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On the Stability of a Differential-Difference Analogue of a Two-Dimensional Problem of Integral Geometry

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Abstract. A differential - difference analogue of a two-dimensional problem of integral geometry with a weight function is studied. A stability estimate for the problem considered is obtained.

1. Introduction

A great deal of work has been devoted to integral geometry problem, that is, the problems of determining a function from its integrals along a given family of curves. One of the stimuli for studying such problems is their connection with multidimensional inverse problems for differential equations [2, 4, 5]. Some inverse problems for hyperbolic equations were shown to reduce to integral geometry problems and, in particular, a problem of integral geometry was considered in the case of shift-invariant curves. Mukhometov [3] showed the uniqueness and estimated the stability of the solution of a two-dimensional integral geometry on the whole. His results were mainly based on the reduction of the two-dimensional integral geometry problem

$$V(\gamma,z)=\int\limits_{K(\gamma,z)} U(x,y)\rho(x,y,z)ds, \gamma\in[0,1], z\in[0,1] \tag{1}$$

where $U\in C^2\left(\overline{D}\right), \rho\left(x,y,z\right)$ is a known function, to the boundary value problem

$$\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial x}\frac{\cos\theta}{\rho}+\frac{\partial W}{\partial y}\frac{\sin\theta}{\rho}\right)=0, (x,y,z)\in\Omega_1, \tag{2}$$

$$W\left(\xi\left(\gamma\right),\eta\left(\gamma\right),z\right)=V\left(\gamma,z\right), V\left(z,z\right)=0, \gamma,z\in\left[0,l\right]. \tag{3}$$

Here D is a bounded simply connected domain in the plane with a smooth boundary Γ :

$$x=\xi\left(z\right),\qquad y=\eta\left(z\right),\qquad z\in\left[0,l\right],\qquad \xi(0)=\xi(l),\qquad \eta(0)=\eta(l), \tag{4}$$

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where the parameter is the length of the curve Γ :

$$\Omega_1 = \Omega \setminus \{(\xi(\gamma), \eta(\gamma), z) : z \in [0, l]\}, \Omega = \overline{D} \times [0, l].$$

$K(x, y, z)$ is the part of the curve $K(\gamma, z)$ included between the points $(x, y) \in \overline{D}$ and $(\xi(z), \eta(z))$, $z \in [0, l]$;

$$W(x, y, z) = \int\limits_{K(x,y,z)} U(x, y)\, \rho(x, y, z)\, ds,$$

$\theta(x, y, z)$ is an angle between the tangent to $K(x, y, z)$ at the point (x, y) and the x -axis and the variable parameter s is the curve length.

2. Differential-Difference Problem

Suppose that the requirements on the family of curves $K(\gamma, z)$ and the domain D necessary for the problem (1) to reduce to problem (2), (3) are met [3]. Assume also that every line parallel to either the $x-$ or the $y-$ axis intersects the boundary of D at no more than two points. Let

$$a_1 = \inf_{(x,y)\in D}\{x\},\ b_1 = \sup_{(x,y)\in D}\{x\},$$

$$a_2 = \inf_{(x,y)\in D}\{y\},\ b_2 = \sup_{(x,y)\in D}\{y\},$$

$$h_j = (b_j - a_j)/N_j, j = 1, 2;\ h_3 = l/N_3,$$

and the N_j, $j = 1, 2, 3$, are natural numbers. Suppose that

$$0 < \varepsilon < \min\Big\{(b_1 - a_1)/3, \quad (b_2 - a_2)/3\Big\},\ D^{\varepsilon} = \left\{(x, y) \in D : \min_{(\alpha,\beta)\in\Gamma} \rho((x, y), (\alpha, \beta)) > \varepsilon\right\},$$

$$R_h = \Big\{\big(x_i, y_j\big) : x_i = a_1 + ih_1, \quad y_j = a_2 + jh_2\ , \quad i = 1, ..., N_1, \quad j = 1, ..., N_2\Big\}.$$

A neighborhood $B(ih_1, jh_2)$ of the point $(a_1 + ih_1, a_2 + jh_2)$ is defined as the five-point set

$$\{(a_1 + ih_1, a_2 + jh_2), (a_1 + (i \pm 1)h_1, a_2 + (j \pm 1)h_2)\}.$$

A set D_h^{ε} consists of all points $(a_1 + ih_1, a_2 + jh_2)$ such that their neighborhoods $B(ih_1, jh_2)$ are contained in $D^{\varepsilon} \cap R_h$. A set Γ_h^{ε} is made of all points $(a_1 + ih_1, a_2 + jh_2) \in D_h^{\varepsilon}$ such that $B(ih_1, jh_2) \cap (D^{\varepsilon} \cap R_h) \setminus D_h^{\varepsilon} \neq 0$. Finally,

$$\Delta_h^{\varepsilon} = \bigcup_{\Gamma_h^{\varepsilon}} B(ih_1, jh_2), \qquad D_h = R_h \cap D\ .$$

From here on we suppose that the coefficients and the solution of problem (2), (3) have the following properties:

$$W(x, y, z) \in C^3(\Omega^{\varepsilon}), \theta(x, y, z) \in C^2(\Omega^{\varepsilon}), \Omega^{\varepsilon} = \overline{D^{\varepsilon}} \times [0, l],$$

$$\rho(x, y, z) \in C^2(\Omega), \rho(x, y, z) > C^* > 0, \frac{\partial\theta}{\partial z} > \left|\frac{\partial\rho}{\partial z} \cdot \frac{1}{\rho}\right|.$$

We consider the differential-difference problem of finding the functions $\Phi_{i,j}(z)$, $U_{i,j}$ which satisfy the equation

$$\Phi_{\overset{0}{x}}\frac{A}{C}+\Phi_{\overset{0}{y}}\frac{B}{C}=U_{i,j},\quad (a_1+ih_1,a_2+jh_2)\in D_h,\quad z\in[0,l]\ , \tag{5}$$

and the boundary condition

$$\Phi_{i,j}(z)=F_{i,j}(z)\,,\quad (a_1+ih_1,a_2+jh_2)\in\Delta_h^{\varepsilon},\quad z\in[0,l]\ , \tag{6}$$

where

$$\Phi_{i,j}(z)=\Phi(x_i,y_j,z)=\Phi(a_1+ih_1,a_2+jh_2,z),$$

$$u_{i,j}=u(x_i,y_j)=u(a_1+ih_1,a_2+jh_2), i=\overline{0,N_1}, j=\overline{0,N_2};$$

$$\Phi_{\overset{0}{x}}=(\Phi_{i+1,j}-\Phi_{i-1,j})/2h_1, \Phi_{\overset{0}{y}}=(\Phi_{i,j+1}-\Phi_{i,j-1})/2h_2,$$

$$A=\cos\theta_{i,j}(z)\,,\qquad B=\sin\theta_{i,j}(z)\ ,\qquad \theta_{i,j}(z)=\theta(a_1+ih_1,a_2+jh_2,z)\qquad ,C=\rho(a_1+ih_1,a_2+jh_2,z)\,.$$

We note that in the differential-difference formulation information on the solution is given not only on the boundary Γ but also in its $\varepsilon-$ neighborhood, because the partial derivatives $\theta_z,\ W_{xz},\ W_{yz},\ W_{xy}$ have singularities of the type $\left[(x-\xi(z))^2+(y-\eta(z))^2\right]^{-\frac{1}{2}}$ in a neighborhood of an arbitrary point $(\xi(z),\eta(z),z)$ (see [1, 3]).

3. The Main Result

Theorem 3.1. *Suppose that problem* (5)-(6)*, with the functions* $\theta_{i,j}(z),\rho(x,y,z)$ *such that* $\theta_{i,j}(0)=\theta_{i,j}(l)$, $\rho(x,y,z)\in C^1(\Omega),\rho(x,y,z)>C*>0$ *and* $\left|\frac{\partial\theta}{\partial z}\right|>\left|\frac{\partial\rho}{\partial z}\frac{1}{\rho}\right|$*, has a solution* $\Phi_{i,j}(z)\in C^1[0,l],\Phi_{i,j}(0)=\Phi_{i,j}(l)$. *Then, for all* $N_j>9, j=1,2$*, the following inequality holds:*

$$\sum_{D_h^{\varepsilon}}U_{i,j}^2h_1h_2\le c_1\int\limits_0^1\sum_{\Delta_h^{\varepsilon}}\left[F_{\overset{\circ}{x}}^2h_1+F_{\overset{\circ}{y}}^2h_2+\left(\frac{\partial F}{\partial z}\right)^2(h_1+h_2)\right]dz,$$

where c_1 *is a constant dependent on the function* $\rho(x,y,z)$ *and the curves family* $K(\gamma,z)$.

Proof. Multiplying both sides of (5) by $C\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\frac{\partial}{\partial z}$, we get

$$C\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\frac{\partial}{\partial z}\left(\frac{A}{C}\Phi_{\overset{0}{x}}+\frac{B}{C}\Phi_{\overset{0}{y}}\right)=0. \tag{7}$$

Using the product differentiation formula, we rewrite (7) as follows:

$$\begin{aligned}0&=C\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\frac{\partial}{\partial z}\left(\frac{A}{C}\Phi_{\overset{0}{x}}+\frac{B}{C}\Phi_{\overset{0}{y}}\right)=\frac{\partial}{\partial z}\left[C\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\left(\frac{A}{C}\Phi_{\overset{0}{x}}+\frac{B}{C}\Phi_{\overset{0}{y}}\right)\right]\\&-\frac{\partial}{\partial z}\left[C\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\right]\left(\frac{A}{C}\Phi_{\overset{0}{x}}+\frac{B}{C}\Phi_{\overset{0}{y}}\right)=\frac{\partial}{\partial z}\left[\left(A\Phi_{\overset{0}{x}}+B\Phi_{\overset{0}{y}}\right)\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\right]-\frac{\partial C}{\partial z}\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\\&\times\left(\frac{A}{C}\Phi_{\overset{0}{x}}+\frac{B}{C}\Phi_{\overset{0}{y}}\right)-\frac{\partial}{\partial z}\left[-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right]\left(A\Phi_{\overset{0}{x}}+B\Phi_{\overset{0}{y}}\right)=\frac{\partial}{\partial z}\left[\left(A\Phi_{\overset{0}{x}}+B\Phi_{\overset{0}{y}}\right)\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\right]\\&+AB\left(\frac{\partial C}{\partial z}\frac{1}{C}\right)\Phi_{\overset{0}{x}}^{\ 2}-\left(\frac{\partial C}{\partial z}\frac{1}{C}\right)A^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}+\left(\frac{\partial C}{\partial z}\frac{1}{C}\right)B^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}-\left(\frac{\partial C}{\partial z}\frac{1}{C}\right)AB\Phi_{\overset{0}{y}}^{\ 2}+\frac{\partial\theta}{\partial z}A^2\Phi_{\overset{0}{x}}^{\ 2}\\&+AB\Phi_{\overset{0}{x}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right)+\frac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}-A^2\Phi_{\overset{0}{x}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right)+\frac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}\\&\qquad+B^2\Phi_{\overset{0}{y}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right)+\frac{\partial\theta}{\partial z}B^2\Phi_{\overset{0}{y}}^{\ 2}-AB\Phi_{\overset{0}{y}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right).\end{aligned} \tag{8}$$

At the same time, rearranging (7) yields

$$
\begin{aligned}
0 &= C\left(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}}\right)\frac{\partial}{\partial z}\left(\frac{A}{C}\Phi_{\overset{0}{x}} + \frac{B}{C}\Phi_{\overset{0}{y}}\right) = C\left(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}}\right)\left[\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right)\frac{A}{C} + \Phi_{\overset{0}{x}}\left(\left(-B\frac{\partial\theta}{\partial z}C - A\frac{\partial C}{\partial z}\right)\frac{1}{C^2}\right)\right.\\
&\times\left[\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right)\frac{A}{C} + \Phi_{\overset{0}{x}}\left(\left(-B\frac{\partial\theta}{\partial z}C - A\frac{\partial C}{\partial z}\right)\frac{1}{C^2}\right) + \frac{B}{C}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \Phi_{\overset{0}{y}}\left(\left(A\frac{\partial\theta}{\partial z}C - B\frac{\partial C}{\partial z}\right)\frac{1}{C^2}\right)\right]\\
&= -AB\Phi_{\overset{0}{x}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) - \frac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} + \frac{\partial C}{\partial z}B^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} - B^2\Phi_{\overset{0}{x}}\frac{\partial}{\partial z}B^2\left(\Phi_{\overset{0}{y}}\right) + \frac{\partial\theta}{\partial z}B^2\Phi_{\overset{0}{x}}{}^2 + \left(\frac{\partial C}{\partial z}\frac{1}{C}\right)AB\Phi_{\overset{0}{x}}{}^2\\
&\quad + A^2\Phi_{\overset{0}{y}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) - \frac{\partial\theta}{\partial z}AB\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} - \left(\frac{\partial C}{\partial z}\frac{1}{C}\right)A^2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}} + AB\Phi_{\overset{0}{y}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \frac{\partial\theta}{\partial z}A^2\Phi_{\overset{0}{y}}{}^2 - \left(\frac{\partial C}{\partial z}\frac{1}{C}\right)AB\Phi_{\overset{0}{y}}{}^2. \quad (9)
\end{aligned}
$$

Sum (9) with (8) and denote

$$K = \sin 2\theta = 2\sin\theta\cos\theta = 2AB, E = \cos 2\theta = \cos^2\theta - \sin^2\theta = A^2 - B^2.$$

As a result, we obtain

$$
\begin{aligned}
&\left(\frac{\partial\theta}{\partial z} + \frac{1}{C}\frac{\partial C}{\partial z}K\right)\Phi_{\overset{0}{x}}{}^2 - 2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}\frac{\partial C}{\partial z}E + \left(\frac{\partial\theta}{\partial z} - \frac{1}{C}\frac{\partial C}{\partial z}K\right)\Phi_{\overset{0}{y}}{}^2\\
&\qquad + \Phi_{\overset{0}{y}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) - \Phi_{\overset{0}{x}}\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) + \frac{\partial}{\partial z}\left[\left(A\Phi_{\overset{0}{x}} + B\Phi_{\overset{0}{y}}\right)\left(-B\Phi_{\overset{0}{x}} + A\Phi_{\overset{0}{y}}\right)\right] = 0. \quad (10)
\end{aligned}
$$

Using the formula

$$(uv)_{\overset{0}{x}} = u_{\overset{0}{x}}v + uv_{\overset{0}{x}} + \frac{h_1^2}{2}[u_x v_x]_{\bar{x}},$$

where

$$f_x = \left(f_{i+1,j} - f_{i,j}\right)/h_1, \qquad f_{\bar{x}} = \left(f_{i,j} - f_{i-1,j}\right)\ /h_1,$$

we have

$$\left[\Phi_{\overset{0}{y}}\frac{\partial\Phi}{\partial z}\right]_{\overset{0}{x}} = \Phi_{\overset{0}{y}\overset{0}{x}}\frac{\partial\Phi}{\partial z} + \Phi_{\overset{0}{y}}\left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{x}} + \frac{h_1^2}{2}\left[\Phi_{\overset{0}{y}x}\left(\frac{\partial\Phi}{\partial z}\right)_x\right]_{\bar{x}}. \quad (11)$$

Similarly, we can write

$$\left[\Phi_{\overset{0}{x}}\frac{\partial\Phi}{\partial z}\right]_{\overset{0}{y}} = \Phi_{\overset{0}{x}\overset{0}{y}}\frac{\partial\Phi}{\partial z} + \Phi_{\overset{0}{x}}\left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{y}} + \frac{h_2^2}{2}\left[\Phi_{\overset{0}{x}y}\left(\frac{\partial\Phi}{\partial z}\right)_y\right]_{\bar{y}}. \quad (12)$$

It follows from (11), (12) that

$$\Phi_{\overset{0}{y}}\left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{x}} - \Phi_{\overset{0}{x}}\left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{y}} = \left[\Phi_{\overset{0}{y}}\left(\frac{\partial\Phi}{\partial z}\right)\right]_{\overset{0}{x}} - \left[\Phi_{\overset{0}{x}}\left(\frac{\partial\Phi}{\partial z}\right)\right]_{\overset{0}{y}} - \frac{h_1^2}{2}\left[\Phi_{\overset{0}{y}x}\left(\frac{\partial\Phi}{\partial z}\right)_x\right]_{\bar{x}} + \frac{h_2^2}{2}\left[\Phi_{\overset{0}{x}y}\left(\frac{\partial\Phi}{\partial z}\right)_y\right]_{\bar{y}}. \quad (13)$$

Taking into account that

$$\frac{\partial}{\partial z}\left(\Phi_{\overset{0}{x}}\right) = \left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{x}}, \qquad \frac{\partial}{\partial z}\left(\Phi_{\overset{0}{y}}\right) = \left(\frac{\partial\Phi}{\partial z}\right)_{\overset{0}{y}}, \quad (14)$$

combining (10) with (13) and (14), we arrive at

$$I_1+\frac{\partial}{\partial z}\left[\left(A\Phi_{\overset{0}{x}}+B\Phi_{\overset{0}{y}}\right)\left(-B\Phi_{\overset{0}{x}}+A\Phi_{\overset{0}{y}}\right)\right]+\left[\Phi_{\overset{0}{y}}\frac{\partial\Phi}{\partial z}\right]_{\overset{0}{x}}-\left[\Phi_{\overset{0}{x}}\frac{\partial\Phi}{\partial z}\right]_{\overset{0}{y}}-\left(\frac{h_1^2}{2}\right)\left[\Phi_{\overset{0}{y}x}\left(\frac{\partial\Phi}{\partial z}\right)_x\right]_{\bar{x}}+\left(\frac{h_2^2}{2}\right)\left[\Phi_{\overset{0}{x}y}\left(\frac{\partial\Phi}{\partial z}\right)_y\right]_{\bar{y}}=0. \quad (15)$$

Here

$$I_1=\left(\frac{\partial\theta}{\partial z}+\frac{1}{C}\frac{\partial C}{\partial z}K\right)\Phi_{\overset{0}{x}}{}^2-2\Phi_{\overset{0}{x}}\Phi_{\overset{0}{y}}\frac{\partial C}{\partial z}E+\left(\frac{\partial\theta}{\partial z}-\frac{1}{C}\frac{\partial C}{\partial z}K\right)\Phi_{\overset{0}{y}}{}^2.$$

The expression for I_1 is a quadratic form with respect to $\Phi_{\overset{0}{x}}$ and $\Phi_{\overset{0}{y}}$, whose determinant is

$$\left(\frac{\partial\theta}{\partial z}\right)^2-\left(\frac{\partial C}{\partial z}\frac{1}{C}\right)^2.$$

Then, from the condition

$$\frac{\partial\theta}{\partial z}>\left|\frac{\partial C}{\partial z}\frac{1}{C}\right|$$

the positive definiteness of the quadratic form I_1 follows. Using the inequality

$$ax^2+2bxy+cy^2\geq\frac{2\left(ac-b^2\right)}{a+c+\left((a-c)^2+4b^2\right)}(x^2+y^2)$$

for a positively definite quadratic form $ax^2+2bxy+cy^2$, we have

$$I_1\geq\left(\frac{\partial\theta}{\partial z}-\left|\frac{\partial C}{\partial z}\frac{1}{C}\right|\right)\left(\Phi_{\overset{0}{x}}{}^2+\Phi_{\overset{0}{y}}{}^2\right). \quad (16)$$

Due to (5), we obtain

$$\left(\Phi_{\overset{0}{x}}{}^2+\Phi_{\overset{0}{y}}{}^2\right)=U_{i,j}^2C^2+\left(B\Phi_{\overset{0}{x}}-A\Phi_{\overset{0}{y}}\right)^2. \quad (17)$$

Since

$$C=\rho(x,y,z),\qquad \rho(x,y,z)>C^*>0,\qquad \frac{\partial\theta}{\partial z}-\left|\frac{\partial C}{\partial z}\frac{1}{C}\right|>0,$$

there exists a positive constant c_1 such that

$$\int_0^1\left(\frac{\partial\theta}{\partial z}-\left|\frac{\partial C}{\partial z}\frac{1}{C}\right|\right)C^2dz\geq\frac{1}{c_1^2}>0. \quad (18)$$

Sum (15) over i,j with the use of (6) and integrate it with respect to z. Then using (16), (17), (18), the inequality $|ab|\leq(a^2+b^2)/2$ and the periodicity of the functions $\Phi_{i,j}(z),\theta_{i,j}(z)$ in z, we can transform the resulting equation so as to obtain the desired estimate

$$\sum_{D_h^\varepsilon}U_{i,j}^2h_1h_2\leq c_1\int_0^1\sum_{\Delta_h^\varepsilon}\left(F_{\overset{0}{x}}h_1^2+F_{\overset{0}{y}}h_2^2+\left(\frac{\partial F}{\partial z}\right)^2\left(h_1^2+h_2^2\right)\right)dz.$$

Here c_1 is a constant dependent on the function $\rho(x,y,z)$ and on the family of curves $K(\gamma,z)$. Thus, the proof of Theorem 3.1 is completed. □

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