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# Minimal structures, generalized topologies, and ascending systems should not be studied without generalized uniformities

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ABSTRACT. By using the Pervin relations, we show that all minimal structures, generalized topologies, and ascending systems can be naturally derived from generalized uniformities. Therefore, they need not be studied separately.

## 0. Introduction

Let $X$ be a set and $\mathcal{P}(X)$ be the family of all subsets of $X$. A subfamily $\mathcal{A}$ of $\mathcal{P}(X)$ is called a minimal structure on $X$ if $\emptyset \in \mathcal{A}$ and $X \in \mathcal{A}$. Minimal structures have been mainly studied by Popa and Noiri [32].

In particular, a minimal structure $\mathcal{A}$ on $X$ is called a generalized topology on $X$ if it is closed under unions in the sense that $\mathcal{B} \subset \mathcal{A}$ implies

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$\bigcup \mathcal{B} \in \mathcal{A}$. Generalized topologies have mainly been studied by Császár [2].

Moreover, a subfamily $\mathcal{A}$ of $\mathcal{P}(X)$ is called a stack on $X$ if it is ascending in the sense that $A \in \mathcal{A}$ and $A \subset B \subset X$ imply $B \in \mathcal{A}$. Stacks, as a common generalization of filters and grills, have been mainly studied by Thron [44].

By using the Pervin [30] relations $R_A = A^2 \cup A^c \times X$, with $A \subset X$, we shall show that all minimal structures, generalized topologies, and proper stacks on $X$ can be naturally derived from generalized uniformities. Therefore, they need not be studied separately.

## 1. A few basic facts on relations

Let $X$ be a set and $X^2 = X \times X$. A subset $R$ of $X^2$ is called a relation on $X$. In particular, $\Delta_X = \{(x, x) : \ x \in X\}$ is called the identity relation on $X$.

For any $x \in X$ and $A \subset X$, the sets $R(x) = \{y \in X : \ (x, y) \in R\}$ and $R[A] = \bigcup_{a \in A} R(a)$ are called the images of $x$ and $A$ under $R$, respectively.

If $R$ is a relation on $X$, then the values $R(x)$, where $x \in X$, uniquely determine $R$ since $R = \bigcup_{x \in X} \{x\} \times R(x)$. Therefore, the inverse relation $R^{-1}$ can be defined such that $R^{-1}(x) = \{y \in X : \ x \in R(y)\}$ for all $x \in X$.

Moreover, if $R$ and $S$ are relations on $X$, then the composition relation $S \circ R$ can be defined such that $(S \circ R)(x) = S[R(x)]$ for all $x \in X$. Now, in particular, we may briefly write $R^2$ instead of $R \circ R$.

If $R$ is a relation on $X$, then the set $D_R = R^{-1}[X] = \{x \in X : R(x) \neq \emptyset\}$ is called the domain of $R$. If in particular $D_R = X$, then we say that $R$ is a total relation on $X$.

A relation $R$ on $X$ is called reflexive, symmetric and transitive if $\Delta_X \subset R$, $R^{-1} \subset R$ and $R^2 \subset R$, respectively. Now, a reflexive and symmetric (transitive) relation may be called a tolerance (preorder) relation.

To feel the importance of tolerance relations, note that if $d$ is a metric on $X$, then for each $r > 0$ the $r$-sized $d$-surrounding $B_r^d = \{(x, y) \in X^2 : \ d(x, y) < r\}$ is, in general, only a tolerance relation on $X$.

**Definition 1.1.** If $A \subset X$, then the relation

$$R_A = A^2 \cup A^c \times X,$$

where $A^c = X \setminus A$, will be called the Pervin relation on $X$ generated by $A$.

**Remark 1.2.** These relations are actually particular cases of the relations $R_{(A,B)} = A\times B \cup A^c\times X$, with $A \subset B \subset X$, considered first by Császár [1, p. 42] in somewhat different forms.

However, their importance could become apparent only with the quasi-uniformization theorem of topological spaces by Pervin [30]. For some closely related results, see also Davis [6], who also used the same relations.

Concerning the relations $R_A$, we can easily prove the following theorems.

**Theorem 1.3.** *If $A \subset X$, then*

$$R_A[B] = A \textit{ if } \emptyset \neq B \subset A \quad \textit{and} \quad R_A[B] = X \textit{ if } A \not\supset B \subset X.$$

*Proof.* Note that $R_A[B] = \bigcup_{x\in B} R_A(x)$, and moreover

$$R_A(x) = A \quad \text{if} \quad x \in A \qquad \text{and} \qquad R_A(x) = X \quad \text{if} \quad x \in A^c.$$

Hence, the required assertions are quite obvious.

**Theorem 1.4.** *If $A \subset X$, then $R_A$ is a preorder relation on $X$ such that $R_A^{-1} = R_{A^c}$.*

*Proof.* It is clear that $R_A$ is reflexive. Moreover, we can easily see that

$$\begin{aligned} R_A^2(x) = R_A[R_A(x)] &= \begin{cases} R_A[A] & \text{if} \quad x \in A \\ R_A[X] & \text{if} \quad x \in A^c \end{cases} \\ &= \begin{cases} A & \text{if} \quad x \in A \\ X & \text{if} \quad x \in A^c \end{cases} = R_A(x) \end{aligned}$$

for all $x \in X$. Therefore, $R_A^2 = R_A$, and thus $R_A$ is transitive.

Furthermore, we can also easily see that

$$R_A^{-1}(x) = \{ y \in X : \quad x \in R_A(y) \} = \begin{cases} X & \text{if} \quad x \in A \\ A^c & \text{if} \quad x \in A^c \end{cases} = R_{A^c}(x)$$

for all $x \in X$. Therefore, $R_A^{-1} = R_{A^c}$ is also true.

**Theorem 1.5.** *If* $A \subset X$, *then for any* $U, V \subset X$, *with* $U \neq \emptyset$ *and* $V \neq X$, *the following assertions are equivalent:*

*(1)* $R_A[U] \subset V$; *(2)* $U \subset A \subset V$.

*Proof.* If $U \not\subset A$, then by Theorem 1.3 we have $R_A[U] = X$. Thus, since $V \neq X$, (1) does not hold. Therefore, (1) implies $U \subset A$.

Moreover, if $U \subset A$, then by Theorem 1.3 and the assumption $U \neq \emptyset$ we have $R_A[U] = A$. Therefore, (1) implies $A \subset V$ too.

On the other hand, if (2) holds, then we evidently have $R_A[U] \subset R_A[A] = A \subset V$. Therefore, (1) also holds.

Now, as an immediate consequence of the latter theorem, we can also state

**Corollary 1.6.** *If* $A \subset X$, *then for any* $V \subset X$, *with* $\emptyset \neq V \neq X$, *we have*

$$R_A[V] = V \iff R_A[V] \subset V \iff V = A.$$

## 2. A few basic facts on relators

A family $\mathcal{R}$ of relations on $X$ is called a relator on $X$. Moreover, the ordered pair $X(\mathcal{R}) = (X, \mathcal{R})$ is called a relator space. For the origins, see [35] and the references therein.

Relator spaces are natural generalizations of ordered sets and uniform spaces [8]. Moreover, all reasonable generalizations of the usual topologal structures can be easily derived from relators according to [36].

For instance, if $\mathcal{R}$ is a relator on $X$, then for any $A, B \subset X$ and $x \in X$ we may naturally write:

(1) $B \in \mathrm{Int}_{\mathcal{R}}(A)$ if $R[B] \subset A$ for some $R \in \mathcal{R}$;

and moreover

(2) $A \in \tau_{\mathcal{R}}$ if $A \in \mathrm{Int}_{\mathcal{R}}(A)$; (3) $x \in \mathrm{int}_{\mathcal{R}}(A)$ if $\{x\} \in \mathrm{Int}_{\mathcal{R}}(A)$;

(4) $A \in \mathcal{T}_{\mathcal{R}}$ if $A \subset \mathrm{int}_{\mathcal{R}}(A)$; (5) $A \in \mathcal{E}_{\mathcal{R}}$ if $\mathrm{int}_{\mathcal{R}}(A) \neq \emptyset$.

The relations $\mathrm{Int}_{\mathcal{R}}$ and $\mathrm{int}_{\mathcal{R}}$ are called the proximal and the topological interiors on $X$ induced by the relator $\mathcal{R}$, respectively. While, the members of the families $\tau_{\mathcal{R}}$, $\mathcal{T}_{\mathcal{R}}$ and $\mathcal{E}_{\mathcal{R}}$ are called the proximally open, the topologically open and the fat subsets of the relator space $X(\mathcal{R})$, respectively.

By using the above definitions, we can easily see that, for any $A \subset X$, we have

(a) $A \in \tau_{\mathcal{R}} \iff \exists\ R \in \mathcal{R}: \quad \forall\ x \in A: \quad R(x) \subset A$;

(b) $A \in \mathcal{T}_{\mathcal{R}} \iff \forall\ x \in A: \quad \exists\ R \in \mathcal{R}: \quad R(x) \subset A$;

(c) $A \in \mathcal{E}_{\mathcal{R}} \iff \exists\ x \in X: \quad \exists\ R \in \mathcal{R}: \quad R(x) \subset A$.

Hence, it is also clear that $\tau_{\mathcal{R}} \subset \mathcal{T}_{\mathcal{R}} \subset \mathcal{E}_{\mathcal{R}} \cup \{\emptyset\}$.

The families $\tau_{\mathcal{R}}$ and $\mathcal{E}_{\mathcal{R}}$ are usually more important tools than $\mathcal{T}_{\mathcal{R}}$. For instance, if $\leq$ is an order relation on $X$, then $\mathcal{T}_{\leq}$ and $\mathcal{E}_{\leq}$ are just the families of all ascending and residual subsets of the ordered set $X(\leq)$, respectively. Moreover, it may occur that $\mathcal{T}_{R} = \{\emptyset, X\}$, but $\mathcal{E}_{R} \neq \{X\}$ for some relation $R$ on $X$.

A relator $\mathcal{R}$ on $X$ may be naturally called total, reflexive, symmetric and transitive if each of its members has the corresponding property. Thus, we may also naturally speak of tolerance and preorder relators. Note that if $d$ is a metric on $X$, then the family $\mathcal{R}_d = \{B_r^d: \quad r > 0\}$ only a tolerance relator on $X$.

**Definition 2.1.** If $\mathcal{A} \subset \mathcal{P}(X)$, then the relator

$$\mathcal{R}_{\mathcal{A}} = \left\{ R_{A}: \quad A \in \mathcal{A} \right\}$$

will be called the Pervin relator on $X$ generated by $\mathcal{A}$.

**Remark 2.2.** In 1961, Pervin [30] proved that if $\mathcal{A}$ is topology on $X$, then $\mathcal{R}_{\mathcal{A}}$ is a subbase for a quasi-uniformity $\mathcal{U}_{\mathcal{A}}$ on $X$ such that $\mathcal{A} = \mathcal{T}_{\mathcal{U}_{\mathcal{A}}}$.

Later, the relationships between $\mathcal{A}$ and $\mathcal{U}_{\mathcal{A}}$ were more fully explored by Levine [13]. The term "Pervin quasi-uniformity" was already used by Murdeshwar and Naimpally [21].

By Theorem 1.4, we evidently have the following

**Theorem 2.3.** *If $\mathcal{A} \subset \mathcal{P}(X)$, then $\mathcal{R}_{\mathcal{A}}$ is a preorder relator on $X$ such that $\mathcal{R}_{\mathcal{A}}^{-1} = \mathcal{R}_{\mathcal{A}^c}$, where now $\mathcal{A}^c = \left\{ A^c: \quad A \in \mathcal{A} \right\}$.*

Moreover, by using Theorems 1.3 and 1.5, we can easily prove the following theorems.

**Theorem 2.4.** *If* $\mathcal{A} \subset \mathcal{P}(X)$, *then*

(1) $\mathcal{A} \subset \tau_{\mathcal{R}_{\mathcal{A}}}$ ; (2) $\mathcal{A} \subset \mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$ ; (3) $\mathcal{A} \setminus \{ \emptyset \} \subset \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$ .

*Proof.* By Theorem 1.3, for any $A \in \mathcal{A}$, we have $R_A[A] = A$, and thus $A \in \operatorname{Int}_{\mathcal{R}_{\mathcal{A}}}(A)$. Therefore, $A \in \tau_{\mathcal{R}_{\mathcal{A}}}$ , and thus (1) is true.

Now, by the inclusions $\tau_{\mathcal{R}_{\mathcal{A}}} \subset \mathcal{T}_{\mathcal{R}_{\mathcal{A}}} \subset \mathcal{E}_{\mathcal{R}_{\mathcal{A}}} \cup \{ \emptyset \}$, it is clear that (2) and (3) are also true.

**Theorem 2.5.** *If* $\mathcal{A} \subset \mathcal{P}(X)$, *then*

(1) $\tau_{\mathcal{R}_{\mathcal{A}}} \setminus \{ \emptyset, X \} \subset \mathcal{A}$;

(2) *if* $V \in \mathcal{T}_{\mathcal{R}_{\mathcal{A}}} \setminus \{ X \}$, *then there exists* $\mathcal{B} \subset \mathcal{A}$ *such that* $V = \bigcup \mathcal{B}$;

(3) *if* $V \in \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$ , *then there exists* $A \in \mathcal{A}$ *such that* $A \subset V$.

*Proof.* If $V \in \tau_{\mathcal{R}_{\mathcal{A}}}$, then there exists $A \in \mathcal{A}$ such that $R_A[V] \subset V$. Hence, if $\emptyset \neq V \neq X$, then by Corollary 1.6 it follows that $V = A$. Therefore, (1) is true.

If $V \in \mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$, then for each $x \in V$ there exists $A_x \in \mathcal{A}$ such that $R_{A_x}(x) \subset V$. Hence, if $V \neq X$, then by Theorem 1.5 it follows that $x \in A_x \subset V$. Therefore, $V = \bigcup_{x \in V} A_x$, and thus (2) is true.

If $V \in \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$ , then there exist $x \in X$ and $A \in \mathcal{A}$ such that $R_A(x) \subset V$. Hence, if $V \neq X$, then by Theorem 1.5 it follows that $x \in A \subset V$. Therefore, (3) is true.

## 3. Applications to generalized topologies and stacks

**Definition 3.1.** If $\mathcal{A} \subset \mathcal{P}(X)$ such that $\emptyset \in \mathcal{A}$ and $X \in \mathcal{A}$, then $\mathcal{A}$ is called a minimal structure on $X$.

**Remark 3.2.** Minimal structures have been mainly studied by Noiri and Popa [24–26, 31–32] with reference to Maki [16]. See also Mocanu [20], and Doignon and Falmagne [7, p. 18].

By the corresponding definitions, we evidently have the following

**Theorem 3.3.** *If* $\mathcal{R}$ *is a nonvoid relator on* $X$, *then* $\tau_{\mathcal{R}}$ *is a minimal structure on* $X$.

Moreover, by using our former results, we can easily prove the following

**Theorem 3.4.** *If* $\mathcal{A} \subset \mathcal{P}(X)$, *then the following assertions are equivalent:*

*(1)* $\mathcal{A}$ *is a minimal structure on* $X$ *;*

*(2)* $\mathcal{A} \neq \emptyset$ *and* $\mathcal{A} = \tau_{\mathcal{R}_{\mathcal{A}}}$ *;*

*(3)* $\mathcal{A} = \tau_{\mathcal{R}}$ *for some nonvoid preorder relator* $\mathcal{R}$ *on* $X$ *;*

*(4)* $\mathcal{A} = \tau_{\mathcal{R}}$ *for some nonvoid relator* $\mathcal{R}$ *on* $X$ *.*

*Proof.* By Theorems 2.4 and 2.5, $\mathcal{A} \subset \tau_{\mathcal{R}_{\mathcal{A}}}$ and $\tau_{\mathcal{R}_{\mathcal{A}}} \setminus \{\emptyset, X\} \subset \mathcal{A}$. Moreover, if (1) holds, then $\{\emptyset, X\} \subset \mathcal{A}$. Therefore, $\tau_{\mathcal{R}_{\mathcal{A}}} \subset \mathcal{A}$, and thus, (2) also holds.

Now, since the implications $(2) \Longrightarrow (3)$ and $(4) \Longrightarrow (1)$ are immediate from Theorems 2.3 and 3.3, and the implication $(3) \Longrightarrow (4)$ trivially holds, the proof is complete.

Now, as an immediate consequence of the latter theorems, we can also state

**Corollary 3.5.** *If* $\emptyset \neq \mathcal{A} \subset \mathcal{P}(X)$, *then* $\tau_{\mathcal{R}_{\mathcal{A}}}$ *is the smallest minimal structure on* $X$ *such that* $\mathcal{A} \subset \tau_{\mathcal{R}_{\mathcal{A}}}$ *.*

*Proof.* By Theorem 3.4, $\tau_{\mathcal{R}_{\mathcal{A}}}$ is a minimal structure on $X$. Moreover, if $\mathcal{B}$ is a minimal structure on $X$ such that $\mathcal{A} \subset \mathcal{B}$, then by using the corresponding definitions and Theorem 3.4, we can easily see that $\mathcal{R}_{\mathcal{A}} \subset \mathcal{R}_{\mathcal{A}}$, and thus $\tau_{\mathcal{R}_{\mathcal{A}}} \subset \tau_{\mathcal{R}_{\mathcal{B}}} = \mathcal{B}$.

**Definition 3.6.** If $\mathcal{A}$ is a minimal structure on $X$ such that $\mathcal{A}$ is closed under arbitrary unions, then $\mathcal{A}$ is called a generalized topology on $X$.

**Remark 3.7.** Generalized topologies have mainly been studied by Császár [2–5]. See also Lugojan [15], Mashhour et al. [19], and Doignon and Falmagne [7, p. 21].

By the corresponding definitions, we evidently have the following

**Theorem 3.8.** *If* $\mathcal{R}$ *is a nonvoid relator on* $X$, *then* $\mathcal{T}_{\mathcal{R}}$ *is a generalized topology on* $X$.

*Proof.* To see that $\mathcal{T}_{\mathcal{R}}$ is closed under unions, suppose that $\mathcal{B} \subset \mathcal{T}_{\mathcal{R}}$, and let $V = \bigcup \mathcal{B}$. Then, for each $x \in V$, there exists $B \in \mathcal{B}$ such that $x \in B$. Moreover, since $B \in \mathcal{T}_{\mathcal{R}}$, there exists $R \in R$ such that $R(x) \subset B$. Hence, since $B \subset V$, it follows that $R(x) \subset V$. This shows that $V \in \mathcal{T}_{\mathcal{R}}$.

Moreover, by using our former results, we can easily prove the following

**Theorem 3.9.** *If* $\mathcal{A} \subset \mathcal{P}(X)$, *then the following assertions are equivalent:*

*(1)* $\mathcal{A}$ *is a generalized topology on* $X$ *;*

*(2)* $\mathcal{A} \neq \emptyset$ *and* $\mathcal{A} = \mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$ *;*

*(3)* $\mathcal{A} = \mathcal{T}_{\mathcal{R}}$ *for some nonvoid preorder relator* $\mathcal{R}$ *on* $X$ *;*

*(4)* $\mathcal{A} = \mathcal{T}_{\mathcal{R}}$ *for some nonvoid relator* $\mathcal{R}$ *on* $X$ *.*

*Proof.* By Theorem 2.4, $\mathcal{A} \subset \mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$. Furthermore, if $V \in \mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$ such that $V \neq X$, then by Theorem 2.5 there exists $\mathcal{B} \subset \mathcal{A}$ such that $V = \bigcup \mathcal{B}$. Moreover, if (1) holds, then $A$ is closed under arbitrary unions. Therefore, $V \in \mathcal{A}$. Hence, since by (1) we also have $X \in \mathcal{A}$, it is clear that $\mathcal{T}_{\mathcal{R}_{\mathcal{A}}} \subset \mathcal{A}$. Thus, (2) also holds.

Now, since the implications $(2) \Longrightarrow (3)$ and $(4) \Longrightarrow (1)$ are immediate from Theorems 2.3 and 3.8, and the implication $(3) \Longrightarrow (4)$ trivially holds, the proof is complete.

Now, analogously to Corollary 3.5, we can also prove

**Corollary 3.10.** *If* $\emptyset \neq \mathcal{A} \subset \mathcal{P}(X)$, *then* $\mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$ *is the smallest generalized topology on* $X$ *such that* $\mathcal{A} \subset \mathcal{T}_{\mathcal{R}_{\mathcal{A}}}$.

**Definition 3.11.** If $\mathcal{A} \subset \mathcal{P}(X)$ such that $\mathcal{A}$ is ascending in $X$, then $\mathcal{A}$ is called a stack on $X$.

**Remark 3.12.** In particular, the stack $\mathcal{A}$ is called proper if $\emptyset \notin \mathcal{A}$, or equivalently $\mathcal{A} \neq \mathcal{P}(X)$.

Stacks, as a common generalization of filters and grills, have been mainly studied by Thron [44] with reference to Schmidt [33] and Grimeisen [10].

By the corresponding definitions, we evidently have the following

**Theorem 3.13.** *If* $\mathcal{R}$ *is a relator on* $X$, *then* $\mathcal{E}_{\mathcal{R}}$ *is a stack on* $X$.

**Remark 3.14.** In addition, we can also easily see that the stack $\mathcal{E}_{\mathcal{R}}$ is proper if and only if the relator $\mathcal{R}$ is total.

Moreover, by using our former results, we can easily prove the following

**Theorem 3.15.** *If* $\mathcal{A} \subset \mathcal{P}(X)$, *then the following assertions are equivalent:*

*(1)* $\mathcal{A}$ *is a proper stack on* $X$ *;*

*(2)* $\mathcal{A} = \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$ *;*

*(3)* $\mathcal{A} = \mathcal{E}_{\mathcal{R}}$ *for some preorder relator* $\mathcal{R}$ *on* $X$ *;*

*(4)* $\mathcal{A} = \mathcal{E}_{\mathcal{R}}$ *for some total relator* $\mathcal{R}$ *on* $X$ *.*

*Proof.* If (1) holds, then $\emptyset \notin \mathcal{A}$. Therefore, by Theorem 2.4, $\mathcal{A} \subset \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$. Furthermore, if $V \in \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$, then by Theorem 2.5 there exists $A \in \mathcal{A}$ such that $A \subset V$. Moreover, if (1) holds, then $A$ is ascending. Therefore, $V \in \mathcal{A}$. Hence, it is clear that $\mathcal{E}_{\mathcal{R}} \subset \mathcal{A}$. Thus, (2) also holds.

Now, since the implications $(2) \Longrightarrow (3)$ and $(4) \Longrightarrow (1)$ are immediate from Theorems 2.3 and 3.13 and Remark 3.14, and the implication $(3) \Longrightarrow (4)$ trivially holds, the proof is complete.

Now, analogously to Corollary 3.5, we can also prove

**Corollary 3.16.** *If* $\mathcal{A} \subset \mathcal{P}(X)$, *then* $\mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$ *is the smallest proper stack on* $X$ *such that* $\mathcal{A} \subset \mathcal{E}_{\mathcal{R}_{\mathcal{A}}}$.

**Remark 3.17.** The above theorems show that minimal structures, generalized topologies, and proper stacks need not be studied without relators, which are evidently more convenient means than the former ones.

These facts, as some immediate consequences of some more general results, have also been established in our former paper [36], which seems to have been completely ignored by the mathematical community, including the editorial boards of reviewing journals.

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