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The Open-Locating-Dominating Number of Some Convex Polytopes

Aleksandar Lj. Savić[a], Zoran Lj. Maksimović[b], Milena S. Bogdanović[c]

[a]*Faculty of Mathematics, University of Belgrade, Studentski trg 16/IV,11 000 Belgrade, Serbia*
[b]*Military Academy, University of Defence, generala Pavla Jurišića Šturma 33, 11 000 Belgrade, Serbia*
[c]*Pedagogical Faculty Vranje, University of Niš, Partizanska 14, 17500 Vranje, Serbia*

Abstract. In this paper we will investigate the problem of finding the open-locating-dominating number for some classes of planar graphs - convex polytopes. We considered D_n, T_n, B_n, C_n, E_n and R_n classes of convex polytopes known from the literature. The exact values of open-locating-dominating number for D_n and R_n polytopes are presented, along with the upper bounds for T_n, B_n, C_n, and E_n polytopes.

1. Introduction

Let $G = (V, E)$ be an arbitrary graph and for any $v \in V$ let us denote $N(v)$ and $N[v]$ the open and closed neighborhoods of v. The open locating dominating set S of graph $G = (V, E)$ is a set of vertices that dominates G and for any $x, y \in V$ holds $N(x) \cap S \neq N(y) \cap S$. Set S will be denoted an *OLD-set of G*. The cardinality of minimal such set S will be denoted as $\gamma_{old}(G)$.

The motivation for introduction of $OLD-set$ and similar sets arose from security and protecting concerns. Different types of networks of facilities, or computer networks or network of routers could be theoretically represented by graphs. The aim is to define and determine the locations in such networks in order to identify and locate any "intruder" or fault in some location in the network. Consider that in any location of the network, which means in any vertex of the corresponding graph there is some detecting device which can detect an intruder in this and in all neighboring locations.

The locating dominating set is a set $L \subset V$, where a detection device in location $x \in L$ can determine if intruder is in that location or in $N(x)$, but could not determine in which element of $N(x)$. It follows, as introduced in [15–17], $L \subset V$ is *locating dominating set* of G if L dominates G (i.e. $\bigcup_{x \in L} N(x) = V$) and for any $x, y \in V \setminus L$ holds $N(x) \cap S \neq N(y) \cap S$.

If a detection device can determine whether there is an intruder in the closed neighborhood of $N[x]$, but could not locate in which location, then we are interested in the *identifying code*. As introduced in [9], identifying code I is a vertex subset of V which dominates G, and for any $x, y \in V$ holds $N[x] \cap S \neq N[y] \cap S$.

Finally, if a detection device can detect an intruder in $N(x)$ without ability to detect it in x we are considering *open neighborhood locating dominating set*, as defined above. The problem of OLD sets was independently introduced by [5] on k-cubes Q_k and generally on graphs in [11, 12].

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Research partially partially supported by Serbian Ministry of Science under the grant no. 174010
Email addresses: asavic@matf.bg.ac.rs (Aleksandar Lj. Savić), zoran.maksimovic@gmail.com (Zoran Lj. Maksimović), mb2001969@beotel.net (Milena S. Bogdanović)

In [10] is presented a bibliography, currently with more than 350 entries, for work on distinguishing sets.

If two vertices $x, y \in V(G)$ such that $N(x) = N(y)$ exist, it follows that $N(x) \cap S = N(y) \cap S$ for any $S \subset V$ and G could not have an *OLD* set. This is proposed in

Proposition 1.1. *[11]. A graph G has an OLD set if and only if G has no isolated vertex and $N(x) \neq N(y)$ for all pairs x, y of distinct vertices.*

For a tree there is more detailed characterization presented in the following proposition.

Proposition 1.2. *[3, 12]. For a tree T of order $n \geq 3$, T has an OLD set if and only if T does not contain a strong support vertex, where a strong support vertex is a vertex which has two or more vertices of degree one as the neighbors.*

Some other connection between values $\gamma_{old}(G)$ and order of G are given in [3].

Proposition 1.3. *Assume $k \geq 2$, and suppose $k + 1 \leq n \leq 2^k - 1$, then there exists a connected graph G of order n with $\gamma_{old}(G) = k$.*

In the special case where graph G is a tree there are following results.

Theorem 1.4. *[12] If tree T of order $n \geq 5$ has an OLD set, then $\lceil n/2 \rceil + 1 \leq \gamma_{old}(T) \leq n - 1$.*

Theorem 1.5. *[13] For $n \geq 5$ and $\lceil n/2 \rceil + 1 \leq j \leq n - 1$ there is a tree $T_{n;j}$ of order n with $\gamma_{old}(T_{n;j}) = j$.*

Naturally, finding $OLD(G)$ is hard, and corresponding optimization problem is NP-hard which was proved in [11].

In paper [3], authors characterize graphs G of order n with $OLD(G) = 2, 3$, or n and graphs with minimum degree $\delta(G) \geq 2$ that are C_4-free with $\gamma_{old}(G) = n - 1$.

In the case of finite graphs G, there are some theoretical results concerning bounds for values of $\gamma_{old}(G)$ in some cases.

Theorem 1.6. *[3] Let G be a connected graph with minimum degree $\delta(G) \geq 3$ and C_4-free. Then $\gamma_{old}(G) \leq n - \rho(G)$, where $\rho(G)$ is the maximum number of vertices which are pairwise at distance at least 3.*

Theorem 1.7. *[3, 11] For a graph G of order n and maximum degree $\Delta(G)$, if G has an OLD set, then $\gamma_{old}(G) \geq \frac{2n}{1+\Delta}$.*

Theorem 1.8. *[4] If G is a cubic graph of order n, then $\gamma_{old}(G) \leq \frac{3n}{4}$.*

There are results for *OLD* sets and values of $\gamma_{old}(G)$ for some classes of infinite graphs but since convex polytopes, the class of graphs considered in this paper, are finite they are out of scope.

In this paper we will consider finding *OLD* sets with minimal cardinality and $\gamma_{old}(G)$ values, as well as bounds for some classes of nontrivial planar graphs. In the literature these classes are for the first time considered in [2] where they were denoted as R_n and Q_n. Some other classes are also considered, denoted B_n and C_n introduced in [8], D_n introduced in [6] while T_n were introduced in [7]. All these classes are called convex polytopes and for all of them in the mentioned papers were given their metric dimensions. As introduced in [14], the problem of binary locating domination is related to that of open locating domination. In [14] the exact values of the binary locating-dominating number for convex polytopes D_n, and R''_n are determined as well as the tight bounds for R_n, Q_n and U_n.

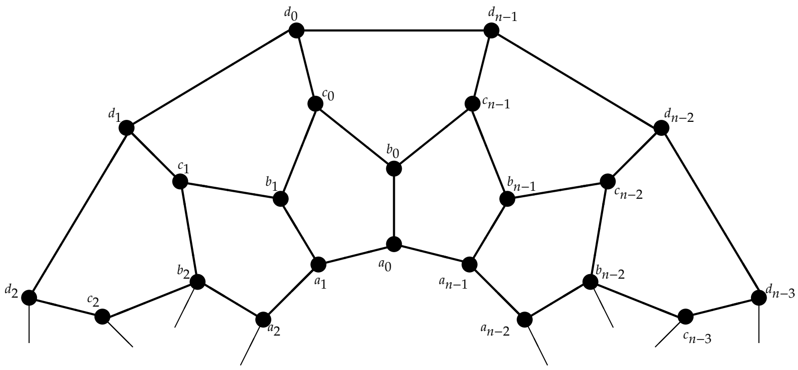

Figure 1: The graph of convex polytope D_n

2. The exact values

2.1. Convex polytope D_n

The graph of convex polytope D_n, in Figure 1, was introduced in [1]. It consists of $2n$ 5-sided faces and a pair of n-sided faces. Mathematically, it has vertex set $V(D_n) = \{a_i, b_i, c_i, d_i \mid i = 0, 1, \ldots, n-1\}$ and edge set $E(D_n) = \{(a_i, a_{i+1}), (d_i, d_{i+1}), (a_i, b_i), (b_i, c_i), (c_i, d_i), (b_{i+1}, c_i) \mid i = 0, 1, \ldots, n-1\}$. Note that indices are taken modulo n.

Theorem 2.1. $\gamma_{old}(D_n) = 2n$.

Proof. It is easy to see that D_n is a regular graph of degree 3, with $4n$ vertices. Then, by Theorem 1.7 it holds $\gamma_{old}(D_n) \geq \left\lceil \frac{2 \cdot 4 \cdot n}{1+3} \right\rceil = 2n$.

Let $S = \{a_i, d_i\} | i = 0, ..., n-1\}$. It is easy to see that all intersections $S \bigcap N(a_i) = \{a_{i-1}, a_{i+1}\}$; $S \bigcap N(b_i) = \{a_i\}$; $S \bigcap N(c_i) = \{d_i\}$ and $S \bigcap N(d_i) = \{d_{i-1}, d_{i+1}\}$ are non-empty and distinct. Since S is a open-locating-dominating set of D_n and $|S| = 2n$ therefore, $\gamma_{old}(D_n) \leq 2n$. Having in mind previous fact that $\gamma_{old}(D_n) \geq 2n$, it is proven that $\gamma_{old}(D_n)$ is equal to $2n$. □

2.2. Convex polytope R_n

Mathematically, the graph of convex polytope R_n have vertex set $V = \{a_i, b_i, c_i | i = 0, \ldots, n-1\}$ and edge set $E = \{(a_i, a_{i+1}), (a_i, b_i), (a_{i+1}, b_i), (b_i, b_{i+1}),$
$(b_i, c_i), (c_i, c_{i+1}) | i = 0, \ldots, n-1\}$.

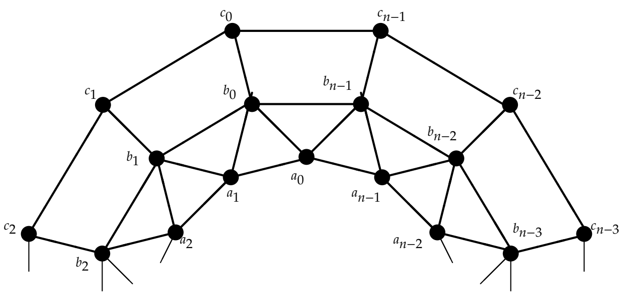

Figure 2: The graph of convex polytope R_n

Theorem 2.2. $\gamma_{old}(R_n) = n$.

Table 1: *OLD*-set S for T_n

Case	S	$\|S\|$
$n = 4k$	$\{b_{4i}, b_{4i+1}, b_{4i+2}, c_{4i}, c_{4i+1}, c_{4i+2}, \|i = 0, \ldots, k-1\}$	$6k$
$n = 4k+1$	$\{b_{4i}, b_{4i+1}, b_{4i+2}, c_{4i}, c_{4i+1}, c_{4i+2}, \|i = 0, \ldots, k-1\} \cup \{b_{4k}, c_{4k}\}$	$6k+2$
$n = 4k+2$	$\{b_{4i}, b_{4i+1}, b_{4i+2}, c_{4i}, c_{4i+1}, c_{4i+2}, \|i = 0, \ldots, k-1\} \cup \{b_{4k}, c_{4k}, b_{4k+1}, c_{4k+1}\}$	$6k+4$
$n = 4k+3$	$\{b_{4i}, b_{4i+1}, b_{4i+2}, c_{4i}, c_{4i+1}, c_{4i+2}, \|i = 0, \ldots, k-1\} \cup \{b_{4k}, c_{4k}, b_{4k+1}, c_{4k+1}, b_{4k+2}, c_{4k+2}\}$	$6k+6$

Proof. Let $S = \{b_i | i = 0, ..., n-1\}$. It is easy to see that all intersections $S \bigcap N(a_i) = \{b_{i-1}, b_i\}$; $S \bigcap N(b_i) = \{b_{i-1}, b_{i+1}\}$ and $S \bigcap N(c_i) = \{b_i\}$ are non-empty and distinct. Since S is an open locating-dominating set of R_n and $|S| = n$ therefore, $\gamma_{old}(R_n) \leq n$.

On the other hand, by Theorem 1.7 it holds $\gamma_{old}(R_n) \geq \left\lceil \frac{2 \cdot 3 \cdot n}{1+5} \right\rceil = n$. Therefore, $\gamma_{old}(R_n) = n$. □

3. The upper bounds

3.1. Convex polytopes T_n

The graph of convex polytope T_n, in Figure 3, was introduced in [7]. It consists of $4n$ 3-sided faces, n 4-side faces and a pair of n-sided faces. Mathematically, it has vertex set $V(T_n) = \{a_i, b_i, c_i, d_i\}$, and the set of edges

$$E(T_n) = \{a_i a_{i+1}, b_i b_{i+1}, c_i c_{i+1}, d_i d_{i+1}, a_i b_i, b_i c_i, c_i d_i, a_{i+1} b_i, c_{i+1} d_i\}$$

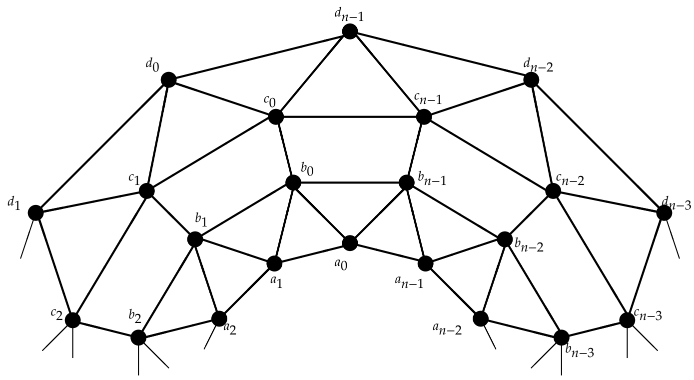

Figure 3: The graph of convex polytope T_n

Theorem 3.1. *$\gamma_{old}(T_n) \leq l_n$, where*

$$l_n = \begin{cases} 6k, & n = 4k; \\ 6k+2, & n = 4k+1; \\ 6k+4, & n = 4k+2; \\ 6k+6, & n = 4k+3. \end{cases}$$

Proof. We shall prove that set S

$$S = \{b_i, c_i | i = 0, 1, \ldots, n-1 \wedge i \not\equiv 3 \pmod 4\}$$

is an *OLD*-set for a graph T_n. The cardinality of set S depends on n, which is described in Table 1, where the congruency of $|S|$ on modulo 4 is given in the first column, elements of set S and its cardinality in the second and third columns, respectively.

Depending on n we will consider the following four cases:

Case $n = 4k$: Intersection of open neighborhood of the vertices and set S are given in Table 2, where vertex v is given in the columns labeled with v and the intersection of open neighborhood of the vertex

Table 2: Intersection of open neighborhoods and set S where $n = 4k$

v	$\mathcal{N}(v) \cap S$	v	$\mathcal{N}(v) \cap S$
a_{4i}	$\{b_{4i}\}$	a_{4i+2}	$\{b_{4i+1}, b_{4i+2}\}$
b_{4i}	$\{b_{4i+1}, c_{4i}\}$	b_{4i+2}	$\{b_{4i+1}, c_{4i+2}\}$
c_{4i}	$\{c_{4i+1}, b_{4i}\}$	c_{4i+2}	$\{c_{4i+1}, b_{4i+2}\}$
d_{4i}	$\{c_{4i}, c_{4i+1}\}$	d_{4i+2}	$\{c_{4i+2}\}$
a_{4i+1}	$\{b_{4i+1}, b_{4i}\}$	a_{4i+3}	$\{b_{4i+2}\}$
b_{4i+1}	$\{b_{4i+2}, b_{4i}, c_{4i+1}\}$	b_{4i+3}	$\{b_{4i+2}, b_{4(i+1)}\}$
c_{4i+1}	$\{c_{4i+2}, c_{4i}, b_{4i+1}\}$	c_{4i+3}	$\{c_{4i+2}, c_{4(i+1)}\}$
d_{4i+1}	$\{c_{4i+1}, c_{4i+2}\}$	d_{4i+3}	$\{c_{4(i+1)}\}$

Table 3: Special cases of intersections of open neighborhoods and set S

n=4k+1		n=4k+2		n=4k+3	
v	$\mathcal{N}(v) \cap S$	v	$\mathcal{N}(v) \cap S$	v	$\mathcal{N}(v) \cap S$
a_0	$\{b_0, b_{4k}\}$	a_0	$\{b_0, b_{4k+1}\}$	a_0	$\{b_0, b_{4k+1}\}$
b_0	$\{b_1, b_{4k}, c_0\}$	b_0	$\{b_1, b_{4k+1}, c_0\}$	b_0	$\{b_1, b_{4k+1}, c_0\}$
c_0	$\{c_1, c_{4k}, b_0\}$	c_0	$\{c_1, c_{4k+1}, b_0\}$	c_0	$\{c_1, c_{4k+1}, b_0\}$
d_0	$\{c_0, c_1\}$	d_0	$\{c_0, c_1\}$	d_0	$\{c_0, c_1\}$
a_{4k}	$\{b_{4k}\}$	a_{4k}	$\{b_{4k}\}$	a_{4k}	$\{b_{4k}\}$
b_{4k}	$\{b_0, c_{4k}\}$	b_{4k}	$\{b_{4k+1}, c_{4k}\}$	b_{4k}	$\{b_{4k+1}, c_{4k}\}$
c_{4k}	$\{c_0, b_{4k}\}$	c_{4k}	$\{c_{4k+1}, b_{4k}\}$	c_{4k}	$\{c_{4k+1}, b_{4k}\}$
d_{4k}	$\{c_0, c_{4k}\}$	d_{4k}	$\{c_{4k+1}, c_{4k}\}$	d_{4k}	$\{c_{4k+1}, c_{4k}\}$
		a_{4k+1}	$\{b_{4k+1}, b_{4k}\}$	a_{4k+1}	$\{b_{4k}, b_{4k+1}\}$
		b_{4k+1}	$\{b_0, b_{4k}, c_{4k+1}\}$	b_{4k+1}	$\{b_{4k+2}, b_{4k}, c_{4k+1}\}$
		c_{4k+1}	$\{c_0, c_{4k}, b_{4k+1}\}$	c_{4k+1}	$\{c_{4k+2}, c_{4k}, b_{4k+1}\}$
		d_{4k+1}	$\{c_0, c_{4k+1}\}$	d_{4k+1}	$\{c_{4k+2}, c_{4k+1}\}$
				a_{4k+2}	$\{b_{4k+1}, b_{4k+2}\}$
				b_{4k+2}	$\{b_0, b_{4k+1}, c_{4k+2}\}$
				c_{4k+2}	$\{c_0, c_{4k+1}, b_{4k+2}\}$
				d_{4k+2}	$\{c_0, c_{4k+2}\}$

v with set S in columns labeled with $\mathcal{N}(v) \cap S$. In Table 2, where $i = 0, \ldots, k-1$, it can be seen that all intersections are nonempty and distinct.

$n = 4k + 1$: The intersections of open neighborhood of a given vertex and the set S are the same as given in Table 2, with exception for the cases with indices $i = 0$ and $i = 4k$. The intersection sets for vertices with indices $i = 0$ and $i = 4k$ are given separately in Table 3. From Tables 2 and 3 it can be concluded that intersection sets are nonempty and distinct.

$n = 4k + 2$: The intersections of open neighborhood of a given vertex and the set S are the same as given in Table 2, with exception for the cases with indices $i = 0$, $i = 4k$ and $i = 4k + 1$. The intersection sets for vertices with indices $i = 0$, $i = 4k$ and $i = 4k + 1$ are given separately in Table 3 . From Tables 2 and 3 it can be concluded that intersection sets are nonempty and distinct.

$n = 4k + 3$: The intersections of open neighborhood of a given vertex and the set S are the same as given in Table 2, with exception for the cases with indices $i = 0$, $i = 4k$, $i = 4k + 1$ and $i = 4k + 2$. The intersection sets for vertices with indices $i = 0$, $i = 4k$, $i = 4k + 1$ and $i = 4k + 2$ are given separately in Table 3 . From Tables 2 and 3 it can be concluded that intersection sets are nonempty and distinct.

From the previous discussion we can conclude that the set S is OLD set for graph T_n and consequently $\gamma_{old}(T_n) \leq |S_n|$. □

3.2. Convex polytope B_n

The graph of convex polytope B_n (Figure 4) is introduced in [2] and consists of $2n$ 4-sided faces, n 3-sided faces, n 5-sided faces and a pair of n-sided faces. The set of vertices is

$$V(B_n) = \{a_i, b_i, c_i, d_i, e_i \,|\, i = 0, \ldots, n-1\}$$

and the set of edges is

$$E(B_n) = \{a_ia_{i+1}, b_ib_{i+1}, d_id_{i+1}, e_ie_{i+1}, a_ib_i, b_ic_i, b_{i+1}c_i, c_id_i, d_ie_i \mid i = 0, \dots, n-1\}$$

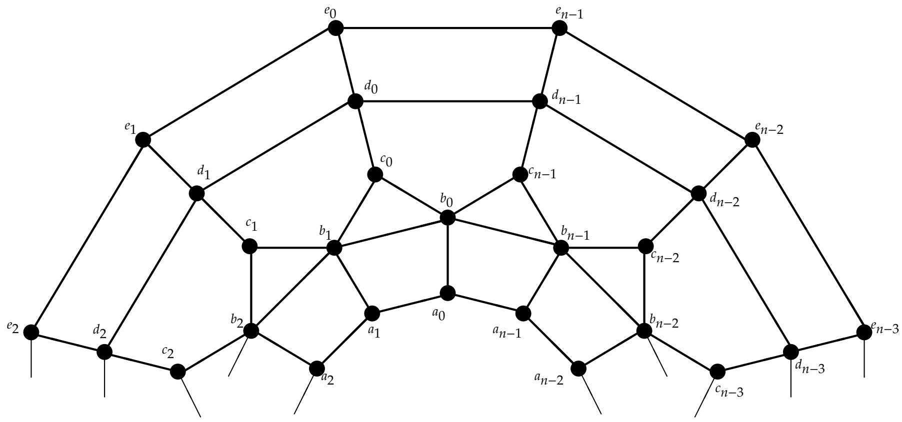

Figure 4: The graph of convex polytope B_n

Theorem 3.2. $\gamma_{old}(B_n) \leq 2n$.

Proof. Let $S = \{b_i, d_i\}|i = 0, ..., n-1\}$. It is easy to see that all intersections $S \bigcap N(a_i) = \{b_i\}$; $S \bigcap N(b_i) = \{b_{i-1}, b_{i+1}\}$; $S \bigcap N(c_i) = \{b_i, b_{i+1}, d_i\}$; $S \bigcap N(d_i) = \{d_{i-1}, d_{i+1}\}$ and $S \bigcap N(e_i) = \{d_i\}$ are non-empty and distinct. Since S is a open-locating-dominating set of B_n and $|S| = 2n$ therefore, $\gamma_{old}(B_n) \leq 2n$. □

3.3. Convex polytope C_n

Convex polytopes C_n (Figure 5) were introduced in [8] consisting of $3n$ 3-sided faces, n 4-sided faces, n 5-sided faces and a pair of n-sided faces. There sets of vertices $V(C_n)$ and sets of edges are given as

$$V(C_n) = \{a_i, b_i, c_i, d_i, e_i \mid i = 0, \dots, n-1\}$$

and

$$E(C_n) = \{a_ia_{i+1}, b_ib_{i+1}, d_id_{i+1}, e_ie_{i+1}, a_ib_i, b_ic_i, b_{i+1}c_i, c_id_i, d_ie_i, d_{i+1}e_i \mid i = 0, \dots, n-1\}.$$

Theorem 3.3. $\gamma_{old}(C_n) \leq 2n$.

Proof. Let $S = \{b_i, d_i\}|i = 0, ..., n-1\}$. It is easy to see that all intersections $S \bigcap N(a_i) = \{b_i\}$; $S \bigcap N(b_i) = \{b_{i-1}, b_{i+1}\}$; $S \bigcap N(c_i) = \{b_i, b_{i+1}, d_i\}$; $S \bigcap N(d_i) = \{d_{i-1}, d_{i+1}\}$ and $S \bigcap N(e_i) = \{d_i, d_{i+1}\}$ are non-empty and distinct. Since S is a open-locating-dominating set of C_n and $|S| = 2n$ therefore, $\gamma_{old}(C_n) \leq 2n$. □

3.4. Convex polytope E_n

The graph of convex polytope E_n (Figure 6) is similar to the C_n and is introduced in [8] consisting of $5n$ 3-sided faces, n 5-sided faces and a pair of n-sided faces, where:

$$V(E_n) = \{a_i, b_i, c_i, d_i, e_i \mid i = 0, \dots, n-1\}$$

$$E(E_n) = \{a_ia_{i+1}, b_ib_{i+1}, d_id_{i+1}, e_ie_{i+1}, a_ib_i, a_{i+1}b_i, b_ic_i, b_{i+1}c_i, c_id_i, d_ie_i, d_{i+1}e_i | i = 0, \dots, n-1\}$$

.

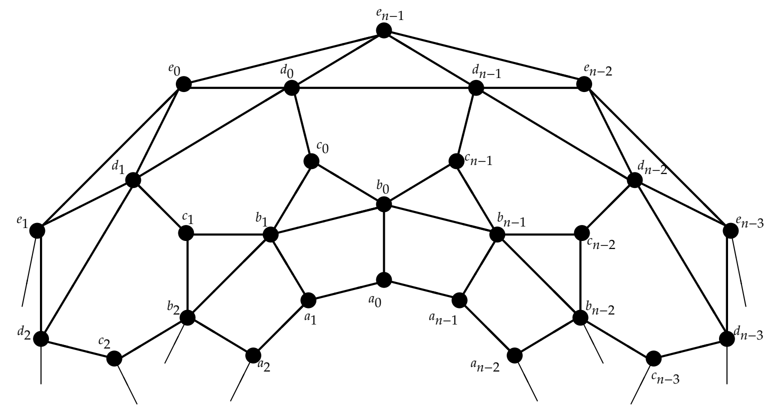

Figure 5: The graph of convex polytope C_n

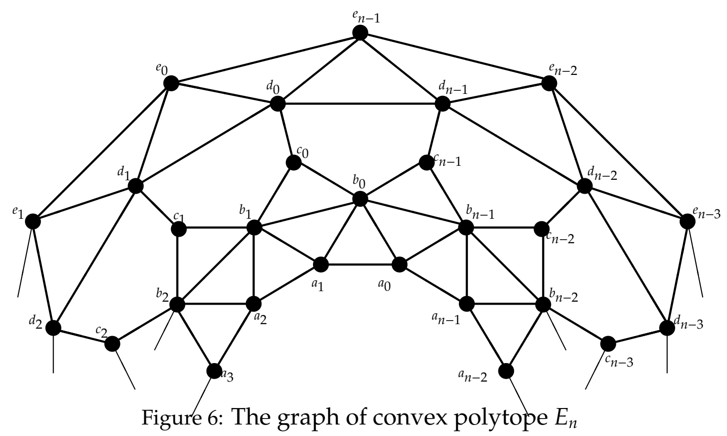

Figure 6: The graph of convex polytope E_n

Theorem 3.4. $\gamma_{old}(E_n) \leq 2n$.

Proof. Let $S = \{b_i, d_i\}|i = 0, ..., n-1\}$. It is easy to see that all intersections $S \bigcap N(a_i) = \{b_{i-1}, b_i\}$; $S \bigcap N(b_i) = \{b_{i-1}, b_{i+1}\}$; $S \bigcap N(c_i) = \{b_i, b_{i+1}, d_i\}$; $S \bigcap N(d_i) = \{d_{i-1}, d_{i+1}\}$ and $S \bigcap N(e_i) = \{d_i, d_{i+1}\}$ are non-empty and distinct. Since S is a open-locating-dominating set of C_n and $|S| = 2n$ therefore, $\gamma_{old}(C_n) \leq 2n$. □

4. Conclusions

In this paper we solved the problem of finding open-locating-dominating number of polytopes D_n and R_n. The upper bound of the open-locating-dominating number for certain classes of convex polytopes is given, along with the appropriate open-locating domination sets.

In the future work the problem of finding open-locating-dominating number for other classes of graphs could be considered. Another direction of future research could be to determine other graph invariants for considered convex polytopes.

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