 0$. Hence, $\psi\,(d(x, Bx)) = 0$ which implies $x = Bx$ and $x \in Fix\,(B)$. Therefore

$$[Fix\,(S) \cap Fix\,(T)] \cap Fix\,(A) \subset [Fix\,(S) \cap Fix\,(T)] \cap Fix\,(B)\,.$$

Similarly, by (2) and (F_1) we obtain

$$[Fix\,(S) \cap Fix\,(T)] \cap Fix\,(B) \subset [Fix\,(S) \cap Fix\,(T)] \cap Fix\,(A)\,.$$

□

Theorems 3.3 and 3.6 imply the following one.

Theorem 3.7. *Let S, T and $\{A_i\}_{i \in \mathbb{N}^*}$ be self mappings of a metric space (X, d) satisfying the inequality*

$$F(\psi\,(d(A_i x, A_{i+1} y))\,, \psi\,(d(Sx, Ty))\,, \psi\,(d(Sx, A_i x))\,, \psi\,(d(Ty, A_{i+1} y))\,, \psi\,(d(Sx, A_{i+1} y))\,, \psi\,(d(Ty, A_i x))) \le 0, \quad (4)$$

for all $x, y \in X$, $F \in \mathfrak{F}$ and ψ is an almost altering distance, $i \in \mathbb{N}^*$.

If (A_1, S) and T satisfy $CLR_{(A,S),T}$ - property and (A_1, S), (A_2, T) are weakly compatible, then S, T and A_i, $i \in \mathbb{N}^*$ have a unique common fixed point.

If $\psi\,(t) = t$ we obtain

Theorem 3.8. *Let S, T and $\{A_i\}_{i \in \mathbb{N}^*}$ be self mappings of a metric space (X, d) satisfying the inequality*

$$F(d(A_i x, A_{i+1} y), d(Sx, Ty), d(Sx, A_i x), d(Ty, A_{i+1} y), d(Sx, A_{i+1} y), d(Ty, A_i x)) \le 0, \quad (5)$$

for all $x, y \in X$ and $i \in \mathbb{N}^*$.

If (A_1, S) and T satisfy $CLR_{(A,S),T}$ - property and (A_1, S) and (A_2, T) are weakly compatible, then S, T and $\{A_i\}_{i \in \mathbb{N}^*}$ have a unique common fixed point.

4. Applications

4.1. Fixed points for mappings satisfying contractive conditions of integral type

In [14], Branciari established the following theorem, which opened the way to the study of fixed points for mappings satisfying contractive conditions of integral type

Theorem 4.1 ([14]). *Let (X,d) be a complete metric space, $c \in (0,1)$ and $f : X \to X$ such that for all $x, y \in X$*

$$\int_0^{d(fx,fy)} h(t)dt \leq c \int_0^{d(x,y)} h(t)dt$$

whenever $h : [0,\infty) \to [0,\infty)$ is a Lebesgue measurable mapping which is summable (i.e., with finite integral) on each compact subset of $[0,\infty)$, such that $\int_0^{\varepsilon} h(t)dt > 0$, for all $\varepsilon > 0$. Then, f has a unique fixed point $z \in X$ such that for all $x \in X$, $z = \lim_{n\to\infty} f^n x$.

Theorem 4.1 has been extended to a pair of compatible mappings in [25].

Theorem 4.2. *Let f, g be compatible self mappings of a complete metric space (X,d), with g - continuous, satisfying the following conditions:*

1) $f(X) \subset g(X)$,

2) $\int_0^{d(fx,gy)} h(t)dt \leq c \int_0^{d(x,y)} h(t)dt$,

for some $c \in (0,1)$, whenever $x, y \in X$ and $h(t)$ as in Theorem 4.1.

Then, f and g have a unique fixed point.

Some fixed point results for mappings satisfying contractive conditions of integral type are obtained in [38], [39], [43] and in other papers.

Lemma 4.3. *Let $h : [0,\infty) \to [0,\infty)$ as in Theorem 4.1. Then $\psi(t) = \int_0^t h(x)dx$ is an almost altering distance.*

Proof. The proof it follows from Lemma 2.5 [39]. □

Theorem 4.4. *Let A, B, S and T be self mappings of a metric space (X,d) such that*

$$\begin{aligned} &F(\int_0^{d(Ax,By)} h(t)dt, \int_0^{d(Sx,Ty)} h(t)dt, \int_0^{d(Sx,Ax)} h(t)dt, \\ &\int_0^{d(Ty,By)} h(t)dt, \int_0^{d(Sx,By)} h(t)dt, \int_0^{d(Ty,Ax)} h(t)dt) \leq 0, \end{aligned} \tag{6}$$

for all $x, y \in X$, $F \in \mathfrak{F}$ and $h(t)$ as in Theorem 4.1.

If A, S, T satisfy $CLR_{(A,S),T}$ - property, then

i) $C(A,S) \neq \emptyset$,

ii) $C(B,T) \neq \emptyset$.

Moreover, if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point.

Proof. By Lemma 4.3, $\psi(t) = \int_0^t h(x)dx$ is an almost altering distance. By (6) we obtain

$$F(\psi(d(Ax,By)), \psi(d(Sx,Ty)), \psi(d(Sx,Ax)), \psi(d(Ty,By)), \psi(d(Sx,By)), \psi(d(Ty,Ax))) \leq 0,$$

which is inequality (2). Hence, the conditions of Theorem 3.3 are satisfied. Theorem 4.4 follows by Theorem 3.3. □

Similarly, by Theorem 4.2 and Example 2.2, we obtain

Theorem 4.5. *Let A, B, S and T be self mappings of a metric space (X,d) such that*

$$\int_0^{d(Ax,By)} h(t)dt \leq k \max\left\{\int_0^{d(Sx,Ty)} h(t)dt, \int_0^{d(Sx,Ax)} h(t)dt, \int_0^{d(Ty,By)} h(t)dt, \int_0^{d(Sx,By)} h(t)dt, \int_0^{d(Ty,Ax)} h(t)dt\right\},$$

where $k \in [0,1)$, for all $x, y \in X$ and $h(t)$ as in Theorem 4.1.

If A, S, T satisfy $CLR_{(A,S),T}$ - property, then

i) $C(A,S) \neq \emptyset$,

ii) $C(B,T) \neq \emptyset$.

Moreover, if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point.

4.2. Fixed points for almost contractive mappings in metric spaces

Definition 4.6. *Let (X,d) be a metric space. A mapping $T:(X,d)\to(X,d)$ is called weak contractive [10], [12] or almost contractive [11] if there exist $\delta\in(0,1)$ and $L\geq 0$ such that*

$$d(Tx,Ty)\leq \delta d(x,y)+Ld(y,Tx).$$

The following theorem is proved in [13].

Theorem 4.7 ([13]). *Let (X,d) be a metric space and $T,S:(X,d)\to(X,d)$ be mappings for which there exists $a\in(0,1)$ and $L\geq 0$ such that*

$$d(Tx,Ty)\leq ad(Sx,Sy)+Ld(Sy,Tx),$$

for all $x,y\in X$.

If $T(X)\subset S(X)$ and $S(X)$ is a complete subspace of X, then T and S have a unique common fixed point.

A similar result is obtained if

$$d(Tx,Ty)\quad\leq\quad ad(Sx,Sy)+L\min\{d(Sx,Tx),d(Sy,Ty),d(Sx,Ty),d(Tx,Sy)\},$$

where $a\in(0,1)$ and $L\geq 0$.

In [7], a similar result is obtained if

$$d(Tx,Ty)\quad\leq\quad \delta m(x,y)+L\min\{d(Sx,Tx),d(Sy,Ty),d(Sx,Ty),d(Tx,Sy)\},$$

where $\delta\in(0,1)$, $L\geq 0$ and

$$m(x,y)\quad=\quad\max\left\{d(Sx,Sy),\frac{d(Tx,Sx)+d(Ty,Sy)}{2},\frac{d(Sx,Ty)+d(Tx,Sy)}{2}\right\}.$$

A general fixed point theorem for almost contractive mappings is obtained in [37].

The following functions $F(t_1,...,t_6):\mathbb{R}_+^6\to\mathbb{R}$ satisfy conditions (F_1), (F_2) and (F_3).

Example 4.8. $F(t_1,...,t_6)=t_1-\delta\max\left\{t_2,\frac{t_3+t_4}{2},\frac{t_5+t_6}{2}\right\}-L\min\{t_3,t_4,t_5,t_6\}$, *where $\delta\in(0,1)$ and $L\geq 0$.*

Example 4.9. $F(t_1,...,t_6)=t_1-at_2-L\min\{t_3,t_4,t_5,t_6\}$, *where $a\in(0,1)$ and $L\geq 0$.*

Example 4.10. $F(t_1,...,t_6)=t_1-k\max\left\{t_2,t_3,t_4,\frac{t_5+t_6}{2}\right\}-L\min\{t_3,t_4,t_5,t_6\}$, *where $k\in(0,1)$ and $L\geq 0$.*

Example 4.11. $F(t_1,...,t_6)=t_1-k\max\{t_2,t_3,t_4,t_5,t_6\}-L\min\{t_3,t_4,t_5,t_6\}$, *where $k\in(0,1)$ and $L\geq 0$.*

Example 4.12. $F(t_1,...,t_6)=t_1-k\max\left\{t_2,\frac{t_3+t_4}{2},\frac{t_5+t_6}{2}\right\}-L\min\left\{t_3,t_4,\sqrt{t_4t_5},\sqrt{t_5t_6}\right\}$, *where $k\in(0,1)$ and $L\geq 0$.*

Example 4.13. $F(t_1,...,t_6)=t_1-k\max\left\{t_2,t_3,\sqrt{t_4t_5},\sqrt{t_5t_6}\right\}-L\min\{t_3,t_4,t_5,t_6\}$, *where $k\in(0,1)$ and $L\geq 0$.*

Example 4.14. $F(t_1,...,t_6)=t_1-\max\{t_2,k(t_3+t_4),k(t_5+t_6)\}-L\min\{t_3,t_4,t_5,t_6\}$, *where $k\in(0,1)$ and $L\geq 0$.*

Example 4.15. $F(t_1,...,t_6)=t_1-\max\left\{t_2,at_3,at_4,\frac{a(t_5+t_6)}{2}\right\}-L\min\{t_3,t_4,t_5,t_6\}$, *where $a\in(0,1)$ and $L\geq 0$.*

By Theorem 3.3 and Example 4.8 we obtain

Theorem 4.16. *Let A, B, S and T be self mappings of a metric space (X, d) such that*

$$\psi\left(d(Ax, By)\right) \leq \delta \max\{\psi(d(Sx,Ty)), \frac{\psi(d(Sx,Ax)) + \psi(d(Ty,By))}{2}, \frac{\psi(d(Sx,By)) + \psi(d(Ty,Ax))}{2}\}$$
$$+L\min\{\psi(d(Sx,Ty)), \psi(d(Ty,By)), \psi(d(Sx,By)), \psi(d(Ty,Ax))\},$$

where $\delta \in (0,1)$, $L \geq 0$ for all $x, y \in X$ and ψ is an almost altering distance.
If (A, S) and T satisfy $CLR_{(A,S),T}$ - property, then
i) $C(A,S) \neq \emptyset$,
ii) $C(B,T) \neq \emptyset$.
Moreover, if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point.

By Example 4.8 and Theorem 4.16 we obtain

Theorem 4.17. *Let A, B, S and T be self mappings of a metric space (X, d) such that*

$$\int_0^{d(Ax,By)} h(t)dt \leq \delta \max\left\{\int_0^{d(Sx,Ty)} h(t)dt, \frac{\int_0^{d(Sx,Ax)} h(t)dt + \int_0^{d(Ty,By)} h(t)dt}{2}, \frac{\int_0^{d(Sx,By)} h(t)dt + \int_0^{d(Ty,Ax)} h(t)dt}{2}\right\}$$
$$+L\min\{\int_0^{d(Sx,Ty)} h(t)dt, \int_0^{d(Sx,Ax)} h(t)dt, \int_0^{d(Ty,By)} h(t)dt, \int_0^{d(Sx,Ty)} h(t)dt, \int_0^{d(Ty,Ax)} h(t)dt\},$$

where $\delta \in (0,1)$, $L \geq 0$ for all $x, y \in X$ and $h(t)$ as in Theorem 4.1.
If A, S and T satisfy $CLR_{(A,S),T}$ - property, then
i) $C(A,S) \neq \emptyset$,
ii) $C(B,T) \neq \emptyset$.
Moreover, if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point.

4.3. Fixed points for mappings satisfying φ - contractive conditions

As in [27], let ϕ be the set of all real nondecreasing continuous functions $\varphi : [0,\infty) \to [0,\infty)$ with $\lim_{n\to\infty} \varphi^n(t) = 0$, for all $t \in [0,\infty)$.
If $\varphi \in \phi$, then
1) $\varphi(t) < t$ for all $t \in (0,\infty)$,
2) $\varphi(0) = 0$.
The following functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ satisfy conditions (F_1), (F_2) and (F_3).

Example 4.18. $F(t_1, ..., t_6) = t_1 - \varphi(\max\{t_2, t_3, t_4, t_5, t_6\})$.

Example 4.19. $F(t_1, ..., t_6) = t_1 - \varphi\left(\max\left\{t_2, t_3, t_4, \frac{t_5+t_6}{2}\right\}\right)$.

Example 4.20. $F(t_1, ..., t_6) = t_1 - \varphi\left(\max\left\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\right\}\right)$.

Example 4.21. $F(t_1, ..., t_6) = t_1 - \varphi\left(\max\left\{t_2, \sqrt{t_3t_4}, \sqrt{t_5t_6}, \sqrt{t_3t_5}, \sqrt{t_4t_6}\right\}\right)$

Example 4.22. $F(t_1, ..., t_6) = t_1 - \varphi(at_2 + bt_3 + ct_4 + dt_5 + et_6)$, *where $a, b, c, d, e \geq 0$ and $a + b + c + d + e \leq 1$.*

Example 4.23. $F(t_1, ..., t_6) = t_1 - \varphi\left(at_2 + b\frac{\sqrt{t_5t_6}}{1+t_3+t_4}\right)$, *where $a, b \geq 0$ and $a + b \leq 1$.*

Example 4.24. $F(t_1, ..., t_6) = t_1 - \varphi\left(at_2 + b\max\{t_3, t_4\} + c\max\left\{\frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\right\}\right)$, *where $a, b, c \geq 0$ and $a+b+c \leq 1$.*

Example 4.25. $F(t_1,...,t_6) = t_1 - \varphi(at_2 + b\max\{2t_4 + t_5, 2t_4 + t_6, t_3 + t_5 + t_6\})$, *where* $a, b \geq 0$ *and* $a + b \leq 1$.

By Theorem 3.3 and Example 4.18 we obtain

Theorem 4.26. *Let* A, B, S *and* T *be self mappings of a metric space* (X, d) *such that*

$$\psi(d(Ax, By)) \leq \varphi(\max\{\psi(d(Sx, Ty)), \psi(d(Sx, Ax)), \\ \psi(d(Ty, By)), \psi(d(Sx, By)), \psi(d(Ty, Ax))),$$

for all $x, y \in X$, $\varphi \in \phi$ *and* ψ *is an almost altering distance.*

If A, S *and* T *satisfy* $CLR_{(A,S),T}$ *- property, then*

i) $C(A, S) \neq \emptyset$,

ii) $C(B, T) \neq \emptyset$.

Moreover, if (A, S) *and* (B, T) *are weakly compatible, then* A, B, S *and* T *have a unique common fixed point.*

By Theorem 4.26, Theorem 4.4 and Example 4.18 we obtain

Theorem 4.27. *Let* A, B, S *and* T *be self mappings of a metric space* (X, d) *such that*

$$\int_0^{d(Ax,By)} h(t)dt \leq \varphi\left(\max\left\{\int_0^{d(Sx,Ty)} h(t)dt, \int_0^{d(Sx,Ax)} h(t)dt, \int_0^{d(Ty,By)} h(t)dt, \int_0^{d(Sx,By)} h(t)dt, \int_0^{d(Ty,Ax)} h(t)dt\right\}\right),$$

for all $x, y \in X$, $\varphi \in \phi$ *and* $h(t)$ *as in Theorem 4.1.*

If (A, S) *and* T *satisfy* $CLR_{(A,S),T}$ *- property, then*

i) $C(A, S) \neq \emptyset$,

ii) $C(B, T) \neq \emptyset$.

Moreover, if (A, S) *and* (B, T) *are weakly compatible, then* A, B, S *and* T *have a unique common fixed point.*

4.4. *Fixed points for* (ψ, φ) *- weakly contractive mappings*

In 1997, Alber and Guerre - Delabriere [4] defined the concept of weak contraction as a generalization of contraction and established the existence of fixed points for self mappings in Hilbert spaces. Rhoades [42] extended this concept in metric spaces. In [9], the authors studied the existence of fixed points for a pair of (ψ, φ) - weakly contractive mappings. New results are obtained in [15], [16], [41]. In [3] and [8], the study of fixed points of (ψ, φ) - weakly contractions with $(E.A)$ - property is initiated. Also, some fixed point theorems for mappings with common limit range property satisfying (ψ, φ) - weak contractive conditions are proved in [23] and [47].

Definition 4.28. *1)* *Let* Ψ *be the subset of all functions* $\psi : [0, \infty) \to [0, \infty)$ *satisfying*

a) ψ *is continuous,*

b) $\psi(0) = 0$ *and* $\psi(t) > 0$, $\forall t > 0$.

2) *Let* Φ *be the set of all functions* $\phi : [0, \infty) \to [0, \infty)$ *satisfying*

a) ϕ *is lower semi - continuous,*

b) $\phi(0) = 0$ *and* $\phi(t) > 0$, $\forall t > 0$.

The following functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ satisfy conditions $(F_1), (F_2)$ and (F_3).

Example 4.29. $F(t_1, ..., t_6) = \psi(t_1) - \psi\left(\max\left\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\right\}\right) + \phi(\max\{t_3, t_4, t_5, t_6\})$.

Example 4.30. $F(t_1, ..., t_6) = \psi(t_1) - \psi(\max\{t_2, t_3, t_4, t_5, t_6\}) + \phi\left(\max\left\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\right\}\right)$.

Example 4.31. $F(t_1, ..., t_6) = \psi(t_1) - \psi\left(\max\left\{t_2, \frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\right\}\right) + \phi(\max\{t_2, t_3, t_4, t_5, t_6\})$.

Example 4.32. $F(t_1,...,t_6) = \psi(t_1) - \psi\left(\max\left\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\right\}\right) + \phi\left(\max\left\{t_3, t_4, \frac{t_5+t_6}{2}\right\}\right)$.

Example 4.33. $F(t_1,...,t_6) = \psi(t_1) - \psi\left(\max\left\{t_2, t_3, t_4, \frac{t_5+t_6}{2}\right\}\right) + \phi\left(\max\left\{\sqrt{t_3t_6}, \sqrt{t_2t_5}, \sqrt{t_5t_6}\right\}\right)$.

Example 4.34. $F(t_1,...,t_6) = \psi(t_1) - \psi\left(\max\left\{\sqrt{t_3t_6}, \sqrt{t_2t_5}, \sqrt{t_5t_6}\right\}\right) + \phi(\max\{t_2, t_3, t_4, t_5, t_6\})$.

Example 4.35. $F(t_1,...,t_6) = \psi(t_1) - \psi\left(\frac{\sqrt{t_3t_6} + \sqrt{t_4t_6} + \sqrt{t_2t_6}}{1 + \sqrt{t_3t_4} + \sqrt{t_4t_6} + \sqrt{t_2t_3}}\right) + \phi(\max\{t_2, t_3, t_4, t_5, t_6\})$.

Example 4.36. $F(t_1,...,t_6) = \psi(t_1) - \psi\left(\sqrt{t_2t_5} + \sqrt{t_2t_6} + \sqrt{t_3t_6} + \sqrt{t_4t_5}\right) + \phi(\max\{t_2, t_3, t_4, t_5, t_6\})$.

By Theorem 3.4 and Example 4.29 we obtain the following

Theorem 4.37. *Let A, B, S and T be self mappings of a metric space (X, d) such that*

$$\psi(d(Ax, By)) \leq \psi(M_1(x,y)) - \phi(M_2(x,y)),$$

for all $x, y \in X$, where

$$M_1(x,y) = \max\{d(Sx,Ty), d(Sx,Ax), d(Ty,By), \frac{d(Sx,By)+d(Ty,Ax)}{2}\}$$

and

$$M_2(x,y) = \max\{d(Sx,Ax), d(Ty,By), d(Sx,By), d(Ty,Ax)\},$$

$\psi \in \Psi$ and $\varphi \in \phi$.

If (A, S) and T satisfy $CLR_{(A,S),T}$ - property, then

i) $C(A,S) \neq \emptyset$,

ii) $C(B,T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point.

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