



LIFE AND WORK OF MIRKO STOJAKOVIĆ (1915-1985) On the centenary of his birth

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Abstract. This paper is devoted to mathematician Mirko Stojaković, member of Serbian Academy of Sciences and Arts on the occasion of 100 years since his birth, but also 30 years since his death. The life path and the main scientific contribution are depicted.

1. Life Path

Mirko Stojaković, mathematician, university professor and full member of SASA, was born on 21 August, 1915 according to the old calendar (3 September according to the new one) in Krepoljin, eastern Serbia. By the age of four he had already lost both his parents and, as a war orphan, he was placed in an orphanage in Požarevac together with his three years older brother Ivan. He was among the younger children at the orphanage and had to go to school with the older children. So, by sheer coincidence and not personal choice, he learned to read and write very early in life, and books and studying became his escape. He finished first grade while he was still at the orphanage and then continued his schooling and growing up with a relative who lived in Krepoljin. After finishing primary school, he moved to Kragujevac to live with his older brother Predrag and this was where he finished lower Gymnasium. He completed his gymnasium education at the Second Belgrade Gymnasium with excellent grades. After being exempt from taking the oral exam, he graduated in June of 1934. By his own admission, during his senior years at the Gymnasium, he was not the best nor the most conscientious student, he was quite restless and did not especially like or study mathematics. This came later. As he became older and more mature, he began to study diligently, to read and draw, participate in sports. Besides mathematics, he was quite fond of history, literature and he showed an interest in geology and ethnology.

In the year 1934, he enrolled in the group for mathematics at the Faculty of Philosophy where he came into contact with eminent older professors, like Mihailo Petrović, Milutin Milanković, Nikola Saltikov and Anton Bilimović, as well as younger mathematicians, like Tadija Pejović, Jovan Karamata, Miloš Radojčić and physicist Vjačeslav Žardecki. They were all people one should aspire to emulate and their influence remained forever embedded in Mirko's future life and work. He graduated in 1938. During the fourth year of his studies he wrote an exceptional paper entitled *Determinante beskonačnog reda i njihova*

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primena na rešavanje jednačina sa beskonačno mnogo promenljivih (Determinants of Infinite Order Used to Solve Equations with an Infinite Number of Variables), which, based on the evaluation and report of our reputed mathematician, Dr. Jovan Karamata, brought him an award at a traditional Saint Sava Prize of the Faculty of Philosophy competition for best thesis in mathematics. It is a less known fact that he was very partial to engineering and that just before World War II he also enrolled at the Faculty of Technical Sciences, only such aspirations were interrupted by the War. After earning a degree in mathematics, he soon started his first teaching job at the Real-Gymnasium in Subotica and remained there until the outbreak of April War in 1941.

After the collapse of the Royal Army, Mirko Stojaković is assigned to a trainee teacher position at the šabac Real-Gymnasium and in September of 1941, he was also allocated to teach, first "without pay" and after September of 1943 permanently at the Fifth Belgrade Gymnasium. In spite of his impaired health after tuberculosis, in August of 1944 he joins the Partisans and as a highly educated civilian and military officer he manages the economics sector of the Valjevo military district until August of 1945 while also teaching mathematical finance at the restored Trade Academy in Valjevo. He received the Order of Merits for the People (III rank) for his military service. After being demobilized, he returns to his teaching position at the Fifth Belgrade Gymnasium and in 1947 transfers to the Technical Faculty in Belgrade to take an assistant professor position. When the Technical Faculty was separated from the University to form the Technical Higher School, later Faculty of Mechanical Engineering in Belgrade, he was appointed professor of mathematics.

In the year 1953, he defended his dissertation, entitled *A Contribution to the Matrix Theory*, at the University of Zagreb. Professor Željko Marković was the president of the examination committee, Djuro Kurepa the rapporteur to the committee and the committee members were Danilo Blanuša, Stanko Bilinski and Vladimir Vranić. The ceremony was held on 30 December, 1953. That same year he was made docent of mathematics at the Faculty of Mechanical Engineering in Belgrade.

The following year, in 1954, Mirko Stojaković transfers to Novi Sad, at the newly-formed Faculty of Philosophy where he spent the greater part of his professional life. He was appointed Associate Professor of all Algebra courses. During the first session of the Faculty Senate, held in October of 1954, Mirko Stojaković was also appointed Head of the Mathematics Department with younger mathematician Bogoljub Stanković as his assistant. The two of them, together with Mileva Prvanović, can be considered pioneers of the development of Mathematics in Novi Sad - Stojaković of the Novi Sad school of Algebra and Mathematical Logic, Stanković of the Novi Sad school of Mathematical Analysis and Mileva Prvanović of Geometry. Mirko Stojaković remained Head of the Mathematics Department even after the Faculty of Natural Sciences and Mathematics was founded in Novi Sad in 1969, until 1976 when he became the first Director of the newly founded Institute of Mathematics at the Faculty of Natural Sciences and Mathematics. He held this position until he went into retirement in 1978, at which time he accepted the position of Director of the Mathematical Institute of the Serbian Academy of Sciences and Arts (SASA).

Stojaković's numerous activities were not limited only to the Department; he also successfully contributed to the development of the Faculty of Philosophy in Novi Sad. In recognition of his overall work he was appointed Dean of the Faculty of Philosophy for three consecutive mandates: 1962-64, 1965-66 and 1966-68. In addition to the Department of Mathematics, during this period the Department of Physics, Chemistry, Biology and Geography had also been formed at the Faculty of Philosophy, and Mirko Stojaković directed his energy towards making them independent and forming the Faculty of Natural Sciences and Mathematics (1969). He was appointed the first Dean of this Faculty.

A particularly striking characteristic of Professor Mirko Stojaković's efforts to modernize teaching and introduce applied mathematics and informatics to the curriculum of the Department of Mathematics is the visionary aspect. At a time when computer science was still in its infancy subjects like elementary computer engineering and informatics had already been added to the curriculum of the Department. Due to his efforts, one of the first computer centers in Vojvodina was formed at the Department of Natural Sciences and Mathematics. Later, his endeavor was crowned with a separate Department of Informatics at the Mathematical Institute. It is interesting to say that in addition to participating at numerous conferences and various mathematical conventions across the world, in 1951 he took part in the First International

Congress on Cybernetics in Paris and the Conference on Cybernetics in Nice (1971).



Mirko Stojaković, Tatomir Andjelić, Mileva Prvanović at PMF Novi Sad, 1975

When on 5 December, 1963 he was elected corresponding member to the SASA he delivered an inaugural address entitled *A Review of Several Methodological Problems in Algebra*. Later, on 7 May 1981, he was elected regular member based on the report signed by academics Radivoje Kašanin, Miodrag Tomić and Slobodan Aljančić and on 23 March, 1982 his inaugural address entitled *Failed Attempts at Reducing Mathematics to Simple Patterns* was delivered by Academic Miodrag Tomić because Stojaković was in an accident due to which he lost his left leg while his right leg was paralyzed, rendering him an invalid.

He was member of the French Mathematical Society, the Mathematical Association of America, the American Mathematical Society and the New York Academy of Sciences. In addition to the already mentioned Order of Merits for the People (III rank), due to his contribution to education, science and society in general, he also received the Novi Sad October Prize (1961) and was awarded Orders of Labor with Golden and Silver Wreaths (1965).

Mirko Stojaković died on 5 July, 1985.

2. Scientific Work

The scientific work of Mirko Stojaković is original, comprehensive and multifaceted. His research incorporated several fields of study which gave his works certain broadness. These works influenced other mathematicians and have remained noted in both monographic and scholarly literature. His results were all the more significant considering he had no predecessors in some of the areas of mathematics that he studied. His areas of study include algebra, automata theory, graph theory, formal language theory, cryptography, model theory and algorithmic theory, mathematical logic, difference and differential equations, though the

greater part of his work deals with algebra. It should also be noted that some of the areas studied by Mirko Stojaković automata theory, formal language theory, model theory and algorithmic theory became subjects of scientific research for the first time in our country, paving the way for new knowledge. And it is due to this sort of research that today we have numerous mathematicians who study these areas of mathematics. He also wrote numerous articles on the subject of mathematics in which he presented his views on certain topics from various areas of mathematics, including the history of mathematics, and commented on current discoveries and activities in mathematics in an interesting and humorous fashion.

Mirko Stojaković published his first article as early as 1939 in *Glasnik*, the bulletin of the Yugoslav Association of Professors, under the title *Jedan lep primer za primenu računa kongruencije* (A Nice Example of Applying Congruence Arithmetic). The onset of the War brought a halt to scientific research and consequently his scientific work took place in the period after liberation. During this period he was drawn to the subject of generalizing the concept of determinants and matrices – determinants of infinite order, determinants of rectangular matrices and the inverse of singular matrices. In a series of scientific papers, he set the foundation for a theory which later underwent rapid development with interesting results, multifarious applications and numerous scientific studies and monographs.

The first attempt to obtain a kind of inverse for noninvertible matrix was in the work of American mathematician E. H. Moore who in 1920 gave a useful extension of the classic notion of inversion for regular matrices. In 1935 this problem was mentioned in Moor's monograph *General Analysis* published posthumously by American Philosophical Society. However, for a long time after this there was no analysis of this problem. Even before Moor's article it had been accepted that solvable systems of equations, although not uniquely solvable, should not be discarded. The problem was to find a way to approach non-uniquely solvable singular systems using the technique applied to regular equations. In order to solve this, it was necessary to find the relations that could be applied to both singular equations and regular equations.

Mirko Stojaković accomplished this in his papers published in 1953 and in a number of subsequent papers he obtained significant results which have their application in solving various problems. He solved the problem of finding inverses of rectangular matrices by using determinants and adjugate matrices. In a series of papers Stojaković introduced the notion of quasi inverse matrices and gave a series of relations connected to them including the relations $ABA = A$ and $BAB = B$, where matrix B is called "quasi-inverse matrix for A ", and matrix BA , denoted in the paper with E , has the role of identity matrix and is called "quasi identity matrix". Similar results were published a few years afterwards by R. Penrose who used the term "pseudo-inverse matrix".

Many authors later cited Penrose's paper and these matrices were named after him despite the fact that Mirko Stojaković has undoubt identity (unit) matrix identity (unit) matrix identity (unit) matrix identity (unit) matrix ed priority. A group of former Yugoslav mathematicians (M. Radić, P. Stanimirović, M. Stanković and S. Prešić) cited Stojaković's results in their papers, which shows that he defined and studied the pseudo-inverse of matrices before Penrose; however, in international mathematical circles the only name mentioned is that of Penrose. The only exception is A. Dragomir, a Romanian mathematician who looked into the issue of priority and established that M. Stojaković was the first to introduce the observed type of matrices. A group of Romanian mathematicians, familiar with the work of Mirko Stojaković, continued to study generalized inverses. B. Krstić, Professor at the Polytechnic University of Timisoara, wrote: " Even though I am loyal to my country one hundred percent, it still somehow pleases me when the work of a colleague from Yugoslavia meets with a good response abroad. By virtue of Professor Arghiriade, you have become a real hero at the Faculty of Mechanics and Mathematics of this University due to your papers on determinants to rectangular matrices and generalized inverses."

Generalized Inverse of Matrices and Its Applications, John Wiley & Sons, New York, 1971, by C. R. Rao and S. K. Mitra, is one of the most famous monographs devoted to generalized inverses. One of the authors of this monograph, S. K. Mitra, later became familiar with Stojaković's studies and in one of his letters wrote: "I note that some statements on the historical development of the concept of generalized inverses we make in our book needs revision in view of earlier work of M. Stojaković in this area which we were unaware of."

Let us remark that work of Mirko Stojaković has a great influence at the Faculty of Sciences and Mathematics from University of Niš. Namely, in the beginning of 80's, mathematician and professor Stojan

Bogdanović came in Niš. With his colleagues, professors Vladimir Rakočević and Miomir Stanković, he organized a seminar that had a great influence for the scientific development at the University of Niš. The main interests of the members of the seminar were the research on generalised inverses. Later this unique seminar splited in two different seminars: the first one were mainly on algebraic researches (the leader was S. Bogdanović, later professor Miroslav Ćirić) and the second one was called Generalized Inverses and Applications (the leaders were V. Rakočević and M. Stanković). Many, today well known mathematicians and professors at the University of Niš, worked or still works in the fields of theory of Generalised inverses and its applications (Dragan Djordjević, Dragana Cvetković Ilić, Predrag Stanimirović, Matko Petković, Petar Protić, Dijana Mojsić, Nebojša Dinić, Jovana Nikolov, Milica Kolundžija, Dragan Rakić and others), and many PhD theses in this field has been done. Also, many papers have been published in very important international journals and a monograph *Lectures on Generalized Inverses* by D. S. Djordjević and V. Rakočević was published at Faculty of Sciences and Mathematics in Niš in 2008.

Of particular interest and importance is the episode related to Stojaković's work on the theory of graphs. We should mention first the monograph *The Four Colour Problem* written by Stojaković and Hungarian-English mathematician Gabriel Andrew Dirac which presents the history and results of the attempts to solve the famous problem of colouring a map in a plane using at most four colours. In his paper *On the decidability of four colour problem*, Stojaković presented a series of reasons why the four colour theorem can be considered decidable, and a method for proving it was suggested. Stojaković himself proposed an algorithm, though it was incomplete since it did not include the so-called periodic case. It is interesting to note that this standpoint on the decidability of the four colour problem expressed in Stojaković's and Dirac's book was deemed wrong by B. Grnbaum, the reviewer from "Mathematical Reviews", but the paper by K. Appel and W. Haken confirmed that Stojaković and Dirac were right.

A. B. Kempe found that the four colour theorem will be correct if it can be proved that every definite 4-coloured planar graph which contains a pentagonal chain inside with other vertices can be four-coloured so that the vertices of the limiting pentagonal chain have at most three different colours in between. His analysis was not complete, so P. J. Heawood and A. Errera constructed counter examples for his claims. Stojaković developed Kempe's model further, although he stated explicitly that the general problem was not solved. In his papers *Abriss eines "Beweises" des vierfarben Satzes and ber die Konstruktion der ebenen Graphen*, Stojaković introduced the technique of mutual replacement of colour pairs, which goes further than Kempe's method and is powerful enough to eliminate the counter-examples of Heawood and Errera, and he also gave a very short and elegant proof of the so-called Wagner and Fáry theorem which claims that any planar graph without multiple branches can always be represented in a plane with branches as straight lines.

We would especially like to point out Professor Stojaković's inclination for the history of mathematics. He wrote a series of papers on the scientific work of Mihailo Petrović. This greatly influenced Prof. Dr. Dragan Trifunović (1930-2012), our first historian of mathematics and science, to choose Petrović's life and work as the topic of his PhD, and to write and defend his dissertation at the University of Novi Sad, with Professor Stojaković as his mentor.

Professor Mirko Stojaković was a charismatic figure, an institution in Novi Sad and Vojvodina mathematics. Even people outside the world of mathematics knew about him and his activities. He was a scientist with his own personal style, often in jeans and jump suits, although quite elegant when the circumstances called for it. The mention of his name evokes memories among many a citizen of Novi Sad of a striking figure and an intelligent, witty and intriguing personality, as well as his lasting successful career in Novi Sad and at the Novi Sad University. Still, in his many interviews regarding the numerous acknowledgments he received during his lifetime, he would always say that there are many other deserving mathematicians and that he is not clear as to why he is the one to receive them and that some mathematicians perhaps overestimate his work. He would often point out that these acknowledgments are not only on behalf of his work but also the undisputed contribution of the Mathematics Department where he was Head.

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