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A Note on the Paper "A Fixed Point Theorems in S_b-Metric Spaces"

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Abstract. Very recently, N. Souayan and N. Mlaiki [Nazir Souayan and Nabil Mlaiki, A fixed point theorem in S_b-metric spaces, J. Math. Comput. Sci. 16 (2016), 131-139] and S. Sedghi et al. [S. Sedghi, A. Gholidahneb, T. Došenović, J. Esfahani, S. Radenović, Common fixed point of four maps in S_b-metric spaces, to appear in J. Linear Topol. Algebra], introduced the concept of S_b-metric space as a generalization of S-metric space. In this paper, we modified the definition of S_b-metric introduced by Souayan and Mlaiki and prove some coupled common fixed point theorems in S_b-metric space. We also present an example to confirm our theoretical results.

To the memory of Professor Lj. Ćirić (1935–2016)

1. Introduction and Preliminaries

S. Sedghi, N. Shobe and A. Aliouche [28] introduced the concept of S-metric space by modifying D-metric and G-metric spaces. The definition of S-metric space is as follows.

Definition 1.1. *Let X be a nonempty set. An S-metric on X is a function $S : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

1. $S(x, y, z) \geq 0$,
2. $S(x, y, z) = 0$ *if and only if* $x = y = z$
3. $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$

The pair (X, S) is called S-metric space.

Sedhi et. al. [28] gave following remarks.

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It is easy to see that every D^*-metric is S-metric, but in general the converse is not true.

In [28] it is also shown the following lemma

Lemma 1.2. *In an S-metric space, we have*

$$S(x, x, y) = S(y, y, x)$$

S. Sedghi and Nguyen van Dung [27] remarked that every S-metric space is topologically equivalent to a metric space.

Bakhtin [3] introduced the concept of b-metric space as a generalization of metric space. S. Czerwick [6, 7] extended the Banach contraction principle is b-metric space.

The definition of b-metric is given below

Definition 1.3. *[3] Let X be a nonempty set. A b-metric on X is a function $d : X^2 \to [0, \infty)$ if there exists a real number $s \geq 1$ such that the following conditions holds for all $x, y \in X$*

(i) $d(x, y) = 0$ *if and only if* $x = y$,

(ii) $d(x, y) = d(y, x)$

(iii) $d(x, z) \leq s[d(x, y) + d(y, z)]$

The pair (X, d) is called a b-metric space.

Inspired by the works of Bakhtin [3] and Sedghi et. al. [28], Nizar and Nabil [35] introduced the concept of S_b-metric space.

Definition 1.4. *[35] Let X be a nonempty set and let $s \geq 1$ be a given number. A function $S_b : X^3 \to [0, \infty)$ is said to be S_b-metric if and only if for all $x, y, z, t \in X$, the following conditions hold:*

(i) $S_b(x, y, z) = 0$ *if and only if* $x = y = z$,

(ii) $S_b(x, x, y) = S_b(y, y, x)$ *for all* $x, y \in X$,

(iii) $S_b(x, y, z) \leq s[S_b(x, x, t) + S_b(y, y, t) + S_b(z, z, t)]$

The pair (X, S_b) is called an S_b-metric space. See also ([29], Definition 1.7).

Remark 1.5. *[35] Note that the class of S_b-metric spaces is larger than the class of S-metric spaces. Indeed, every S-metric space is an S_b-metric space with $s = 1$. However, the converse is not always true.*

Example 1.6. *[35] Let X be a nonempty set and card(X) ≥ 5. Suppose $X = X_1 \cup X_2$ a partition of X such that card(X_1) ≥ 4. Let $s \geq 1$. Then*

$$S_b(x, y, z) = \begin{cases} 0, & \text{if } x = y = z = 0; \\ 3s, & \text{if } (x, y, z) \in X_1^3; \\ 1, & \text{if } (x, y, z) \notin X_1^3. \end{cases}$$

for all $x, y, z \in X$. S_b is a S_b-metric on X with coefficient $s \geq 1$.

Definition 1.7. *[35] Let (X, S_b) be an $S_b$metric space and $\{x_n\}$ be a sequence in X. Then*

(i) *A sequence $\{x_n\}$ is called convergent if and only if there exists $z \in X$ such that $S_b(x_n, x_n, z) \to 0$ as $n \to \infty$. In this case we write $\lim_{n\to\infty} x_n = z$.*

(ii) *A sequence* $\{x_n\}$ *is called Cauchy sequence if and only if* $S_b(x_n, x_n, x_m) \to 0$ *as* $n, m \to \infty$.

(iii) (X, S_b) *is said to be a complete* S_b*-metric space if every Cauchy sequence* $\{x_n\}$ *converges to a point* $x \in X$ *such that*

$$\lim_{n,m\to\infty} S_b(x_n, x_n, x_m) = \lim_{n\to\infty} S_b(x_n, x_n, x) = S_b(x, x, x)$$

(iv) *Define the diameter of a subset Y of X by*

$$diam(Y) := \sup\{S_b(x, y, z)|x, y, z \in X\}.$$

Sedghi et.al. [28] claimed that S-metric space is a generalization of a G-metric, that is, every G-metric is an S-metric. But Dung et.al. [8] explained with following examples that the assersion of Sedghi et.al [28] is not correct. Dung et.al. [8] further claimed that the class of all S-metric and the class of all G-metric are all distinct.

In definition 1.4 condition (ii) is not true in general. In order to make definition 1.4 a general one, we modify as follows.

Definition 1.8. *Let X be a nonempty set and let* $s \geq 1$ *be a given number. A function* $S_b : X^3 \to [0, \infty)$ *is said to be* S_b*-metric if and only if for all* $x, y, z, t \in X$*, the following conditions holds:*

(i) $S_b(x, y, z) = 0$ *if and only if* $x = y = z$,

(ii) $S_b(x, y, z) \leq s[S_b(x, x, t) + S_b(y, y, t) + S_b(z, z, t)]$

the pair (X, S_b) *is called an* S_b*-metric space.*

Definition 1.9. *A* S_b*-metric* S_b *is said to be symmetric if*

$$S_b(x, x, y) = S_b(y, y, x) \text{ for all } x, y \in X.$$

Now we recall one important definitions.

Definition 1.10. *Let X be a non-empty set and let* $f, g : X \times X \to X$ *be two mappings. A point* $(x, y) \in X \times X$ *is called coupled coincidence point of f and g if* $f(x, y) = g(x, y)$ *and* $f(y, x) = g(y, x)$*; then* $(g(x, y), g(y, x))$ *is called a coupled point of coincidence; a common coupled fixed point of f and g if* $f(x, y) = g(x, y) = x$ *and* $f(y, x) = g(y, x) = y$*, ([1], [2], [4], [5], [9],[11],[13]-[21], [23]-[25], [30], [31], [34]).*

2. Main Results

Now we prove our first common coupled fixed point result in the framework of complete symmetric S_b metric spaces:

Theorem 2.1. *Let* (X, S_b) *be a complete symmetric* S_b*-metric space with parameter* $s \geq 1$ *and let the mappings* $f, g : X^2 \to X$ *satisfying*

$$\begin{aligned} S_b(f(x, y), f(x, y), g(u, v)) \leq\ & a_1 \frac{S_b(x, x, u) + S_b(y, y, v)}{2} \\ & + a_2 \frac{S_b(f(x, y), f(x, y), g(u, v))S(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\ & + a_3 \frac{S_b(f(x, y), f(x, y), g(u, v))S_b(y, y, v)}{1 + S_b(x, x, u) + S_b(y, y, v)} \\ & + a_4 \frac{S_b(x, x, f(x, y))S_b(x, x, u)}{1 + S_b(x, x, u) + S_b(y, y, v)} \end{aligned}$$

$$+a_5\frac{S_b(x,x,f(x,y))S_b(y,y,v)}{1+S_b(x,x,u)+S_b(y,y,v)}$$
$$+a_6\frac{S_b(u,u,g(u,v))S_b(x,x,u)}{1+S_b(x,x,u)+S_b(y,y,v)}$$
$$+a_7\frac{S_b(u,u,g(u,v))S_b(y,y,v)}{1+S_b(x,x,u)+S_b(y,y,v)} \quad (1)$$

for all $x,y,u,v \in X$ and $a_1,a_2,...,a_7 \geq 0$ with $a_1+a_2+a_3+a_4+a_5+a_6+a_7 < 1$ and $s < \frac{1-a_2-a_3-a_6-a_7}{a_1+a_4+a_5}$. Then f and g have a unique common coupled fixed point in X.

Proof. Let $x_0, y_0 \in X$ be arbitrary points. Define

$$x_{2k+1} = f(x_{2k}, y_{2k}) \quad , \quad y_{2k+1} = f(y_{2k}, x_{2k})$$
$$x_{2k+2} = g(x_{2k+1}, y_{2k+1}) \quad , \quad y_{2k+2} = g(y_{2k+1}, x_{2k+1})$$

for $k = 0, 1, 2, , ...$ Then

$$\begin{aligned}
S_b(x_{2k+1},x_{2k+1},x_{2k+2}) &= S_b(f(x_{2k},y_{2k}),f(x_{2k},y_{2k}),g(x_{2k+1},y_{2k+1})) \\
&\leq a_1\frac{S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})}{2} \\
&\quad +a_2\frac{S_b(f(x_{2k},y_{2k}),f(x_{2k},y_{2k}),g(x_{2k+1},y_{2k+1}))S_b(x_{2k},x_{2k},x_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_3\frac{S_b(f(x_{2k},y_{2k}),f(x_{2k},y_{2k}),g(x_{2k+1},y_{2k+1}))S_b(y_{2k},y_{2k},y_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_4\frac{S_b(x_{2k},x_{2k},f(x_{2k},y_{2k}))S_b(x_{2k},x_{2k},x_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_5\frac{S_b(x_{2k},x_{2k},f(x_{2k},y_{2k}))S_b(y_{2k},y_{2k},y_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_6\frac{S_b(x_{2k+1},x_{2k+1},g(x_{2k+1},y_{2k+1}))S_b(x_{2k},x_{2k},x_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_7\frac{S_b(x_{2k+1},x_{2k+1},g(x_{2k+1},y_{2k+1}))S_b(y_{2k},y_{2k},y_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&= a_1\frac{S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})}{2} \\
&\quad +a_2\frac{S_b(x_{2k+1},x_{2k+1},x_{2k+2})S_b(x_{2k},x_{2k},x_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_3\frac{S_b(x_{2k+1},x_{2k+1},x_{2k+2})S_b(y_{2k},y_{2k},y_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_4\frac{S_b(x_{2k},x_{2k},x_{2k+1})S_b(x_{2k},x_{2k},x_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_5\frac{S_b(x_{2k},x_{2k},x_{2k+1})S_b(y_{2k},y_{2k},y_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_6\frac{S_b(x_{2k+1},x_{2k+1},x_{2k+2})S_b(x_{2k},x_{2k},x_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})} \\
&\quad +a_7\frac{S_b(x_{2k+1},x_{2k+1},x_{2k+2})S_b(y_{2k},y_{2k},y_{2k+1})}{1+S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})}
\end{aligned}$$

$$\leq a_1\frac{S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})}{2}+(a_2+a_3)S_b(x_{2k+1},x_{2k+1},x_{2k+2})$$
$$+(a_4+a_5)S_b(x_{2k},x_{2k},x_{2k+1})+(a_6+a_7)S_b(x_{2k+1},x_{2k+1},x_{2k+2})$$

So,

$$(1-a_2-a_3-a_6-a_7)S_b(x_{2k+1},x_{2k+1},x_{2k+2})\leq(\frac{a_1}{2}+a_4+a_5)S_b(x_{2k},x_{2k},x_{2k+1})+\frac{a_1}{2}S_b(y_{2k},y_{2k},y_{2k+1})$$

and therefore,

$$\begin{aligned}S_b(x_{2k+1},x_{2k+1},x_{2k+2}) &\leq \frac{\frac{a_1}{2}+a_4+a_5}{1-a_2-a_3-a_6-a_7}S_b(x_{2k},x_{2k},x_{2k+1})\\ &+ \frac{\frac{a_1}{2}}{1-a_2-a_3-a_6-a_7}S_b(y_{2k},y_{2k},y_{2k+1}).\end{aligned} \tag{2}$$

Proceeding similarly one can prove that

$$\begin{aligned}S_b(y_{2k+1},y_{2k+1},y_{2k+2}) &\leq \frac{\frac{a_1}{2}+a_4+a_5}{1-a_2-a_3-a_6-a_7}S_b(y_{2k},y_{2k},y_{2k+1})\\ &+\frac{\frac{a_1}{2}}{1-a_2-a_3-a_6-a_7}S_b(x_{2k},x_{2k},x_{2k+1})\end{aligned} \tag{3}$$

Adding (2) and (3) we have

$$\begin{aligned}S_b(x_{2k+1},x_{2k+1},x_{2k+2}) &+ S_b(y_{2k+1},y_{2k+1},y_{2k+2})\\ &\leq \frac{a_1+a_4+a_5}{1-a_2-a_3-a_6-a_7}[S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})]\end{aligned}$$

Therefore,

$$\begin{aligned}S_b(x_{2k+1},x_{2k+1},x_{2k+2}) &+ S_b(y_{2k+1},y_{2k+1},y_{2k+2})\\ &\leq h[S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})]\end{aligned}$$

where $h=\frac{a_1+a_4+a_5}{1-a_2-a_3-a_6-a_7}<1$.

Also, we can show that

$$\begin{aligned}S_b(x_{2k+2},x_{2k+2},x_{2k+3}) &+ S_b(y_{2k+2},y_{2k+2},y_{2k+3})\\ &\leq h[S_b(x_{2k+1},x_{2k+1},x_{2k+2})+S_b(y_{2k+1},y_{2k+1},y_{2k+2})]\\ &\leq h^2[S_b(x_{2k},x_{2k},x_{2k+1})+S_b(y_{2k},y_{2k},y_{2k+1})]\end{aligned}$$

Continuing this way, we have

$$\begin{aligned}S_b(x_n,x_n,x_{n+1}) &+ S_b(y_n,y_n,y_{n+1})\\ &\leq h[S_b(x_{n-1},x_{n-1},x_n)+S_b(y_{n-1},y_{n-1},y_n)]\\ &\leq h^2[S_b(x_{n-2},x_{n-2},x_{n-1})+S_b(y_{n-2},y_{n-2},y_{n-1})]\\ &\leq ...\leq h^n[S_b(x_0,x_0,x_1)+S_b(y_0,y_0,y_1)]\end{aligned}$$

If $S_b(x_n,x_n,x_{n+1})+S_b(y_n,y_n,y_{n+1})=S_{b_n}$, then

$$S_{b_n}\leq hS_{b_{n-1}}\leq h^2S_{b_{n-2}}\leq ...\leq h^nS_{b_0}.$$

So for $m>n$,

$$\begin{aligned}
S_b(x_n,x_n,x_m)+S_b(y_n,y_n,y_m) &\le s[2S_b(x_n,x_n,x_{n+1})+S_b(x_{n+1},x_{n+1},x_m)\\
&\quad +2S_b(y_n,y_n,y_{n+1})+S_b(y_{n+1},y_{n+1},y_m)]\\
&= 2s[S_b(x_n,x_n,x_{n+1})+S_b(y_n,y_n,y_{n+1})]\\
&\quad +s[S_b(x_{n+1},x_{n+1},x_m)+S_b(y_{n+1},y_{n+1},y_m)]\\
&\le 2s[S_b(x_n,x_n,x_{n+1})+S_b(y_n,y_n,y_{n+1})]\\
&\quad +s^2[2S_b(x_{n+1},x_{n+1},x_{n+2})+S_b(x_{n+2},x_{n+2},x_m)\\
&\quad +2S_b(y_{n+1},y_{n+1},y_{n+2})+S_b(y_{n+2},y_{n+2},y_m)\\
&= 2s[S_b(x_n,x_n,x_{n+1})+S_b(y_n,y_n,y_{n+1})]\\
&\quad +2s^2[S_b(x_{n+1},x_{n+1},x_{n+2})+S_b(y_{n+1},y_{n+1},y_{n+2})\\
&\quad +s^2[S_b(x_{n+2},x_{n+2},x_m)+S_b(y_{n+2},y_{n+2},y_m)]\\
&\le 2s[S_b(x_n,x_n,x_{n+1})+S_b(y_n,y_n,y_{n+1})]\\
&\quad +2s^2[S_b(x_{n+1},x_{n+1},x_{n+2})+S_b(y_{n+1},y_{n+1},y_{n+2})\\
&\quad +...+2s^{m-n-1}[S_b(x_{m-2},x_{m-2},x_{m-1})+S_b(y_{m-2},y_{m-2},y_{m-1})]\\
&\quad +s^{m-n}[S_b(x_{m-1},x_{m-1},x_m)+S_b(y_{m-1},y_{m-1},y_m)]\\
&\le 2s[S_b(x_n,x_n,x_{n+1})+S_b(y_n,y_n,y_{n+1})]\\
&\quad +2s^2[S_b(x_{n+1},x_{n+1},x_{n+2})+S_b(y_{n+1},y_{n+1},y_{n+2})\\
&\quad +2s^3[S_b(x_{n+2},x_{n+2},x_{n+3})+S_b(y_{n+2},y_{n+2},y_{n+3})]\\
&\quad +...+2s^{m-n-1}[S_b(x_{m-2},x_{m-2},x_{m-1})+S_b(y_{m-2},y_{m-2},y_{m-1})]\\
&\quad +2s^{m-n}[S_b(x_{m-1},x_{m-1},x_m)+S_b(y_{m-1},y_{m-1},y_m)]\\
&\le 2(sh^n+s^2h^{n+1}+s^3h^{n+2}+...+s^{m-n}h^{m-1})S_{b_0}\\
&< 2sh^n[1+sh+(sh)^2+...]S_{b_0}\\
&= \frac{2sh^n}{1-sh}S_{b_0}\to 0 \text{ as } n\to\infty
\end{aligned}$$

which shows that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X. As X is complete S_b-metric space, so there exists $x, y \in X$ such that $x_n \to x$ and $y_n \to y$ as $n \to \infty$.

Now we will prove that $x = f(x, y)$ and $y = f(y, x)$. On the contrary suppose that $x \neq f(x, y)$ or $y \neq f(y, x)$. Then $S_b(x, x, f(x, y)) = l_1 > 0$ or $S_b(y, y, f(y, x)) = l_2 > 0$.

Using inequality (1)

$$\begin{aligned}
l_1 &= S_b(x,x,f(x,y))\\
&\le s[2S_b(x,x,x_{n+1})+S_b(x_{n+1},x_{n+1},f(x,y))]\\
&= s[2S_b(x,x,x_{n+1})+S_b(f(x_n,y_n),f(x_n,y_n),f(x,y))]\\
&\le 2sS_b(x,x,x_{n+1})+s\Big[a_1\frac{S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}{2}+a_2\frac{S_b(f(x_n,y_n),f(x_n,y_n),f(x,y))S_b(x_n,x_n,x)}{1+S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}\\
&\quad +a_3\frac{S_b(f(x_n,y_n),f(x_n,y_n),f(x,y))S_b(y_n,y_n,y)}{1+S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}+a_4\frac{S_b(x_n,x_n,f(x_n,y_n))S_b(x_n,x_n,x)}{1+S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}\\
&\quad +a_5\frac{S_b(x_n,x_n,f(x_n,y_n))S_b(y_n,y_n,y)}{1+S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}+a_6\frac{S_b(x,x,f(x,y))S_b(x_n,x_n,x)}{1+S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}+a_7\frac{S_b(x,x,f(x,y))S_b(y_n,y_n,y)}{1+S_b(x_n,x_n,x)+S_b(y_n,y_n,y)}\Big].
\end{aligned}$$

Since $\{x_n\}$ and $\{y_n\}$ are convergent to x and y, therefore by taking limit as $n \to \infty$ we get $l_1 \le 0$, which is a contradiction, so $S_b(x, x, f(x, y) = 0)$ which gives $x = f(x, y)$.

Similarly, we can prove that $y = f(y, x)$. Also, we can prove that $x = g(x, y)$ and $y = g(y, x)$. Hence (x, y) is a common coupled fixed point of f and g.

In order to prove the uniqueness of the coupled fixed point, if possible let (p,q) be the second common coupled fixed point of f and g.

Then by using inequality (1), we have

$$\begin{aligned}
S_b(x,x,p) &= S_b(f(x,y),f(x,y),g(p,q))\\
&\leq \frac{a_1}{2}(S_b(x,x,p)+S_b(y,y,q))\\
&\quad + a_2\frac{S_b(f(x,y),f(x,y),g(p,q))S_b(x,x,p)}{1+S_b(x,x,p)+S_b(y,y,q)} + a_3\frac{S_b(f(x,y),f(x,y),g(p,q))S_b(y,y,q)}{1+S_b(x,x,p)+S_b(y,y,q)}\\
&\quad + a_4\frac{S_b(x,x,f(x,y))S_b(x,x,p)}{1+S_b(x,x,p)+S_b(y,y,q)} + a_5\frac{S_b(x,x,f(x,y))S_b(y,y,q)}{1+S_b(x,x,p)+S_b(y,y,q)}\\
&\quad + a_6\frac{S_b(p,p,g(p,q))S_b(x,x,p)}{1+S_b(x,x,p)+S_b(y,y,q)} + a_7\frac{S_b(p,p,g(p,q)))S_b(y,y,q)}{1+S_b(x,x,p)+S_b(y,y,q)}\\
&= \frac{a_1}{2}(S_b(x,x,p)+S_b(y,y,q)) + a_2\frac{S_b(x,x,p)S_b(x,x,p)}{1+S_b(x,x,p)+S_b(y,y,q)}\\
&\quad + a_3\frac{S_b(x,x,p)S_b(y,y,q)}{1+S_b(x,x,p)+S_b(y,y,q)} + a_4\frac{S_b(x,x,x)S_b(x,x,p)}{1+S_b(x,x,p)+S_b(y,y,q)}\\
&\quad + a_5\frac{S_b(x,x,x)S_b(y,y,q)}{1+S_b(x,x,p)+S_b(y,y,q)} + a_6\frac{S_b(p,p,p)S_b(x,x,p)}{1+S_b(x,x,p)+S_b(y,y,q)}\\
&\quad + a_7\frac{S_b(p,p,p)S_b(y,y,q)}{1+S_b(x,x,p)+S_b(y,y,q)}
\end{aligned}$$

Accordingly,

$$S_b(x,x,p) \leq \frac{a_1}{2}(S_b(x,x,p)+S_b(y,y,q)) + a_2S_b(x,x,p) + a_3S_b(x,x,p),$$

which implies

$$S_b(x,x,p) \leq \frac{a_1}{2-a_1-2a_2-2a_3}S_b(y,y,q). \tag{4}$$

Similarly,

$$S_b(y,y,q) \leq \frac{a_1}{2-a_1-2a_2-2a_3}S_b(x,x,p). \tag{5}$$

Adding (4) and (5) we have

$$S_b(x,x,p)+S_b(y,y,q) \leq \frac{a_1}{2-a_1-2a_2-2a_3}[S_b(x,x,p)+S_b(y,y,q)],$$

and we get

$$\frac{2(1-a_1-a_2-a_3)}{2-a_1-2a_2-a_3}[S_b(x,x,p)+S_b(y,y,q)] \leq 0.$$

Since $a_1+a_2+a_3<1$, $\frac{2(1-a_1-a_2-a_3)}{2-a_1-2a_2-2a_3} > 0$ we have that $S_b(x,x,p)+S_b(y,y,q)=0$, which implies that $x=p$ and $y=q$ i.e., $(x,y)=(p,q)$.

Thus f and g have unique coupled common fixed point. This completes the proof. □

Corollary 2.2. *Let (X, S_b) be a complete symmetric S_b-metric space with parameter $s \geq 1$ and let the mapping $f : X^2 \to X$ satisfying*

$$\begin{aligned}
&S_b(f(x,y), f(x,y), f(u,v)) \\
&\leq a_1 \frac{S_b(x,x,u) + S_b(y,y,v)}{2} \\
&+a_2 \frac{S_b(f(x,y), f(x,y), f(u,v))S_b(x,x,u)}{1 + S_b(x,x,u) + S_b(y,y,v)} + a_3 \frac{S_b(f(x,y), f(x,y), f(u,v))S_b(y,y,v)}{1 + S_b(x,x,u) + S_b(y,y,v)} \\
&+a_4 \frac{S_b(x,x,f(x,y))S_b(x,x,u)}{1 + S_b(x,x,u) + S_b(y,y,v)} + a_5 \frac{S_b(x,x,f(x,y))S_b(y,y,v)}{1 + S_b(x,x,u) + S_b(y,y,v)} \\
&+a_6 \frac{S_b(u,u,f(u,v))S_b(x,x,u)}{1 + S_b(x,x,u) + S_b(y,y,v)} + a_7 \frac{S_b(u,u,f(u,v))S_b(y,y,v)}{1 + S_b(x,x,u) + S_b(y,y,v)}
\end{aligned}$$

for all $x, y, u, v \in X$ and $a_1, a_2, ..., a_7 \geq 0$ with $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 < 1$ and $s < \frac{1-a_2-a_3-a_6-a_7}{a_1+a_4+a_5}$. Then f has a unique coupled fixed point in X.

Our second common coupled fixed point result is the following:

Theorem 2.3. *Let (X, S_b) be a complete symmetric S_b-metric space with parameter $s \geq 1$ and let the mappings $f, g : X^2 \to X$ satisfy*

$$\begin{aligned}
S_b(f(x,y), f(x,y), g(u,v)) \leq\ & \beta_1 \frac{S_b(x,x,u) + S_b(y,y,v)}{2} \\
&+\beta_2 \frac{S_b(x,x,f(x,y))S_b(u,u,g(u,v))}{1 + s[S_b(x,x,g(x,y) + S_b(u,u,f(u,v)) + S_b(x,x,u) + S_b(y,y,v))]}
\end{aligned} \tag{6}$$

for all $x, y, u, v \in X$ and β_1, β_2 are non-negative real numbers with $\beta_1 + \beta_2 < 1$ and $s < \frac{1-\beta_2}{\beta_1}$. Then f and g have unique common coupled fixed point.

Proof. Let $x_0.y_0$ be arbitrary points. Define

$$\begin{aligned}
x_{2k+1} &= f(x_{2k}, x_{2k}) \quad , \quad y_{2k+1} = f(y_{2k}, x_{2k}) \\
x_{2k+2} &= g(x_{2k+1}, y_{2k+1}) \quad , \quad y_{2k+2} = g(y_{2k+1}, x_{2k+1})
\end{aligned}$$

for $k = 0, 1, 2, ...$ Then

$$\begin{aligned}
&S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \\
&= S_b(f(x_{2k}, y_{2k}), f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1})) \\
&= \beta_1 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})}{2} \\
&\quad +\beta_2 \frac{S_b(x_{2k}, x_{2k}, x_{2k+1})S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})}{1 + s[S_b(x_{2k}, x_{2k}, x_{2k+2}) + S_b(x_{2k+1}, x_{2k+1}, x_{2k+1}) + S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]} \\
&\leq \frac{\beta_1}{2}(S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})) + \beta_2 S_b(x_{2k+1}, x_{2k+1}, x_{2k+2})
\end{aligned}$$

Accordingly we have,

$$S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) \leq \frac{\beta_1}{2(1-\beta_2)}[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})]. \tag{7}$$

Similarly we can show that

$$S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) \leq \frac{\beta_1}{2(1-\beta_2)}[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] \tag{8}$$

Adding (7) and (8) we have

$$\begin{aligned} S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) &+ S_b(y_{2k+1}, y_{2k+1}, y_{2k+2}) \\ &\le \frac{\beta_1}{1-\beta_2}[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] \\ &= k[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] \end{aligned}$$

where $k = \frac{\beta_1}{1-\beta_2}$.
Analogously we can show that

$$\begin{aligned} S_b(x_{2k+2}, x_{2k+2}, x_{2k+3}) &+ S_b(y_{2k+2}, y_{2k+2}, y_{2k+3}) \\ &\le k[S_b(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S_b(y_{2k+1}, y_{2k+1}, y_{2k+2})] \\ &\le k^2[S_b(x_{2k}, x_{2k}, x_{2k+1}) + S_b(y_{2k}, y_{2k}, y_{2k+1})] \end{aligned}$$

Now, if $S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1}) = S_{b_n}$ then

$$S_{b_n} \le kS_{b_{n-1}} \le k^2 S_{b_{n-2}} \le \dots \le k^n S_{b_0}$$

So, for $m > n$ we have

$$\begin{aligned} S_b(x_n, x_n, x_m) + S_b(y_n, y_n, y_m) &\le s[2S_b(x_n, x_n, x_{n+1}) + S_b(x_{n+1}, x_{n+1}, x_m) \\ &\quad +2S_b(y_n, y_n, y_{n+1}) + S_b(y_{n+1}, y_{n+1}, y_m)] \\ &= s[2S_b(x_n, x_n, x_{n+1}) + 2S_b(y_n, y_n, y_{n+1})] \\ &\quad +s[S_b(x_{n+1}, x_{n+1}, x_m) + S_b(y_{n+1}, y_{n+1}, y_m)] \\ &\le 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ &\quad +s^2[2S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(x_{n+2}, x_{n+2}, x_m) \\ &\quad +2S_b(y_{n+1}, y_{n+1}, y_{n+2}) + S_b(y_{n+2}, y_{n+2}, y_m) \\ &= 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ &\quad +2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2}) \\ &\quad +s[S_b(x_{n+2}, x_{n+2}, x_m) + S_b(y_{n+2}, y_{n+2}, y_m)] \\ &\le 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ &\quad +2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2}) \\ &\quad +\dots + 2s^{m-n-1}[S_b(x_{m-2}, x_{m-2}, x_{m-1}) + S_b(y_{m-2}, y_{m-2}, y_{m-1})] \\ &\quad +s^{m-n}[S_b(x_{m-1}, x_{m-1}, x_m) + S_b(y_{m-1}, y_{m-1}, y_m)] \\ &\le 2s[S_b(x_n, x_n, x_{n+1}) + S_b(y_n, y_n, y_{n+1})] \\ &\quad +2s^2[S_b(x_{n+1}, x_{n+1}, x_{n+2}) + S_b(y_{n+1}, y_{n+1}, y_{n+2}) \\ &\quad +\dots + 2s^{m-n-1}[S_b(x_{m-2}, x_{m-2}, x_{m-1}) + S_b(y_{m-2}, y_{m-2}, y_{m-1})] \\ &\quad +2s^{m-n}[S_b(x_{m-1}, x_{m-1}, x_m) + S_b(y_{m-1}, y_{m-1}, y_m)] \\ &\le 2(sk^n + s^2k^{n+1} + \dots + s^{m-n}k^{m-1})S_{b_0} \\ &< 2sk^n[1 + sk + (sk)^2 + \dots]S_{b_0} \\ &= \frac{2sk^n}{1-sk}S_{b_0} \to 0 \text{ as } n \to \infty \end{aligned}$$

Therefore $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X. Since X is complete S_b-metric space, there exist $x, y \in X$ such that $x_n \to x$ and $y_n \to y$ as $n \to \infty$.

Now, we will show that $x = f(x, y)$ and $y = f(y, x)$. Suppose on contrary that $x \neq f(x, y)$ or $y \neq f(y, x)$, so that $S_b(x, x, f(x, y)) = l_1 > 0$ or $S_b(y, y, f(y, x)) = l_2 > 0$. Consider the following and using inequality (6), we get

$$\begin{aligned}
l_1 &= S_b(x, x, f(x, y) \\
&\leq s[2S_b(x, x, x_{n+1}) + S_b(x_{n+1}, x_{n+1}, f(x, y))] \\
&= sS_b(x, x, x_{n+1}) + sS_b(f(x_n, y_n), f(x_n, y_n), f(x, y)) \\
&\leq sS_b(x, x, x_{n+1}) + s\Big[\beta_1 \frac{S_b(x_n, x_n, x) + S_b(y_n, y_n, y)}{2} \\
&\quad + \beta_2 \frac{S_b(x_n, x_n, f(x_n, y_n))S_b(x, x, f(x, y))}{1 + s[S_b(x_n, x_n, f(x, y)) + S_b(x, x, f(x_n, y_n)) + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)} \\
&= sS_b(x, x, x_{n+1}) + \frac{s\beta_1}{2}[S_b(x_n, x_n, x) + S_b(y_n, y_n, y)] \\
&\quad + \beta_2 \frac{S_b(x_n, x_n, x_{n+1})S_b(x, x, f(x, y))}{1 + s[S_b(x_n, x_n, f(x, y)) + S_b(x, x, x_{n+1}) + S_b(x_n, x_n, x) + S_b(y_n, y_n, y)}
\end{aligned}$$

Taking the limit as $n \to \infty$ we get

$$S_b(x, x, f(x, y)) \leq 0$$

Therefore

$$S_b(x, x, f(x, y)) = 0$$

which implies that $x = f(x, y)$. Similarly, we can prove that $y = f(y, x)$. Also, we can prove that $x = g(x, y)$ and $y = g(y, x)$. Hence, (x, y) is a common coupled fixed point of f and g.

In order to prove the uniqueness of the common coupled fixed point of f and g, if possible let (p, q) be the second common coupled fixed point of f and g.

Then by using inequality (6), we have

$$\begin{aligned}
S_b(x, x, p) &= S_b(f(x, y), f(x, y), g(p, q)) \\
&\leq \frac{\beta_1}{2}(S_b(x, x, p) + S_b(y, y, q)) \\
&\quad + \beta_2 \frac{S_b(x, x, f(x, y))S_b(p, p, g(p, q))}{1 + s[S_b(x, x, g(p, q)) + S_b(p, p, f(x, y)) + S_b(x, x, p) + S_b(y, y, q)]}
\end{aligned}$$

Therefore,

$$S_b(x, x, p) \leq \frac{\beta_1}{2}(S_b(x, x, p) + S_b(y, y, q)),$$

and

$$(1 - \frac{\beta_1}{2})S_b(x, x, p) \leq \frac{\beta_1}{2}S_b(y, y, q).$$

So,

$$S_b(x, x, p) \leq \frac{\beta_1}{2 - \beta_1}S_b(y, y, q) \tag{9}$$

Similarly,

$$S_b(y, y, q) \leq \frac{\beta_1}{2 - \beta_1}S_b(x, x, p) \tag{10}$$

Adding (9) and (10) we have

$$S_b(x, x, p) + S_b(y, y, q) \leq \frac{\beta_1}{2 - \beta_1}[S_b(x, x, p) + S_b(y, y, q)]$$

i.e.,

$$\frac{2(1-\beta_1)}{2-\beta_1}[S_b(x,x,p)+S_b(y,y,q)] \leq 0.$$

But $\frac{2(1-\beta_1)}{2-\beta_1} > 0$. Therefore $S_b(x,x,p)+S_b(y,y,q)=0$. Which implies that $x=p$ and $y=q$, i.e., $(x,y)=(p,q)$. Thus f and g have a unique common coupled fixed point. □

This completes the proof.

Corollary 2.4. *Let (X, S_b) be a complete symmetric S_b-metric space with parameter $s \geq 1$ and let the mapping $f : X^2 \to X$ satisfying*

$$\begin{aligned} S_b(f(x,y), f(x,y), f(u,v)) \quad &\leq \beta_1 \frac{S_b(x,x,u)+S_b(y,y,v)}{2} \\ &+\beta_2 \frac{S_b(x,x,f(x,y))(u,u,f(u,v))}{1+s[S_b(x,x,f(u,v)+S_b(u,u,f(x,y))+S_b(x,x,u)+S_b(y,y,v))]} \end{aligned}$$

for all $x,y,u,v \in X$ and β_1, β_2 are non-negative real numbers with $\beta_1+\beta_2 < 1$ and $s < \frac{1-\beta_2}{\beta_1}$. Then f has a unique coupled fixed point.

Example 2.5. *Let $X = \mathbb{R}$ be the set of real numbers,*

$$S_b(x,y,z) = |x-z| + |y-z| \text{ for all } x,y,z \in \mathbb{R}$$

where $s = 2 < 3$. Then (X, S_b) is an S_b-metric space. Let $f(x,y) = g(x,y) = \frac{2x-y+11}{12}$.

For the pair (f,g) as well as for all $x,y,u,v \in \mathbb{R}$ we have

$$\begin{aligned} S_b(f(x,y), f(x,y), g(u,v)) &= 2\left|\frac{2x-y+11}{12} - \frac{2u-v+11}{12}\right| \\ &= \frac{|x-u|}{3} + \frac{|y-v|}{6} \\ &\leq \frac{1}{3}(|x-u|+|y-v|) \\ &= \frac{1}{3}\Big(\frac{S(x,x,u)+S(y,y,v)}{2}\Big) \end{aligned}$$

In equation (1) *is satisfied with $a_1 = \frac{1}{3}$ and $a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 0$. Note that corollary 2.2 is also satisfied. Therefore the pair (f,g) has a unique coupled fixed point $(x,y) = (1,1)$.□*

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally in preparation of the final manuscript. All the authors read and approved the final manuscript.

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References

[1] M. Abbas, Ali Khan, S. Radenović: Common coupled fixed point theorems in cone metric spaces for w-compatible mappings, *Applied Mathematics and Computation,* **217** (2010) 195-202.
[2] A. Aghajani, M. Abbas, E.P. Kallehbasti: Coupled fixed point theorems in partially ordered metric spaces and application, *Math. Commun.*, **17** (2012) 497-509.
[3] I.A. Bakhtin: The contraction mapping principle in quasimetric spaces, *Functional Analysis,* **30** (1989) 26-37.
[4] R. Batra, S. Vashistha, R. Kumar: Coupled coincidence point theorems for mappings without mixed monotone property under c-distance in cone metric spaces, *J. Nonlinear Sci. Appl.,* **7** (2014) 345-358.
[5] T.G. Bhaskar, V. Lakshmikantham: Fixed Point Theory in partially ordered metric spaces and applications, *Nonlinear Analysis,* **65** (2006) 1379-1393.
[6] S. Czerwik: Nonlinear set-valued contraction mappings in b-metric spaces., *Atti Semin. Mat. Fis. Univ. Modena,* **46** (1998) 263-276.
[7] S. Czerwik: Contraction mapping in b-metric spaces, *Acta Math. Inform. Univ. Ostrav.,* **1** (1993) 5-11.
[8] N.V. Dung, N. T. Hieu and S. Radojević: Fixed point theorems for g-monotone maps on partially ordered S-metric spaces, *Filomat,* **28:9** (2014) 1885-1898.
[9] D. Guo, V. Lakshmikantham: Coupled Fixed Points of Nonlinear Operators with Applications, *Nonlinear Analysis:TMA,* **11** (1987) 623-632.
[10] A. Gupta: Cyclic contraction on S-metric space, *International Journal of Analysis and Applications*, **3**(2) (2013) 119-130.
[11] V. Gupta, R. Deep: Some coupled fixed point theorems in partially ordered S-metric spaces, *Miskloc Mathematical Notes,* **16**(1) (2015) 181-194.
[12] N. Hussain, V. Parvaneh, J.R. Roshan, Z. Kadelburg: Fixed points of cyclic weakly $(\phi;\psi;L;A;B)$contractive mappings in ordered b-metric spaces with applications, *Fixed Point Theory Appl.,* **2013**2013, Article ID 256.
[13] Z. Kadelburg, S. Radenović: Coupled Fixed point results under TVS-cone metric and w-cone-distance, *Advances in Fixed point Theory,* **2** (2012) 29-46.
[14] E. Karapinar: Coupled Fixed Point on Cone Metric Spaces, *Gazi University Journal of Science,* **1** (2011) 51-58.
[15] E. Karapinar, D. Turkoglu: Best Approximations Theorems for a couple in Cone Banach Space, *Fixed Point Theory and Applications,* **2010** doi: 10.1155/2010/784578.
[16] B. Khomdram, Y. Rohen, T.C. Singh: Coupled fixed point theorems in G_b-metric space satisfying some rational contractive conditions, *Springer Plus,* (2016) **5:1261**.
[17] V. Lakshmikantham and Lj. Ćirić, Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaes, *Nonlinear Anal. 70 (2009), 4341-4349*
[18] W. Long, B.E. Rhoades, M. Rajović: Coupled Coincidence points for two mappings in metric spaces and cone metric spaces, *Fixed Point Theory and Applications,* **2012** (2012) 1-9.
[19] N. Malhotra, B. Bansal: Some common coupled fixed point theorems for generalized contraction in b-metric space, *J. Nonlinear Sci. Appl.*, **8** (2015) 8-16.
[20] N. Malhotra, B. Bansal: Some common coupled fixed point theorems for generalised contraction in $b-$metric spaces, *J. Nonlinear Sci. Appl.*, **8** (2015) 8-16.
[21] J.G. Mehta, M.L. Joshi: On Coupled Fixed Point Theorem in Partially Ordered Complete Metric Space, *Int. J. Pure Appl. Sci. Technol.,* **1** (2010) 87-92.
[22] Z. Mustafa, J.R. Roshan, V. Parvaneh: Common fixed Point Theorems in Complex Valued Metric Spaces, *Numerical Functional Analysis and Optimization,* **32** (2011) 243-253.
[23] Z. Mustafa, J.R. Roshan, V. Parvaneh: Coupled Coincidence Point Results for $(\psi,\phi)-$Weakly Contarctive Mappings in Partially Ordered G_b-Metric Spaces, *Fixed Point Theory and Applications,* **206** (2013) 21 pages.
[24] Nguyen Van Dung: On coupled common fixed points for mixed weakly monotone maps in partially ordered S-metric spaces, *Fixed Point Theory Appl.,* (2013) **2013:48**.
[25] S. Sedghi, N. Shobkolaei, J.R. Roshan, W. Shantanawi: Coupled fixed point theorems in G_b-metric spaces, *Mathematički Vesnik,* **66**(2) (2014) 190-201.
[26] S. Sedghi, S, N. Shobe: A common unique random fixed point theorems in S-metric spaces, *Journal of Prime Research in Mathematics*, **7** (2011) 25-34.
[27] S. Sedghi, Nguyen Van Dung, Fixed point theorems on S-metric spaces, *Matematički Vesnik,* **66**(1) (2014) 113-124.
[28] S. Sedghi, N. Shobe, N, A. Aliouche: A generalization of fixed point theorems in S-metric spaces, *Matematički Vesnik,* **64**(3) (2012) 258-266.
[29] S. Sedghi, A. Gholidahneh, T. Došenović, J. Esfahani, S. Radenović: Common fixed point of four maps in S_b-metric spaces, *J. Linear Topol. Algebra,* **5** (2) 2016, 93–104.
[30] R.J. Shahkoohi, S.A. Kazemipour, A.R. Eyvali: Tripled coincidence point under $\phi-$contractions in ordered G_b-metric spaces, *Journal of Linear and Topological Algebra ,* **3** (2011) 131-147.
[31] W. Shantanawi: Some Common Coupled Fixed Point results in cone metric spaces, *Int. Journal of Math. Analysis,* **4** (2010) 2381-2388.
[32] T.C. Singh , Y.R. Singh: A comparative study of relationship among various type of spaces, *Int J Appl Math,* **25** (2015) 29-36.
[33] T.C. Singh, Y.R. Singh: Triple fixed points theorems on cone Banach space, *J. Glob res Math Arch,* **2** (2014) 43-49.
[34] A. Singh, N. Hooda: Coupled fixed point theorems in S-metric spaces, *International Journal of Mathematics and Statistics Invention,* **2**(4) (2014) 33-39.
[35] N. Souayah N. Mlaiki, A fixed point theorem in S_b-metric spaces, *J. Math. Computer Sci.,* **16** (2016) 131-139.