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COMMON FIXED POINT THEOREM FOR FOUR MAPPINGS IN NON-ARCHIMEDEAN PM-SPACES

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ABSTRACT. We define the concept of weakly f-compatible pair (f,S) in non-Archimedean Menger probabilistic metric spaces and obtain a common fixed point theorem for four maps which improves a theorem of Y.J.Cho.et.al.

Introduction

Recently Y.J.Cho et.al [4] introduced the concepts of compatible mappings and compatible mappings of type (A) in non-Archimedean Menger probabilistic metric spaces and obtained some common fixed point theorems in the space. In this paper we prove a common fixed point theorem which generalizes a theorem of Y.J.Cho et.al [4] by introducing the notion of weakly compatible pair of mappings in non -Archimedean PM-Space. For terminologies, notations and properties of probabilistic metric spaces, refer to [1], [2], [3] and [4].
DEFINITION 1: A distribution function is a mapping F: $IR^+ \to IR^+$ which is non decreasing and left continuous with inf F = 0 and sup F = 1. We will denote D by the set of all distribution functions.
DEFINITION 2: Let X be any non empty set. An ordered pair $(X, \mathbb{F})$ is called a non-Archimedean probabilistic metric space (briefly a N.A. PM-space) if $\mathbb{F}$ is a mapping from $X \times X$ into D satisfying the following conditions (We shall denote the distribution function $\mathbb{F}(x, y)$ by $F(x, y)$ for all $x, y \in X$):
(2.1) $F(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,

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(2.2) $F(x,y) = F(y,x)$,
(2.3) $F(x,y,0) = 0$,
(2.4) If $F(x,y,t_1) = 1$ and $F(y,z,t_2) = 1$ then $F(x,y,max\{t_1,t_2\}) = 1$.
DEFINITION 3: A t-norm is a function Δ: [0,1] $\times$[0,1] $\rightarrow$[0,1] satisfying the following conditions:
(3.1) $\Delta(a,b) \geq \Delta(c,d)$ for a $\geq$c, b $\geq$d ,
(3.2) $\Delta(a,b)$=$\Delta(b,a)$
(3.3) $\Delta(a,1) = a$,
(3.4) $\Delta(\Delta(a,b),c)$= $\Delta(a,\Delta(b,c)$
DEFINITION 4: A non-Archimedean Menger PM-space is an ordered triplet $(X,\mathbb{F},\Delta)$ where Δ is t-norm and $(X,\mathbb{F})$ is a non-Archimedean PM-space satisfying the following condition:
(4.1)$F(x,z,max\{t_1,t_2\}) \geq \Delta(F(x,y,t_1),F(y,z,t_2))$ for all $x,y,z \in X$ and t_1,$t_2 \geq 0$.
DEFINITION 5: A PM-space $(X,\mathbb{F})$ is said to be type $(C)_g$ if there exists a $g \in \Omega$ such that
(5.1) $g(F(x,y,t)) \leq g(F(x,z,t)) + g(F(z,y,t))$ for all $x,y,z \in X$ and $t \geq 0$ where $\Omega = \{g/g : [0,1] \rightarrow [0,\infty)$ is continuous, strictly decreasing, g(1)=0 $\}$.
DEFINITION 6: A non-Archimedean Menger PM-space $(X,\mathbb{F},\Delta)$ is said to be type $(D)_g$ if there exists a $g \in \Omega$ such that
(6.1) $g(\Delta(s,t)) \leq g(s) + g(t)$ for all $s,t \in [0,1]$.
Note : If a N.A. PM-space $(X,\mathbb{F},\Delta)$ is of type $(D)_g$ then it is of type $(C)_g$. Throughout this paper, let $(X,\mathbb{F},\Delta)$ be a N.A. PM-space of type $(D)_g$ with a continuous strictly increasing t-norm Δ. Here afterwards we denote $g(F(x,y,t))$ by $\theta(x,y,t)$.
DEFINITION 7: Let $f,S : X \rightarrow X$ be mappings. The pair (f,S) is said to be partially commuting (or coincidentally commuting or weak-compatible) at z if $fz = Sz$ provided there exists $w \in X$ such that $fw = Sw = z$.
DEFINITION 8 ([4]) : Let $f,S : X \rightarrow X$ be mappings. f and S are said to be compatible if $\lim_{n\to\infty} \theta(\mathrm{fSx_n},\mathrm{Sfx_n},\mathrm{t}) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} \mathrm{fx_n} = \lim_{n\to\infty} \mathrm{Sx_n}$ for some $z \in X$.
DEFINITION 9 ([4]) : Let $f,S : X \rightarrow X$ be mappings. f and S are said to be compatible of type(A)if $\lim_{n\to\infty} \theta(\mathrm{fSx_n},\mathrm{SS_n},\mathrm{t}) = 0$ and $\lim_{n\to\infty} \theta(\mathrm{Sfx_n},\mathrm{ffx_n},\mathrm{t}) = 0$ for all $t > 0$, whenever $\{x_n\}$ is asequence in X such that $\lim_{n\to\infty} \mathrm{fx_n} = \lim_{n\to\infty} \mathrm{Sx_n}$ for some $z \in X$.
Now we give the following definition.
DEFINITION 10: Let $f,S : X \rightarrow X$ be mappings. The ordered pair (f,S) is said to be weakly f-compatible at z if either $\lim_{n\to\infty} \theta(Sfx_n,fz,t) = 0$ or $\lim_{n\to\infty} \theta(SSx_n,fz,t) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = z$ and $\lim_{n\to\infty} fSx_n = \lim_{n\to\infty} ffx_n = fz$ for some $z \in X$.

REMARK 11: (i) If (f, S) is weakly f-compatible at z then it is partially commuting at z.

(ii) If f and S are compatible or compatible of type (A) then the ordered pair (f, S) is weakly f-compatible. The converse need not be true in view of the following example in metric space.

EXAMPLE 12: Let X = [0,1] with usual metric d. Define $f, S : X \to X$ by $fx = 1 - x$ and

$$Sx = \begin{cases} x & \text{if} \quad 0 \leq x \leq 1/2, \\ 1 & \text{if} \quad 1/2 < x \leq 1. \end{cases}$$

Let $\{x_n\}$ be a sequence in X such that $x_n < 1/2 \ \forall n$ and $x_n \to 1/2$.
Then $fx_n = 1 - x_n \to 1/2$ and $Sx_n = x_n \to 1/2$.
Also $fSx_n = 1 - x_n \to 1/2 = f(1/2), ffx_n = x_n \to 1/2 = f(1/2)$,
$Sfx_n = 1, SSx_n = x_n \to 1/2$.
Clearly (f, S) is weakly f-compatible at $1/2$.
Since $d(fSx_n, Sfx_n) = x_n \to 1/2$, it follows that f and S are not compatible.
Since $d(Sfx_n, ffx_n) = 1 - x_n \to 1/2$, it follows that f and S are not compatible of type (A).
We need the following Lemma.

LEMMA 13(Lemma 1.2.of Cho.et.al.[4]): Let $\{y_n\}$ be a sequence in X such that $F(y_n, y_{n+1}, t)$= 1 for all $t > 0$. If the sequence $\{y_n\}$ is not a Cauchy sequence in X, then there exist $\varepsilon_0 > 0$, $t_0 > 0$, two sequences $\{m_k\}$, $\{n_k\}$ of positive integers such that
(13.1) $m_k > n_k + 1$ and $n_k \to \infty$ as $k \to \infty$,
(13.2) $F(y_{m_k}, y_{n_k}, t_0) < 1 - \varepsilon_0$ and $F(y_{m_k-1}, y_{n_k}, t_0) \geq 1 - \varepsilon_0$, k = 1,2, ...

Main Theorem:

THEOREM 14: Let A, B, S and T be self maps on X satisfying
(14.1)$\theta(Ax, By, t) \leq \Psi(\theta(Sx, Ty, t))$ for all $t > 0$ and for all $x, y \in X$ with $Ax = Ty$ or $By = Sx$ and
(14.2) $\theta(Ax, By, t) \leq$

$$\leq \Psi(max\{\theta(Sx, Ty, t) + \theta(Ax, Sx, t) + \theta(By, Ty, t), \theta(Ax, Sx, t) + \theta(Sx, By, t), \theta(By, Ty, t) + \theta(Ax, Ty, t)\})$$

for all $t > 0$ and for all $x, y \in X$, where $\Psi : IR^+ \to IR^+$ is monotonically increasing and $\Psi(t+) < t$ for all $t > 0$.
Suppose that for some $x_0 \in X$, there exists a sequence $\{x_n\}$ in X such that $Ax_{2n} = Tx_{2n+1}(= y_{2n}, say)$ and $Bx_{2n+1} = Sx_{2n+2}(= y_{2n+1}, say)$ for n =0,1,.. Then $\{y_n\}$ is a Cauchy sequence in X.
Further assume that $\{y_n\}$ converges to some $z \in X$. Then z is the unique common fixed point of A, B, S and T if one of the following statements is true.

(i) (A, S) is A-continuous at z and (A, S) is weakly A-compatible at z, (B, T) is partially commuting at z, $Az \in T(X)$ and $Bz \in S(X)$.

(ii) (B, T) is B-continuous at z and (B, T) is weakly B-compatible at z, (A, S)

is partially commuting at z, $Az \in T(X)$ and $Bz \in S(X)$.
(iii) (A,S) is S-continuous at z and (A,S) is weakly S-compatible at z, (B,T)
is partially commuting at z and $Az \in T(X)$.
(iv) (B,T) is T-continuous at z and (B,T) is weakly T-compatible at z, (A,S)
is partially commuting at z and $Bz \in S(X)$.
PROOF: Since $Ax_{2n} = Tx_{2n+1}$ from (14.1) we have
$\theta(y_{2n}, y_{2n+1}, t) = \theta(Ax_{2n}, Bx_{2n+1}, t) \leq \Psi(\theta(y_{2n-1}, y_{2n}, t))$.
Since $Sx_{2n} = Bx_{2n-1}$ from (14.1) we have
$\theta(y_{2n}, y_{2n-1}, t) = \theta(Ax_{2n}, Bx_{2n-1}, t) \leq \Psi(\theta(y_{2n-1}, y_{2n-2}, t))$.
Thus $\theta(y_n, y_{n+1}, t) \leq \Psi(\theta(y_{n-1}, y_n, t))$ for n = 1,2,..
Hence $\theta(y_n, y_{n+1}, t) \leq \Psi^n(\theta(y_0, y_1, t))$ for n = 1,2,..
Since Ψ is monotonically increasing and $\Psi(t+) < t$ for all $t > 0$ it follows that $\Psi^n(t) \to 0$ as $n \to \infty$ for any $t > 0$. Hence
(I) $\theta(y_n, y_{n+1}, t) \to 0$ as $n \to \infty$.
Suppose $\{y_n\}$ is not a Cauchy sequence. Since g is strictly decreasing, by Lemma (13), there exist $\varepsilon_0 > 0$, $t_0 > 0$ and two sequences $\{m_k\}, \{n_k\}$ of positive integers such that
(a)$m_k > n_k + 1$ and $n_k \to \infty$ as $k \to \infty$,
(b)$\theta(y_{m_k}, y_{n_k}, t_0)>$ g$(1-\varepsilon_0)$ and $\theta(y_{m_k-1}, y_{n_k}, t_0) \leq$ g$(1-\varepsilon_0)$ for k = 1,2,...
Now

$$g(1-\varepsilon_0) < \theta(y_{m_k}, y_{n_k}, t_0 0) \leq$$
$$\leq \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{m_k-1}, y_{n_k}, t_0) \leq \theta(y_{m_k}, y_{m_k-1}, t_0) + g(1-\varepsilon_0).$$

Letting $k \to \infty$ we get
(II) $\lim\limits_{n\to\infty} \theta(y_{m_k}, y_{n_k}, t_0) = g(1-\varepsilon_0)$
On the otherhand, we have
(III) $g(1-\varepsilon_0) < \theta(y_{m_k}, y_{n_k}, t_0) \leq \theta(y_{m_k}, y_{n_k+1}, t_0) + \theta(y_{n_k+1}, y_{n_k}, t_0)$
Without loss of generality assume that both m_k and n_k are even.

$$\begin{aligned}
\theta(y_{m_k}, y_{n_k+1}, t_0) &= \theta(Ax_{m_k}, Bx_{n_k+1}, t_0) \\
&\leq \Psi(\max\{\theta(y_{m_k-1}, y_{n_k}, t_0) + \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{n_k+1}, y_{n_k}, t_0), \\
&\qquad \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{m_k-1}, y_{n_k+1}, t_0), \\
&\qquad \theta(y_{n_k+1}, y_{n_k}, t_0) + \theta(y_{m_k}, y_{n_k}, t_0)\}) \\
&\leq \Psi(\max\{g(1-\varepsilon_0) + \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{n_k+1}, y_{n_k}, t_0), \\
&\qquad \theta(y_{m_k}, y_{m_k-1}, t_0) + g(1-\varepsilon_0) + \theta(y_{n_k}, y_{n_k+1}, t_0), \\
&\qquad \theta(y_{n_k+1}, y_{n_k}, t_0) + \theta(y_{m_k}, y_{n_k}, t_0)\})
\end{aligned}$$

Substituting this in (III), letting $k \to \infty$and using (I),(II)
we get $g(1-\varepsilon_0) \leq \Psi(g(1-\varepsilon_0)) < g(1-\varepsilon_0)$ which is a contradiction.
Hence $\{y_n\}$ is a Cauchy sequence in X.
Further assume that $\{y_n\}$ converges to some $z \in X$.
(i) Suppose that the statement (i) is true.
Since $\{Ax_{2n}\}$ and $\{Sx_{2n}\}$ converge to z and (A,S) is A-continuous at z we have $\{AAx_{2n}\}$ and $\{ASx_{2n}\}$ converge to Az.

Since (A, S) is weakly A-compatible at z we have either $\{SAx_{2n}\}$ or $\{SSx_{2n}\}$ converge to Az.
Case :- Suppose $\{SAx_{2n}\}$ converges to Az.

$\theta(AAx_{2n}, Bx_{2n+1}, t) \leq \Psi(max\{\theta(SAx_{2n}, Tx_{2n+1}, t) + \theta(AAx_{2n}, SAx_{2n}, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(AAx_{2n}, SAx_{2n}, t) + \theta(SAx_{2n}, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(AAx_{2n}, Tx_{2n+1}, t)\})$.

Letting $n \to \infty$ we get

$\theta(Az, z, t) \leq \Psi(max\{\theta(Az, z, t) + \theta(Az, Az, t) + \theta(z, z, t), \theta(Az, Az, t) + +\theta(Az, z, t), \theta(z, z, t) + \theta(Az, z, t)\})$

Case:- Suppose $\{SSx_{2n}\}$ converges to Az.

$\theta(ASx_{2n}, Bx_{2n+1}, t) \leq \Psi(max\{\theta(SSx_{2n}, Tx_{2n+1}, t) + \theta(ASx_{2n}, SSx_{2n}, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(ASx_{2n}, SSx_{2n}, t) + \theta(SSx_{2n}, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(ASx_{2n}, Tx_{2n+1}, t)\})$.

Letting $n \to \infty$ we get

$\theta(Az, z, t) \leq \Psi(max\{\theta(Az, z, t) + \theta(Az, Az, t) + \theta(z, z, t), \theta(Az, Az, t) + \theta(Az, z, t), \theta(z, z, t) + \theta(Az, z, t)\})$
$= \Psi(\theta(Az, z, t))$ which implies that $Az = z$.

Since $z = Az =\in T(X)$, there exists $w \in X$ such that $z = Tw$.

$\theta(Ax_{2n}, Bw, t) \leq \Psi(max\{\theta(Sx_{2n}, Tw, t) + \theta(Ax_{2n}, Sx_{2n}, t) + \theta(Bw, Tw, t), \theta(Ax_{2n}, Sx_{2n}, t) + \theta(Sx_{2n}, Bw, t), \theta(Bw, Tw, t) + \theta(Ax_{2n}, Tw, t)\})$

Letting $n \to \infty$ we get

$\theta(z, Bw, t) \leq \Psi(max\{\theta(z, z, t)+\theta(z, z, t)+\theta(Bw, z, t), \theta(z, z, t)+\theta(z, Bw, t), \theta(Bw, z, t) + \theta(z, z, t)\})$
$= \Psi(\theta(z, Bw, t))$ which implies that $Bw = z$.

Since (B, T) is partially commuting at z and $Bw = Tw = z$. We have $Bz = Tz$.

$\theta(Ax_{2n}, Bz, t) \leq \Psi(max\{\theta(Sx_{2n}, Tz, t) + \theta(Ax_{2n}, Sx_{2n}, t) + \theta(Bz, Tz, t), \theta(Ax_{2n}, Sx_{2n}, t) + \theta(Sx_{2n}, Bz, t), \theta(Bz, Tz, t) + \theta(Ax_{2n}, Tz, t)\})$.

Letting $n \to \infty$ we get

$\theta(z, Bz, t) \leq \Psi(max\{\theta(z, Bz, t)+\theta(z, z, t)+\theta(Bz, Bz, t), \theta(z, z, t)+\theta(z, Bz, t), \theta(Bz, Bz, t) + \theta(z, Bz, t)\})$
$= \Psi(\theta(z, Bz, t))$ which implies that $Bz = z$.

Thus $Bz = z = Tz$.

Now $z = Bz \in S(X)$, there exists $v \in X$ such that $Sv = z$.

$\theta(Av, Bx_{2n+1}, t) \leq \Psi(max\{\theta(Sv, Tx_{2n+1}, t)+ +\theta(Av, Sv, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(Av, Sv, t)+ +\theta(Sv, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(Av, Tx_{2n+1}, t)\})$.

Letting $n \to \infty$ we get

$\theta(Av, z, t) \leq \Psi(max\{\theta(z, z, t) + \theta(Av, z, t) + \theta(z, z, t), \theta(Av, z, t) + \theta(z, z, t), \theta(z, z, t) + \theta(Av, z, t)\})$
$= \Psi(\theta(Av, z, t))$ which implies that $Av = z$.

Thus $Av = Sv = z$.

Since (A, S) is weakly A-compatible at z it is partially commuting at z.

Hence $Az = Sz$ so that $z = Az = Sz$.
Thus z is a common fixed point of A, B, S and T.
Uniqueness of common fixed point follows easily from (14.2).
(ii) Proof follows as in (i).
(iii) Suppose the statement (iii) is true.
Since $\{Ax_{2n}\} and \{Sx_{2n}\}$ converge to z and (A, S) is S-continuous at z we have $\{SAx_{2n}\}$ and $\{SSx_{2n}\}$ converge to Sz.
Since (A, S) is weakly S-compatible at z it follows that $\{ASx_{2n}\}$ or $\{AAx_{2n}\}$ converges to Sz.
Case:- Suppose $\{ASx_{2n}\}$ converges to Sz.

$$\theta(ASx_{2n}, Bx_{2n+1}, t) \leq \Psi(max\{\theta(SSx_{2n}, Tx_{2n+1}, t) + \theta(ASx_{2n}, SSx_{2n}, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(ASx_{2n}, SSx_{2n}, t) + \theta(SSx_{2n}, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(ASx_{2n}, Tx_{2n+1}, t)\})$$

Letting $n \to \infty$ we get

$$\theta(Sz, z, t) \leq \Psi(max\{\theta(Sz, z, t)+\theta(Sz, Sz, t)+\theta(z, z, t), \theta(Sz, Sz, t)+\theta(Sz, z, t), \theta(z, z, t) + \theta(Sz, z, t)\})$$
$$= \Psi(\theta(Sz, z, t)) \text{ which implies that } Sz = z.$$

Case:- Suppose $\{AAx_{2n}\}$ converges to Sz.

$$\theta(AAx_{2n}, Bx_{2n+1}, t) \leq \Psi(max\{\theta(SAx_{2n}, Tx_{2n+1}, t) + \theta(AAx_{2n}, SAx_{2n}, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(AAx_{2n}, SAx_{2n}, t) + \theta(SAx_{2n}, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t)+\theta(AAx_{2n}, Tx_{2n+1}, t)\}).$$

Letting $n \to \infty$ we get

$$\theta(Sz, z, t) \leq \Psi(max\{\theta(Sz, z, t)+\theta(Sz, Sz, t)+\theta(z, z, t), \theta(Sz, Sz, t)+\theta(Sz, z, t), \theta(z, z, t) + \theta(Sz, z, t)\})$$
$$= \Psi(\theta(Sz, z, t)) \text{ which implies that } Sz = z.$$

Now

$$\theta(Az, Bx_{2n+1}, t) \leq \Psi(max\{\theta(Sz, Tx_{2n+1}, t) + \theta(Az, Sz, t)+ +\theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(Az, Sz, t) + \theta(Sz, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(Az, Tx_{2n+1}, t)\}).$$

Letting $n \to \infty$ we get

$$\theta(Az, z, t) \leq \Psi(max\{\theta(z, z, t) + \theta(Az, z, t) + \theta(z, z, t), \theta(Az, z, t) + \theta(z, z, t), \theta(z, z, t) + \theta(Az, z, t)\})$$
$$= \Psi(\theta(Az, z, t)) \text{ which implies that } Az = z.$$

Since $z = Az \in T(X)$ and (B, T) is partially commuting at z it follows as in (i) that $Bz = Tz = z$.
Thus z is a common fixed point of A,B,S and T.
(iv) Proof follows as in (iii).
Theorem 14 is an improvement of the following theorem.
THEOREM 15 (Theorem 3.2 of [4]) : Let $A, B, S, T : X \to X$ be mappings satisfying
(15.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,
(15.2) $\theta(Ax, By, z) \leq \Psi(max\{\theta(Sx, Ty, t), \theta(Ax, Sx, t), \theta(By, Ty, t), 1/2[\theta(Sx, By, t) + \theta(Ty, Ax, t)]\})$

for all $t > 0$ and for all $x, y \in X$ where $\Psi : IR^+ \to IR^+$ is upper semi continuous from the right and $\Psi(t) < t$ for all $t > 0$.
(15.3) S or T is continuous,
(15.4) the pairs (A, S) and (B, T) are compatible of type (A).
Then A, B, S and T have a unique common fixed point in X.

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