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ON FUZZY GENERALIZED α-CLOSED SET AND ITS APPLICATIONS

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Abstract

In this paper, we define and study fuzzy generalized α-closed sets and r-open sets of a given L-fuzzy topological space and prime element $r \in P(L)$ and coprime element $\alpha \in M(L)$. The concept of L-fuzzy r-open sets was introduced in [10], and it was proved that all r-open sets for L-fuzzy topological space form a new L-fuzzy topology, which is called stratiform L-fuzzy topology. Making use of the fuzzy generalized α-closed sets, fuzzy generalized α-continuous map is presented.

1 Introduction

In this paper, we study fuzzy generalized α-closed sets, fuzzy r-open sets, fuzzy α-continuous functions and their applications. In section 2 we give preliminaries and some definitions. Section 3 is devoted to studying generalized fuzzy α-closed sets and their properties. Section 4 is devoted to studying generalized fuzzy α-continuous mappings and their properties. In section 5 we introduce fuzzy gαc-irresolute maps and their properties.

2 Preliminaries

Throughout this paper, $L = (\leq, \vee, \wedge, \prime)$ always denotes a fuzzy lattice, and 0 and 1 are the smallest and the greatest element of L, respectively. By a fuzzy lattice we mean a completely distributive lattice L, if L has an order-reversing involution $\prime : L \to L$. Let X be a nonempty crisp set. A mapping

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from X into L is said to be an L-fuzzy set on X. The collection of all L-fuzzy sets on X, denoted by L^X, can be naturally seen as a fuzzy lattice $(L^X, \leq, \vee, \wedge, \prime)$. For $\alpha \in L, \alpha_X$ denotes a constant value L-fuzzy set on X, i.e., $\alpha_X(x) \equiv \alpha, \forall x \in X$.

Definition 2.1 Let X be a nonempty ordinary set, L a fuzzy lattice, $\delta \subset L^X$. δ is called a L-fuzzy topology on X, and (L^X, δ) is called an L-fuzzy topological space, or L-fts for short, if δ satisfies the following three condotions:
(LFT1) $\underline{0}, \underline{1} \in \delta$;
(LFT2) $\forall \mathcal{A} \subset \delta, \bigvee \mathcal{A} \in \delta$;
(LFT3) $\forall U, V \in \delta, U \wedge V \in \delta$.

Particularly, when $L = [0, 1]$, call an L-fuzzy topological space (L^X, δ) a $F - topological\ space$ or a F-ts for short, and simply denote it by (X, δ).

Each fuzzy mapping $f\ : L^X \to L^Y$ considered in this paper is induced from a crisp mapping $f\ : X \to Y$ as usual, i.e., for $A \in L^X, B \in L^Y, x \in X, y \in Y$,

$$f(A)(y) = \bigvee \{A(x) :\ x \in X, f(x) = y\},$$
$$f^{-1}(B)(x) = B(f(x)).$$

Definition 2.2 Let L be a fuzzy lattice. $\alpha \in L$ is called a union-irreducible element (or a molecule [6])of L, if for arbitrary $a, b \in L$ we have $\alpha \leq a \vee b \Rightarrow \alpha \leq a$ or $\alpha \leq b$. The set of all the nonzero union- irreducible elements of L is denoted by $M(L)$. The set of all molecules of a fuzzy lattice L^X is denoted by $M^*(L^X)$.

Definition 2.3 Let L be a fuzzy lattice. $r \in L$ is called a prime element of L, if for arbitrary $a, b \in L$ we have $a \wedge b \leq r \Rightarrow a \leq r$ or $b \leq r$. The set of all the prime elements which are not 1 of L is denoted by $P(L)$. Clearly, $r \in P(L)$ iff $r' \in M(L)$.

Definition 2.4 Let (L^X, δ) be an L-fts, and $r \in P(L)$, and $A \in L^X$. A is called an r-open set, if for any $x \in X$, $A^\circ(x) \leq r \Rightarrow A(x) \leq r$ [11].

The set of all r-open sets in (L^X, δ) is denoted by $O_r(\delta)$. Clearly, $\forall r \in P(L), \delta \subset O_r(\delta)$.

Definition 2.5 Let (L^X, δ) be an L-fts, and $\alpha \in M(L)$, and $A \in L^X$. A is called an α-closed set, if for any $x \in X$, $\overline{A}(x) \geq \alpha \Rightarrow A(x) \geq \alpha$.

The set of all α-closed sets in (L^X, δ) is denoted by $C_\alpha(\delta)$. Clearly, $\forall \alpha \in M(L), \delta' \subset C_\alpha(\delta)$.

Theorem 2.6 Let (L^X, δ) be an L-fts, $\alpha \in M(L), A \in L^X$. Then A is α-closed iff A' is α'-open [11].

Definition 2.7 Let (L^X, δ) be an L-fts, $\alpha \in M(L), r \in P(L)$. $(L^X, O_r(\delta))$ and $(L^X, C_\alpha(\delta))$ are called stratiform L-fuzzy topological spaces of (L^X, δ).

Theorem 2.8 [10] Let (L^X, δ) be an L-fts, and $A \in L^X$. Then
(1) $A \in \delta$ iff $\forall r \in P(L), A \in O_r(\delta)$.
(2) $A \in \delta'$ iff $\alpha \in M(L), A \in C_\alpha(\delta)$.

It is clear that $A \in C_\alpha(\delta)$, implies $A' \in O_{\alpha'}(\delta)$. Also $A \in O_r(\delta)$, implies $A' \in C_{r'}(\delta)$.

3 Fuzzy generalized α-closed sets in L-fuzzy topological spaces

In this section, we study α-closed set and its properties. After that we want to introduce open r-cover, fuzzy r-compact, fuzzy r-regular and fuzzy α-closed map.

Definition 3.1 If λ is an L-fuzzy set in a L-fts L^X and $\alpha \in M(L)$ then $cl_\alpha(\lambda) = \bigcap\{\mu \ : \ \mu \geq \lambda\}$, μ "is fuzzy α-closed set", is called a fuzzy α-closure of λ.

An L-fuzzy set λ in a L-fts (L^X, δ) is fuzzy α-closed if and only if $\lambda = cl_\alpha(\lambda)$.

Definition 3.2 Let (L^X, δ) be an L-fts, and $\alpha \in M(L)$, and $\lambda \in L^X$. λ is called an fuzzy generalized α-closed set (in short Fgα-closed set), if $cl_\alpha(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy α'-open.

Remark 3.3 Let (L^X, δ) be an L-fts, $\alpha \in M(L)$, and $\lambda \in L^X$. If λ is fuzzy α-closed set then λ' is fuzzy α'-open set.

Theorem 3.4 If λ_1 and λ_2 are Fgα-closed sets then $\lambda_1 \vee \lambda_2$ is a $Fg\alpha$-closed set.

Proof : Follows from the definition of $Fg\alpha$-closed and the fact that $cl_\alpha(\lambda_1 \vee \lambda_2) = cl_\alpha(\lambda_1) \vee cl_\alpha(\lambda_2)$. □

However, the intersection of two $Fg\alpha$-closed sets is not fuzzy generalized α-closed set as the following example shows.

Example 3.5 The intersection of two $Fg\alpha$-closed sets is not generally a $Fg\alpha$-closed set.
Let $X = \{x_1, x_2, x_3\}$. Define $f_1, f_2, f_3 : X \to [0, 1]$ as follows.
$f_1 = 0_X;\ f_2 = 1_X;\ f_3(x_2) = f_3(x_3) = 0, f_3(x_1) = 1$.

Clearly, $\delta = \{f_1, f_2, f_3\}$ is a fuzzy topology on X. Define $\lambda_1, \lambda_2 : X \to [0, 1]$ as follows:
$\lambda_1(x_1) = \lambda_1(x_2) = 1,\ \lambda_1(x_3) = 0.$
$\lambda_2(x_1) = \lambda_2(x_3) = 1,\ \lambda_2(x_2) = 0.$

Since $Cl_\alpha(\lambda_1) \leq 1_X, 1_X \in \delta$ and $\delta \subseteq O_{\alpha'}(\delta)$, it follows that λ_1 is Fgα-closed set. Similarly λ_2 is $Fg\alpha$-closed set but $\lambda_1 \wedge \lambda_2 = f_3$ is not $Fg\alpha$-closed set.

Theorem 3.6 If λ is $Fg\alpha$-closed set and $\lambda \leq \mu \leq \overline{\lambda}$, then μ is $Fg\alpha$-closed set.

Proof : Let β be a fuzzy α'-open set such that $\beta \geq \mu$. Since $\mu \geq \lambda, \beta \geq \lambda$,and λ is Fgα-closed set. It follows that $\beta \geq \overline{\lambda}$. But $\overline{\lambda} \geq \overline{\mu}$, since $\overline{\lambda} \geq \mu$ and so $\beta \geq \overline{\mu}$. Since $\overline{\mu}$ is α-closed set, $cl_\alpha(\mu) \leq \beta$. Hence μ is $Fg\alpha$-closed set.

Definition 3.7 Let $(L^X, O_r(\delta))$ for $r \in P(L)$ be an L-fts, $\lambda \in L^X$, $\mathcal{A} \subset O_r(\delta)$. $\mathcal{A}$ is called an open r-cover of λ, if $\bigvee \mathcal{A} \geq \lambda$; particularly, $\mathcal{A}$ is called an open r-cover of $(L^X, O_r(\delta))$, if $\mathcal{A}$ is an open r-cover of $\underline{1}$.

Definition 3.8 Let $(L^X, O_r(\delta))$ for $r \in P(L)$ be an L-fts, $\lambda \in L^X, \lambda$ is called r-compact, if every open r-cover of λ of $(L^X, O_r(\delta))$ has a finite subcover.

Theorem 3.9 Let $(L^X, O_r(\delta))$ be a fuzzy r-compact (Lindelof, countably r-compact) space and suppose that λ is a Fgr'-closed set of L^X. Then λ is fuzzy r-compact (Lindlof, countably r-compact).

Proof : We prove the case of fuzzy r-compactness as the proof is similar for other cases. Let $\{\mu_t\}_{t\in T}$ be a family of r-open sets in L^X such that $\lambda \leq \bigvee_{t\in T} \mu_t$. Since λ is Fgr'-closed set and $\bigvee_{t\in T} \mu_t$ is fuzzy r-open, it follows that $\overline{\lambda} \leq \bigvee_{t\in T} \mu_t$. But $\overline{\lambda}$ is fuzzy r-compact and therefore it follows that $\lambda \leq \overline{\lambda} \leq \mu_{r_1} \vee \mu_{r_2} \vee \cdots \vee \mu_{r_n}$ for some natural number $n \in N$.

Definition 3.10 Let $(L^X, O_r(\delta))$ be an L-fts and $r \in P(L)$. $(L^X, O_r(\delta))$ is called r-regular, if for $\mu \in O_r(\delta)$ there exsists $\vartheta \subset O_r(\delta)$ such that $\bigvee \vartheta = \mu$ and $\overline{\lambda} \leq \mu$ for every $\lambda \in \vartheta$.

Theorem 3.11 If $(L^X, O_r(\delta)), r \in P(L)$, is fuzzy r-regular and λ is fuzzy r-compact, then λ is Fg**??**-closed set.

Proof : Suppose that $\lambda \leq \mu$, and $\lambda, \mu \in O_r(\delta)$, since $(L^X, O_r(\delta))$ is fuzzy r-regular, we can write $\mu = \bigvee_{t \in T} \mu_t$, T being an index set, $\mu_t \in O_r(\delta), \overline{\mu_t} \leq \mu$. Hence for some finite subset $T_\circ \subset T$ we will have

$$\lambda \leq \bigvee_{t \in T_\circ} \mu_t \leq \overline{\bigvee_{t \in T_\circ} \mu_t} \leq \mu.$$

This shows that $\overline{\lambda} \leq \mu$. Since $\overline{\lambda} \in C_{r'}(\delta)$, thus $\overline{\lambda} = cl_{r'}(\lambda) \leq \mu$ i.e. λ is Fgr'-closed set. □

Definition 3.12 An L-fuzzy set λ is called fuzzy generalized α'-open (in short Fgα'-open) iff λ' is Fgα-closed set for $\alpha \in M(L)$.

We shall now prove some properties of fuzzy generalized r-open sets.

Definition 3.13 If λ is an L-fuzzy set in a L-fts (L^X, δ) and $r \in P(L)$, then $rInt(\lambda) = Int_r(\lambda) = \bigcup\{\mu \ : \ \lambda \geq \mu\}$, μ is fuzzy r-open set, is called a fuzzy r-Interior of λ.

An L-fuzzy set λ in a L-fts (L^X, δ) is fuzzy r-open if and only if $\lambda = Int_r(\lambda)$.

Theorem 3.14 Let $\alpha \in M(L)$. An L-fuzzy set λ is Fgα'-open set $\Leftrightarrow$ $\mu \leq Int_{\alpha'}(\lambda)$ whenever μ is fuzzy α-closed set and $\mu \leq \lambda$.

Proof : Let λ be a Fgα'-open set and μ be a fuzzy α-closed set such that $\mu \leq \lambda$. Now $\mu \leq \lambda \Rightarrow \mu' \geq \lambda'$ and λ' is $Fg\alpha$-closed set, that is, $(\overline{\lambda'})' \geq (\mu')' = \mu$. But $(\overline{\lambda'})' = Int_{\alpha'}(\lambda)$ [3].

Thus we obtain that $\mu \leq Int_{\alpha'}(\lambda)$.

Conversly, suppose that λ is a fuzzy set such that $\mu \leq Int_{\alpha'}(\lambda)$ whenever μ is fuzzy α-closed and $\mu \leq \lambda$. We claim λ' is Fgα-closed set. So let $\lambda' \leq \mu$ where μ is fuzzy α'-open.

Now $\lambda' \leq \mu$, then $\mu' \leq \lambda$. Hence, by assumption we must have :

$\mu' \leq Int_{\alpha'}(\lambda)$, i.e. $(Int_{\alpha'}(\lambda))' \leq \mu$. But $(Int_{\alpha'}(\lambda))' = (\overline{\lambda'})$ [3]. Hence $(\overline{\lambda'}) \leq \mu$. This shows λ' is Fgα-closed set. □

Remark 3.15 (1) The union of two Fgα'-open sets is not generally Fgα'-open set.
(2) The intersection of any two Fgα'-open sets is Fgα'-open set.

Theorem 3.16 Let $\alpha \in M(L)$. If $Int_{\alpha'}(\lambda) \leq \mu \leq \lambda$ and λ is Fgα'-open set, then μ is Fgα'-open set.

Proof : Given $Int_{\alpha'}(\lambda) \leq \mu \leq \lambda$, we have $\lambda' \leq \mu' \leq (Int_{\alpha'}(\lambda))' = \overline{\lambda'}$. Since λ is Fgα'-open, λ' is Fgα-closed and so it follows by Theorem (3.6) that μ' is Fgα-closed set, i.e. μ is Fgα'-open set. □

Definition 3.17 A map $f : L^X \rightarrow L^Y$ is called fuzzy α-closed (in short $F\alpha$-closed) if the image of every α-closed set in L^X is α-closed set in L^Y.

Theorem 3.18 If λ is a Fgα-closed set in L^X and if $f : L^X \rightarrow L^Y$ is F-continuous and Fα-closed, then $f(\lambda)$ is Fgα-closed set in L^Y.

Proof : If $f(\lambda) \leq \mu$ where μ is Fα'-open in L^Y, then $\lambda \leq f^{-1}(\lambda)$. Since λ is Fgα-closed and $f^{-1}(\mu)$ is Fα'-open, $cl_{\alpha}(\lambda) = \overline{\lambda} \leq \overline{f^{-1}}(\mu)$ i.e. $f(\overline{\lambda}) \leq \mu$. Now by assumption, $f(\overline{\lambda})$ is Fα-closed and $\overline{f(\lambda)} \leq \overline{f(\overline{\lambda})} = f(\overline{\lambda}) \leq \mu$. We know that $\overline{f(\lambda)} = cl_{\alpha}(f(\lambda))$, thus $cl_{\alpha}(f(\lambda)) \leq \mu$. This means $f(\lambda)$ is Fgα-closed set. □

Example 3.19 Under Fα-closed, F-continuous maps Fgα'-open sets are generally not taken into Fgα'-open sets.

Let $X = \{a\}, Y = \{b, c\}, T_1 = \{0_X, 1_X\}, T_2 = \{0_Y, 1_Y, f_1\}$ where $f_1 : Y \rightarrow [0, 1]$ is such that :
$f_1(b) = 1 \quad f_1(c) = 0.$

Clearly, T_1 and T_2 are fuzzy topologies on X and Y, respectively. Define $f : X \rightarrow Y$ as follows:
$a \longmapsto f(a) = c.$

One can verfiy that f is F-continuous and Fα-closed. Now we shall show that f does not take Fgα'-open sets to Fgα'-open set.

Clearly, 1_X is Fgα'-open on X. But $f(1_X) = 1 - f_1$ which is not Fgα'-open on Y.

4 Fuzzy generalized α-continuous mapping and its properties

In this section, by means of fuzzy α-closed set and Fgα-closed set, we study fuzzy generalized α-continuous mapping and its properties.

Definition 4.1 Let (L^X, δ) and (L^Y, σ) be two L-fuzzy topological spaces. A map $f : L^X \rightarrow L^Y$ is called fuzzy generalized α-continuous (in short Fgα-continuous) if the invers image of every fuzzy α-closed set in L^Y is Fgα-closed in L^X. The following are the properties of Fgα-continuous functions.

Theorem 4.2 If $f : L^X \to L^Y$ is fuzzy α-continuous then it is Fgα-continuous. The convers of Theorem 4.2 is not true.

Example 4.3 let $X = \{a, b, c\}, Y = \{p, q\}$. Define $\delta_1 = \{0_X, 1_X, \lambda\}$ where $\lambda : X \to [0, 1]$ is such that $\lambda(a) = 1,\ \lambda(b) = 0,\ \lambda(c) = 0$, and $\delta_2 = \{0_Y, 1_Y, \mu\}$ where $\mu : Y \to [0, 1]$ is such that $\mu(p) = 0, \mu(q) = 1$. Also define $f : X \to Y$ as $f(a) = f(c) = q; f(b) = p$. Then f is not fuzzy α-continuous since $f^{-1}(\mu)$ is not in δ_1 for $\mu \in \delta_2$. But however f is Fgα-continuous.

Theorem 4.4 Let $f : (L^X, \delta) \to (L^Y, \sigma)$ be a map. Then the following statements are eqivalent :
(a) f is Fgα-continuous.
(b) The inverse image of each fuzzy α'-open set in L^Y is Fgα'-open in L^X.

We define the fuzzy generalized α-closure operator cl^*_α for any fuzzy set λ in (X, δ) as follows:
$cl^*_\alpha(\lambda) = \bigwedge \{\mu \mid \lambda \leq \mu\}$, μ is Fgα-closed.

Theorem 4.5 Let $f : L^X \to L^Y$ be Fgα-continuous. Then $f[cl^*_\alpha(\lambda)] \leq cl_\alpha[f(\lambda)]$ where λ is any L-fuzzy set in L^X.

Proof : Now $cl_\alpha f(\lambda)$ is a fuzzy α-closed set in L^Y. Since f is Fgα-continuous, $f^{-1}(cl^*_\alpha(\lambda))$ is Fgα-closed in L^X. $\lambda \leq f^{-1}[cl_\alpha f(\lambda)]$ and so $cl^*_\alpha(\lambda) \leq f^{-1}[cl_\alpha f(\lambda)]$. Hence, $f[cl^*_\alpha(\lambda)] \leq cl_\alpha f(\lambda)$. □

The converse of Theorem 4.5 is not true.

Example 4.6 Let $X = \{a, b, c\}$. Define $\delta_1 = \{1_X, 0_X, \lambda\}$ where $\lambda : X \to [0, 1]$ is such that $\lambda(a) = 1; \lambda(b) = \lambda(c) = 0$, $\delta_2 = \{1_X, 0_X, \mu\}$ where $\mu : X \to [0, 1]$ is such that $\mu(a) = \mu(c) = 1, \mu(b) = 0$. Define $f : (X, \delta_1) \to (X, \delta_2)$ as $f(a) = b, f(b) = a, f(c) = c$. Then for any fuzzy set $\lambda, f(cl^*_\alpha(\lambda)) \leq cl_\alpha(f(\lambda))$, but f is not Fgα-continuous. (Since μ' is a fuzzy α-closed set in Y but $f^{-1}(\mu')$ is not Fgα-closed in X.)

Definition 4.7 An L-fuzzy topological space (L^X, δ) is said to be fuzzy $\alpha - T_{1/2}$, if every fuzzy generalized α-closed set in L^X is fuzzy α-closed in L^X.

Theorem 4.8 Let $f : L^X \to L^Y$ and $g : L^Y \to L^Z$ be mappings and L^Y

be fuzzy α-$T_{1/2}$. If f and g are Fgα-continuous, then the composition $g.f$ is Fgα-continuous.

Theorem 4.8 is not valid if L^Y is not fuzzy $\alpha - T_{1/2}$.

Example 4.9 Put $X = \{a, b, c\}$. Define $\delta_1 = \{0_X, 1_X, \lambda\}$ where $\lambda : X \to [0, 1]$ is such that $\lambda(a) = \lambda(c) = 0$ and $\lambda(b) = 1$. $\delta_2 = \{0_X, 1_X, \mu, \nu\}$ where $\mu : X \to [0, 1]$ is such that $\mu(b) = \mu(c) = 1, \mu(a) = 0, \nu : X \to [0, 1]$ is such that $\nu(a) = 1, \nu(b) = \nu(c) = 0$. $\delta_3 = \{0_X, 1_X, \rho\}$ where $\rho : X \to [0, 1]$ is such that $\rho(a) = \rho(c) = 1; \rho(b) = 0$. Also define $f : (X, \delta_1) \to (X, \delta_2)$ as $f(a) = f(c) = c; f(b) = b$ and let $g : (X, \delta_2) \to (X, \delta_3)$ be the identity map. Then f and g are Fgα-continuous but $g.f$ is not Fgα-continuous; since ρ' $\in \delta_3' \subset C_\alpha(\delta_3)$ is α-closed, $g^{-1}(\rho') = \rho'$ and $f^{-1}(g^{-1}(\rho')) = f^{-1}(\rho') = \lambda$ is not Fgα-closed in (X, δ_1). Therefore, $g.f$ is not Fgα-continuous. Further (X, δ_2) is not fuzzy α-$T_{1/2}$.

5 Fuzzy gαc-irresolute maps and their properties

In this section, we use from fuzzy generalized α-closed concept to introduce fuzzy generalized α closed-irresolute map (in short Fgαc-irresolute) map.

Definition 5.1 Let $\alpha \in M(L)$. A map $f : L^X \to L^Y$ is called fuzzy gαc-irresolute, if the inverse image of every Fgα-closed set in L^Y is Fgα-closed in L^X.

The following are the propreties of fuzzy gαc-irresolute maps.

Theorem 5.2 $f : L^X \to L^Y$is fuzzy gαc-irresolute iff the inverse image of every Fgα-open set in L^Y is Fgα-open set in L^X.

Theorem 5.3 If $f : L^X \to L^Y$ is fuzzy gαc-irresolute then it is Fgα-continuous.

The converse of Theorem 5.3 is not true.

Example 5.4 Let $X = \{a, b, c\}$. Define $\delta_1 = \{0_X, 1_X, \lambda_1, \lambda_2, \lambda_3\}$ where $\lambda_1 : X \to [0, 1]$ such that $\lambda_1(a) = 1, \lambda_1(b) = \lambda_1(c) = 0; \lambda_2 : X \to [0, 1]$ such that $\lambda_2(a) = \lambda_2(b) = 0, \lambda_2(c) = 1; \lambda_3 : X \to [0, 1]$ such that $\lambda_3(a) = \lambda_3(c) = 1, \lambda_3(b) = 0$; and also define $\delta_2 = \{0_X, 1_X, \lambda_1\}$.

Define $f : (X, \delta_1) \to (X, \delta_2)$ as follows: $f(a) = a; f(b) = b; f(c) = a$. Then f is Fgα-continuous but f is not fuzzy Fgα-irresolute. For λ_3 is Fgα-

closed in (X, δ_2) but $f^{-1}(\lambda_3) = \lambda_3$ is not Fgα-closed in (X, δ_1).

Theorem 5.5 Suppose $f : L^X \to L^Y,\ g : L^Y \to L^Z$ be maps. Assume f is fuzzy gαc-irresolute and g is Fgα-continuous. Then $g.f$ is Fgα-continuous.

Proof : Let λ be a α-closed set in L^Z, since g is Fgα-continuous, it follows that $g^{-1}(\lambda)$ is a Fgα-closed set in L^Y. Now by assumption, $f^{-1}(g^{-1}(\lambda))$ is a Fgα-closed set in L^X. This show that $g.f$ is Fgα-continuous. □

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