Filomat 31:2 (2017), 513–522
DOI 10.2298/FIL1702513D

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Fuzzy Covering Location Problems with Different Aggregation Operators

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Abstract. Covering location problems is well-known and very important class of combinatorial optimization problems. Standard models for covering location problems cannot encompass real-life problems, because real-life problems contain some degree of uncertainty. The use of fuzzy sets in modeling covering location problems allows the implementation of these conditions. Depending on the type of problems, it is necessary to use different aggregation operators in calculating solution's quality. The aim of this study is introducing of fuzzy sets with different corresponding conorms in modeling most known types of covering location problems.

1. Introduction

Covering location problems represents a class of well-known problems of combinatorial optimization. The main task of covering location problem is finding optimal places for facilities in given set of locations, depending on the coverage conditions. The concept of coverage problem was introduced by Church, Toregas and ReVelle in [1], [2] and [3]. It is applicable in various branches of science and technology, especially in transportation and location sciences. Mathematical models and algorithms for solving covering location problems help planners to find optimal positions for different facilities - from emergency services till dumps. Depending on the conditions, they use various types of covering location problems - they differ in numbers of facilities (fixed or unlimited), types of coverage (maximal or minimal) and the problem space (network, hub, etc.). In the real-life problems, conditions could not be described well in binary space since the variables are usually more complex. For example, radius of coverage or travel time could be "around 10 kilometers" or "between 8 and 12 minutes" and modeling in classical sense could not describes problem as well. In line with this, researchers have made many improvements of covering location models by introducing uncertainties and probabilities.

Many authors have done some research in this area and illustrative approaches could be found in [4], [5], [8], [9], [10] and [14]. Detailed review of previous results for each type of covering location problem will be presented in the section 3. In previous researches, the authors have studied fuzzification of a single

2010 *Mathematics Subject Classification*. Primary 93A30
Keywords. Covering location problems, fuzzy sets, fuzzy modeling
Received: 10 October 2014; Accepted: 11 July 2015
Communicated by Predrag Stanimirović
Research supported by Ministry of Science and Technological Development of Republic of Serbia, project 174009
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condition, usually radius of coverage. Using fuzzy numbers for modeling of coverage radius is very clear, locations close to the border of coverage should have some impact to the solution. The second imprecision in classical models are modeling a distance between location. Distances between locations are fixed values, but in real-life problems travel times between locations are not the same for equal distances. Travel times depends on many external factors, like traffic jam, terrain, weather etc.

The main aim of this paper is the introduction of simultaneous fuzzyfication of two mentioned variabilities - radius of coverage and travel time between locations with the application of different aggregation operators (limited sum, maximum and ordered weighted average). New approaches will be illustrated on the most known types of covering location problems, with detailed description of practical background for using different models.

2. Definitions and Preliminaries

Fuzzy Set Theory was introduced by L. Zadeh in [12]. A good overview of fuzzy sets and operations with them is given in [11]. The basic types of fuzzy sets are triangular, trapezoidal and a left(right) shoulder. Generally these three types of fuzzy sets are called fuzzy numbers. Triangular fuzzy sets or fuzzy numbers are used to model approximate values for example 5±1. Similarly a trapezoidal fuzzy number approximates an interval and finally left and right shoulder fuzzy sets are used to model sets bounded only from one end for example "tall or short people".

Definition 2.1. *A fuzzy number on the set* $\mathbb{R}$ *is a mapping* $u : \mathbb{R} \to [0, 1]$ *such that:*

- *u is continuous monotone non-decreasing on* $(-\infty, b)$*,*
- *u is continuous monotone non-increasing on* (c, ∞)*,* $c \geq b$*,*
- $u(x) = 1, x \in [b, c]$.

In the case that $b = -\infty$ a right-shoulder fuzzy number is obtained. Similarly if $c = \infty$ a left-shoulder fuzzy number is obtained.

If the function is linear and strictly increasing on (a, b) and linear strictly decreasing on (c, d) a trapezoidal fuzzy number is obtained. Moreover of $b = c$ then a triangular fuzzy number is obtained.

In this study, variabilities are considered to be fuzzy numbers. Right shoulder fuzzy number represents a radius of coverage ($S + r$, where S is radius of full coverage and r radius of partial coverage) and triangular fuzzy number is used to represent the travel time between locations. Degrees of location's coverage is defined as an intersection of these fuzzy numbers, illustrated on figure 2, given by variable: c_{ij} - degree of coverage of location i by facility located in site j, given by formula

$$c_{ij} = \begin{cases} 1, t_{ij} \leq S \\ 0, t_{ij} > S + r \\ \dfrac{1}{2}(\mu(v_1) + \mu(v_2)), \text{otherwise.} \end{cases} \tag{1}$$

where $\mu(v_i)$ represents intersection of fuzzy right shoulder fuzzy number for coverage radius and triangular fuzzy number for travel time.

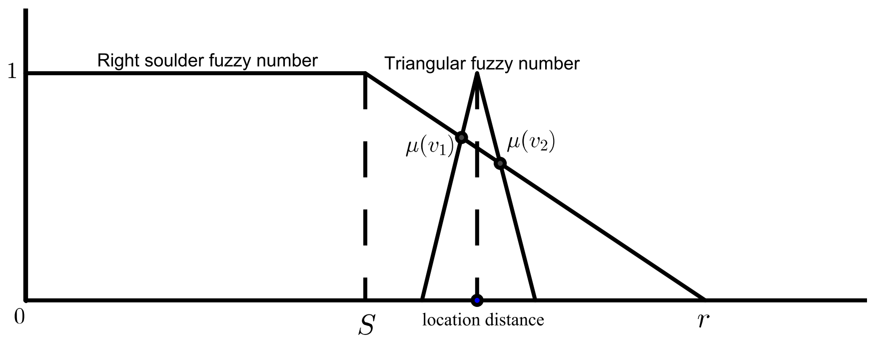

Figure 1: Intersection of fuzzy numbers that represent a radius and a travel time between locations

The conditions in real-life problems (e.g. locating emergency service) are the reasons why we chose these types of fuzzy numbers. The radius of coverage could be separated into three areas: the area of full coverage (value 1) where the emergency service has full effect and it decreases while the radius increases (from 1 to 0), until it exceed a limit where there is no effect (value 0). The right shoulder fuzzy number best describes this behaviour. A triangle fuzzy number is the most appropriate for representing travel times between locations due to it is always in some interval ($t_{ij} \pm t$). The arithmetic mean of intersection points is used to determine the degree of partial coverage of a location, but some other classes of aggregation operators could be also used, like OWA operators.

Let us now give the definition of aggregation operators.

Definition 2.2. *A is an aggregation operator on the unit interval* $A : \cup_{n\in\mathcal{N}}[0,1]^n \to [0,1]$ *if the following conditions hold:*

(A1) $A(0,\ldots,0) = 0$,

(A2) $A(1,\ldots,1) = 1$,

(A3) *if* $(\forall i = 1,2,\ldots,n)(x_i \le y_i)$ *then* $A(x_1,x_2,\ldots,x_n) \le A(y_1,y_2,\ldots,y_n)$.

Conditions **(A1)** and **(A2)** are called boundary conditions, and **(A3)** resembles the monotonicity property of the operator A. Let us now briefly define the notion of a fuzzy set. One of the most common aggregation operators are t-norms and t-conorms.

Definition 2.3. *A mapping* $T : [0,1]^2 \to [0,1]$ *is called a t-norm if the following conditions are satisfied for all* $x,y,z \in [0,1]$*(see [11]):*

(T1) $T(x,y) = T(y,x)$

(T2) $T(x,T(y,z)) = T(T(x,y),z)$

(T3) if $y \le z$ *then* $T(x,y) \le T(x,z)$

(T4) $T(x,1) = x$.

One of the most common t-norms is the $T_M(x,y) = \min(x,y)$. Besides T_M, we have $T_P(x,y) = xy$ and $T_L(x,y) = \max(x+y-1,0)$. In order to generalize disjunction, t-conorms or s-norms are used.

Definition 2.4. *A mapping* $S : [0,1]^2 \to [0,1]$ *is called an s-norm or a t-conorm if the following conditions (T1), (T2), (T3) from the previous definition and the condition (S4) are satisfied for all* $x,y,z \in [0,1]$.

(S4) $S(x,0) = x$.

These operators are dual i.e. for each t-norm T there exists a t-conorm such that

$$T(x,y) = 1 - S(1-x,1-y).$$

Thus, the dual t-conorm for T_M is $S_M(x, y) = max(x, y)$. Similarly, for T_P the dual is the probabilistic sum $S_P(x, y) = x + y - xy$. Finally, the dual for T_L is the limited sum

$$S_L(x, y) = min(x + y, 1).$$

which we will use in this paper.

Since t-norms and t-conorms are associative they are extendable to n-ary operators. For each t-conorm $S : [0, 1]^2 \to [0, 1]$ its n-ary extension is obtained for each $n \in \mathcal{N}$:

$$S_n(x_1, \dots, x_n) = \begin{cases} x_1, & \text{if } n = 1, \\ S(x_1, x_2), & \text{if } n = 2, \\ S(S_{n-1}(x_1, \dots, x_{n-1}), x_n). \end{cases}$$

Definition 2.5. *An ordered weighted sum (OWS) operator is a mapping $F : [0, 1]^n \to [0, 1]$ that has an associated collection of weights $W = \{w_1, \dots, w_n\}$ from the unit interval with properties $w_1 = 1$ and $w_k \leq w_{k-1}$ for $k = 2, 3, ..., n$, given with:*

$$F_{w_1,w_2,...,w_n}(a_1, \dots, a_n) = \min(1, \sum_{j=1}^{n} w_j b_j),$$

where b_j is the j-th largest of the a_i.

3. Fuzzy Location Covering Models with Different Aggregation Operators

As mentioned before, the main goal of this paper is the introduction of fuzzy sets with different aggregation operators in modelling of location problems, with the aim of describing of uncertainties in real-life problems in a better way. In this chapter, we present some new methods for modeling three well-known location problems - Location set covering problem (LSCP), Maximal covering location problem (MCLP) and Minimal covering location problem (MinCLP).

For all three models, following definitions will be used:

$$I \text{ - set of locations (indexed by } i), \tag{2}$$

$$J \text{ - set of eligible facility sites (indexed by } j), \tag{3}$$

$$x_j = \begin{cases} 1, \text{if facility is located at location } j, \\ 0, \text{otherwise}, \end{cases} \tag{4}$$

$$t_{ij} \text{ - travel time from location i to location } j, \tag{5}$$

$$S \text{ - radius of coverage}, \tag{6}$$

$$P \text{ - number of facilities}. \tag{7}$$

3.1. Location set covering problem (LSCP)

Location set covering problem (LSCP) represents one very important class of problems in location science. LSCP sought minimal number of facilities in such way that every location has been covered with at least one facility within given distance or travel time. In the context of medical services, every patient represents a location and each hospital represents a facility. Together, patients and hospitals represent vertices on the network with edges represented by paths between them. Many applications of the LSCP are presented in [15].

Mathematical model for LSCP, introduced in [3], with definitions (2) - (7) is:

$$P = \text{minimize} \sum_{j\in J} x_j,$$
$$\sum_{j\in N_i} x_j \geq 1, \forall i \in I,$$
$$x_j \in \{0,1\}, \forall j \in J,$$
where:
$N_i = \{j|t_{ij} \leq S\}$ - set of all facilities j which cover location i.

This original model is also known as deterministic LSCP and it does not take into consideration the possibilities that a server facility may be busy at the time the call arrives or the possibilities for a traffic jam. It also does not consider possibilities for using partial coverage. Many authors have been doing some research on the introduction of uncertainty and probability in LSCP models and brief a overview follows.

Daskin in [16] formulates the Maximum Expected Covering Location Problem (MEXCLP), which seeks to maximize the expected value of the coverage within a time standard. Daskin assumed a single system-wide busy fraction in his formulation, and Bianchi and Church modified his model to consider the location of depots as well [17]. Probabilistic Location Set Covering Problem (PLSCP) was introduced by ReVelle and Hogan and it utilizes a region-specific busy fraction, instead of a system-wide busy fraction [18] and [19] and it was a base for Marianov and ReVelle work on a probabilistic model for LSCP, where each location has a constraint for a server availability [20]. Apart from using probability in modeling of LSCP, some authors presented models with variable radius of coverage. Berman at all. in [21] presented a covering problem where the covering radius of a facility is controlled by the decision-maker. Davari et. all [22] and Zarandi at. all [23] presented covering problem with variable radius, where travel times between locations are considered to be fuzzy variables. A detailed review of various models of LSCP and its applications could be found [21], [6] and [7].

In existing models, authors presented various methods for modeling variable coverage radii or variable travel time between locations. The goal of the following model is simultaneous variabilities of both conditions - coverage radius and travel times between locations. Rationality for coverage radius is very clear - locations which are very close to the coverage border affect the improvement of quality of solution and they must be taken into consideration On the other hand, traffic jam (in transportation problems) or different air flows (in pollution problems) also affect to the quality of solution. As mentioned before, fuzzy numbers will be used for both variabilities.

A method for calculating the degree of location coverage is given by formulae (1). The next question in how to calculate degrees for locations that are partially covered by more facilities. This decision depends on the characteristics of the problem. Because LSCP requires that each location must be fully covered, in this model just a limited sum conorm is allowed. More precisely, a limited sum conorm provides that the degree of coverage for each location must be at least 1.

Before formulating of a new mathematical model for fuzzy LSCP (FLCSP), it is necessary to redefine preliminary variables:

I - set of locations (indexed by i), $n = \text{card}I$, (8)

J - set of eligible facility sites (indexed by j,) (9)

$$x_j = \begin{cases} 1, \text{if facility is located at location } j, \\ 0, \text{otherwise}, \end{cases} \tag{10}$$

$t_{ij} \pm t$ - fuzzy travel time from location i to location j, (11)

$S + r$ - fuzzy radius of coverage, (12)

$$c_{ij} = \begin{cases} 1, t_{ij} \leq S, \\ 0, t_{ij} > S + s, \\ \frac{1}{2}(\mu(v_1) + \mu(v_2)), \text{otherwise}. \end{cases} \tag{13}$$

as introduced in (1).

The new model of Fuzzy LCSP with the limited sum conorm is as follows:

$$\text{minimize} \sum_{j \in J} x_j,$$

subject to:

$$S_L(x_{i_1} \cdot c_{i_1 j}, x_{i_2} \cdot c_{i_2 j}, \ldots, x_{i_n} \cdot c_{i_n j}) = 1, \forall i_k \in I, k \in \{1, 2, \ldots, n\},$$
$$x_j \in \{0, 1\}, \forall j \in J.$$

3.2. Maximal covering location problem (MCLP)

The Maximal Covering Location Problem (MCLP) was originally introduced by Church and ReVelle in 1974. [1]. The aim of MLCP is finding best positions for the given number of facilities, in which as more as possible locations are covered by them. MCLP could be used in many real-life problems, as finding the best positions for the shops, petrol stations, emergency services, heat sources, light sources etc. The original MCLP is defined on a network and since then a lot of extensions of MCLP has been presented.

Hogan and ReVelle [24] introduced a backup coverage as a decision criterion for location problems on a network. Moore and ReVelle [25] introduced hierarchical model for MCLP, Berman and Krass [26] introduced a model with partial coverage and Qu and Weng [27] introduced the problem of multiple allocation hub maximal covering problem. MCLP is originally defined as network problem, but some authors introduces a MCLP on the plane with given Euclidian distances between them [28]. Many papers described applications of MCLP in solving real-life problems, like maximal covering model for network design [29], finding the distribution of police patrol areas in Dallas [30], models for improving accessibility to rural health services in Ghana [31] and the model for locating emergency medical services in Istanbul [32]. In the beginning, MCLP was defined just in a deterministic sense, but in later studies, MCLP with uncertainty parameters are presented. Most papers use stochastic and fuzzy models for uncertain conditions. Louveaux in [33] and Weaver and Church in [34] used probability distribution for modeling vagueness. A fuzzy multi-objective covering-based location model with applications and different solutions is presented by Araz et. all in [14]. Perez at al. in [8] involved a fuzzy model for real-world problems with linguistic vagueness, Darzentas in [35] presented using a fuzzy set partitioning for modeling MCLP. Batanovic at al. [9] described using fuzzy sets in modelling problems on networks in uncertain environments and Davari at al. in [5] used fuzzy variables for travel times between nodes.

Using variables (2)-(7), model of MCLP, introduced by Church and ReVelle in [1], is:

$$\text{maximize} \sum_{i \in I} y_i,$$

subject to

$$\sum_{j \in N_i} x_j \geq y_i, \forall i \in I,$$
$$\sum_{j \in J} x_j = P,$$
$$x_j \in \{0, 1\}, \forall j \in J,$$
$$y_i \in \{0, 1\}, \forall i \in I,$$

where:

$N_i = \{j | t_{ij} \leq S\}$ - set of all facilities j which cover location i.

As in LSCP, the improvement of the MCLP model can be also achieved by using fuzzy numbers for radius of coverage and travel times between locations. Because of the nature of FMCLP, using conorms in fuzzy MCLP (FMCLP) models for calculation of coverage degree is more complex than in FSCLP. In MCLP for finding locations medical services, each node must be assigned to the nearest facility and quality of solution will not be increased if location is covered by more that one facility. That reason is very clear - a patient from some location must be assigned to the nearest hospital and other close hospitals do not have any importance for him. In other types of problems, like in locating fire stations, multiple partial coverage increases quality of solution - if more fire stations partially cover some location, that location is safer. The reasons for this is also clear - if more fire trucks come to the locations, damage will be less. At the end, in some problems, like location of shops for example the coverage is not simply the sum but the weighted sum of coverage degreed. If one shop covers a location via a certain degree the second shop can only add to the quality in the parts that the first shop does not cover, the third shop can only add to the part which the first two shops do not cover etc... For this case we propose an OWS operator with decreasing weights $w_1 \geq w_2 \cdots \geq w_n$ with $w_1 = 1$. Considering this and using notations (8)-(13), anew models of FMCLP are:

FMCLP model with maximum conorm:

$$\text{maximize} \sum_{i \in I} y_i,$$

with conditions:

$$\sum_{j \in J} x_j = P,$$

$$\max_{j \in J} x_j \cdot c_{ij} \geq y_i, \forall i \in I,$$

$$x_i \in \{0, 1\},$$

$$y_i \in [0, 1].$$

FMCLP model with the limited sum conorm:

$$\text{maximize} \sum y_i,$$

with conditions:

$$\sum_{j \in J} x_j = P,$$

$$S_L(x_1 \cdot c_{i_1}, x_2 \cdot c_{i_2}, \ldots, x_n \cdot c_{in}) \geq y_i, \forall j \in J, i_k \in I$$

$$x_i \in \{0, 1\},$$

$$y_i \in [0, 1].$$

FMCLP model with the OWS operator:

$$\text{maximize} \sum y_i,$$

with conditions:

$$\sum_{j \in J} x_j = P,$$

$$F_W(x_1, x_2, \ldots, x_n) \geq y_i, \forall j \in J, \ W - \text{sorted elements from } \{c_{1j}, c_{2j}, \ldots, c_{nj}\},$$

$$x_i \in \{0, 1\},$$

$$y_i \in [0, 1].$$

3.3. *Minimal covering location problem (MinCLP)*

The aim of minimal covering location problem (MinCLP) is finding the best positions for facilities, such as less as possible locations will be covered. MinCLP could be used for modeling problems in finding the undesirable facility location, eg. finding locations for pollutants (like power plants), potential dangerous facilities (like nuclear plants or chemical installations) or unwanted facilities (as garbage collectors). MinCLP requires predefined minimal distance between facilities, because without this condition best solution is locating all facilities on the same place. Minimal covering problem was not much studied in the past, the first study was by Drezner and Wesolowsky [36] with the aim to find a circle containing the minimum weight of points in the plane. Brimberg and ReVelle [37] improved this model to the multi-facility version by allowing a partial satisfaction of demand at the fixed points. Ozan and Wesolowsky using Minimal covering location problem solved a planar expropriation problem with non-rigid rectangular facilities [38]. Takaci at al. introduced a fuzzy model for MinCLP [39] with proposed method for solving it.

MinCLP needs to minimize facility influence to locations, so maximal, limited sum and average conorms could be used in models. A limited sum conorm is appropriate for problems with one facility type, where overall influence is equal to the sum of all. For example, total influence caused by smoke pollutants is equal to the sum of all partial pollutants. Maximal conorm could be used for problems where is the maximal influence is given as condition and average sum could be used for problems with different facility types, where importance could be equal to maximal or average influence. The example for this could be location smoke and water pollutants, where sum or weighted sum of pollution could be used, depending on the nature of the problem.

As mentioned before, it is necessary to define additional variable in existing definitions (2) - (7):

$$D - \text{minimal distance between facilities} \tag{14}$$

With this, three models of MinCLP are as follows.

MinCLP with the maximum conorm:

$$\begin{aligned}
&\text{minimize} \sum y_i,\\
&\text{with conditions:}\\
&\sum_{j \in J} x_j = P,\\
&d(x_i, x_j) \geq D, \forall i \neq j,\\
&\max_{j \in J} x_j \cdot c_{ij} \leq y_i, \forall i \in I,\\
&x_i \in \{0, 1\},\\
&y_i \in [0, 1].
\end{aligned}$$

MinCLP with the limited sum conorm:

$$\begin{aligned}
&\text{minimize} \sum y_i,\\
&\text{with conditions:}\\
&\sum_{j \in J} x_j = P,\\
&d(x_i, x_j) \geq D, \forall i \neq j,\\
&S_L(x_1 \cdot c_{i_1}, x_2 \cdot c_{i_2}, \dots, x_n \cdot c_{in}) \leq y_i, \forall j \in J, i_k \in I,\\
&x_i \in \{0, 1\},\\
&y_i \in [0, 1].
\end{aligned}$$

MinCLP with the OWS aggregation operator:

$$\begin{aligned}
&\text{minimize} \sum y_i,\\
&\text{with conditions:}\\
&\sum_{j \in J} x_j = P,\\
&d(x_i, x_j) \geq D, \forall i \neq j\\
&F_W(x_1, x_2, \dots, x_n) \leq y_i, \forall j \in J,\ W - \text{sorted elements from } \{c_{1j}, c_{2j}, \dots, c_{nj}\},\\
&x_i \in \{0, 1\},\\
&y_i \in [0, 1].
\end{aligned}$$

4. Conclusion and Next Steps

This paper generalizes the use of fuzzy sets in modelling covering location problems. Seven new fuzzy location covering models with different aggregation operators, together with practical validity are presented. A novel fuzzy approach is presented applying fuzzyfication not only to the distance but for the travel times also. All three types of location problems minimal, maximal and set are covered and models for each application type are suggested. These results open the possibilities for further research in development and adaptation of algorithms for their solution.

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