

WEAK AND STRONG FORM OF \tilde{g} -IRRESOLUTE FUNCTIONS

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Abstract

The purpose of this paper is to give two new types of irresolute functions called, completely \tilde{g} -irresolute functions and weakly \tilde{g} -irresolute functions. We obtain their characterizations and their basic properties.

1 Introduction and Preliminaries

Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science. In 1972, Crossley and Hildebrand [3] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences.

Recently, as generalization of closed sets, the notion of \tilde{g} -closed sets were introduced and studied by Jafari et al [8]. This notion was further studied by Rajesh and Ekici [18, 16, 15, 17]. In this paper, we will continue the study of related irresolute functions with \tilde{g} -open sets. We introduce and characterize the concepts of completely \tilde{g} -irresolute and weakly \tilde{g} -irresolute functions.

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X, \tau) \rightarrow (Y, \sigma)$ (or simply $f : X \rightarrow Y$) denotes a function f of a space (X, τ) into a space

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(Y, σ) . Let A be a subset of a space X . The closure and the interior of A are denoted by $\text{Cl}(A)$ and $\text{Int}(A)$, respectively.

Definition 1.1. A subset A of a space (X, τ) is called semi-open [9] (resp. regular open [22]) if $A \subset \text{Cl}(\text{Int}(A))$ (resp. $A = \text{Int}(\text{Cl}(A))$). The complement of semi-open (resp. regular open) set is called semi-closed (resp. regular closed). The family of all regular open (resp. regular closed) subsets of (X, τ) is denoted by $RO(X)$ (resp. $RC(X)$).

The semi-closure [4] of a subset A of X , denoted by $s\text{Cl}(A)$ is defined to be the intersection of all semi-closed sets containing A in X .

Definition 1.2. A subset A of a space X is called:

- (i) \hat{g} -closed [25] if $\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is semi-open in X . The complement of \hat{g} -closed set is called \hat{g} -open.
- (ii) $*g$ -closed [24] if $\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is \hat{g} -open in X . The complement of $*g$ -closed set is called $*g$ -open.
- (iii) $\#g$ -semi-closed [26] if $s\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is $*g$ -open in X . The complement of $\#g$ -semi-closed is called $\#g$ -semi-open.
- (iv) \tilde{g} -closed [8] if $\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is $\#g$ -semi-open in X . The complement of \tilde{g} -closed set is called \tilde{g} -open.

The family of all \tilde{g} -open subsets of (X, τ) is denoted by $\tilde{G}O(X)$. We set $\tilde{G}O(X, x) = \{V \in \tilde{G}O(X) | x \in V\}$ for $x \in X$.

The union (resp. intersection) of all \tilde{g} -open (resp. \tilde{g} -closed) sets, each contained in (resp. containing) a set A in a space X is called the \tilde{g} -interior (resp. \tilde{g} -closure) of A and is denoted by $\tilde{g}\text{-Int}(A)$ (resp. $\tilde{g}\text{-Cl}(A)$) [15].

Note that the family of \tilde{g} -open subsets of (X, τ) forms a topology [8].

Definition 1.3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called:

- (i) strongly continuous [10] (resp. strongly \tilde{g} -continuous [20]) if $f^{-1}(V)$ is both open (resp. \tilde{g} -open) and closed (resp. \tilde{g} -closed) in X for each subset V of Y ;
- (ii) completely continuous [1] if $f^{-1}(V)$ is regular open in X for every open set V of Y ;
- (iii) \tilde{g} -irresolute [18] if $f^{-1}(V)$ is \tilde{g} -closed (resp. \tilde{g} -open) in X for every \tilde{g} -closed (resp. \tilde{g} -open) subset V of Y .

2 Completely \tilde{g} -irresolute functions

Definition 2.1. A function $f : X \rightarrow Y$ is called completely \tilde{g} -irresolute if the inverse image of each \tilde{g} -open subset of Y is regular open in X .

Clearly, every strongly continuous function is completely \tilde{g} -irresolute and every completely \tilde{g} -irresolute function is \tilde{g} -irresolute.

Remark 2.2. The converses of the above implications are not true in general as seen from the following examples.

Example 2.3. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Clearly, the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely \tilde{g} -irresolute but not strongly continuous.

Example 2.4. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is \tilde{g} -irresolute but not completely \tilde{g} -irresolute.

Theorem 2.5. The following statements are equivalent for a function $f : X \rightarrow Y$

- (i) f is completely \tilde{g} -irresolute;
- (ii) $f : (X, \tau) \rightarrow (Y, \tilde{G}O(X))$ is completely continuous;
- (iii) $f^{-1}(F)$ is regular closed in X for every \tilde{g} -closed set F of Y .

Proof. (i) \Leftrightarrow (ii): Follows from the definitions. (i) \Leftrightarrow (iii): Let F be any \tilde{g} -closed set of Y . Then $Y \setminus F \in \tilde{G}O(Y)$. By (i), $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F) \in RO(X)$. We have $f^{-1}(F) \in RC(X)$.

Converse is similar. □

Lemma 2.6. [11] Let S be an open subset of a space (X, τ) . Then the following hold:

- (i) If U is regular open in X , then so is $U \cap S$ in the subspace (S, τ_s) .
- (ii) If $B \subset S$ is regular open in (S, τ_s) , then there exists a regular open set U in (X, τ) such that $B = U \cap S$.

Theorem 2.7. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a completely \tilde{g} -irresolute function and A is any open subset of X , then the restriction $f|_A : A \rightarrow Y$ is completely \tilde{g} -irresolute.

Proof. Let F be a \tilde{g} -open subset of Y . By hypothesis $f^{-1}(F)$ is regular open in X . Since A is open in X , it follows from the previous Lemma that $(f|_A)^{-1}(F) = A \cap f^{-1}(F)$, which is regular open in A . Therefore, $f|_A$ is completely \tilde{g} -irresolute. \square

Theorem 2.8. *The following hold for functions $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$:*

- (i) *If f is completely \tilde{g} -irresolute and g is strongly \tilde{g} -continuous, then $g \circ f$ is completely continuous;*
- (ii) *If f is completely \tilde{g} -irresolute and g is \tilde{g} -irresolute, then $g \circ f$ is completely \tilde{g} -irresolute;*
- (iii) *If f is completely continuous and g is completely \tilde{g} -irresolute functions, then $g \circ f$ is completely \tilde{g} -irresolute function.*

Proof. The proof of the theorem is easy and hence omitted. \square

Definition 2.9. *A space X is said to be almost connected [6] (resp. \tilde{g} -connected [16]) if there does not exist disjoint regular open (resp. \tilde{g} -open) sets A and B such that $A \cup B = X$.*

Theorem 2.10. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely \tilde{g} -irresolute surjective function and X is almost connected, then Y is \tilde{g} -connected.*

Proof. Suppose that Y is not \tilde{g} -connected. Then there exist disjoint \tilde{g} -open sets A and B of Y such that $A \cup B = Y$. Since f is completely \tilde{g} -irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X . Moreover, $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \emptyset$ and $f^{-1}(B) \neq \emptyset$. This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected. By contradiction, Y is \tilde{g} -connected. \square

Definition 2.11. *A space X is said to be*

- (i) *nearly compact [13, 21] if every regular open cover of X has a finite subcover;*
- (ii) *nearly countably compact [7] if every countable cover by regular open sets has a finite subcover;*
- (iii) *nearly Lindelof [6] if every cover of X by regular open sets has a countable subcover;*
- (iv) *\tilde{g} -compact [16] if every \tilde{g} -open cover of X has a finite subcover;*

- (v) countably \tilde{g} -compact [14] if every \tilde{g} -open countable cover of X has a finite subcover;
- (vi) \tilde{g} -Lindelof [14] if every cover of X by \tilde{g} -open sets has a countable subcover.

Theorem 2.12. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a completely \tilde{g} -irresolute surjective function. Then the following statements hold:*

- (i) *If X is nearly compact, then Y is \tilde{g} -compact.*
- (ii) *If X is nearly Lindelof, then Y is \tilde{g} -Lindelof.*
- (i) *If X is nearly countably compact, then Y is countably \tilde{g} -compact.*

Proof. (i) Let $f : X \rightarrow Y$ be a completely \tilde{g} -irresolute function of nearly compact space X onto a space Y . Let $\{U_\alpha : \alpha \in \Delta\}$ be any \tilde{g} -open cover of Y . Then, $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X . Since X is nearly compact, there exists a finite subfamily, $\{f^{-1}(U_{\alpha_i}) | i = 1, 2, \dots, n\}$ of $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ which cover X . It follows that $\{U_{\alpha_i} : i = 1, 2, \dots, n\}$ is a finite subfamily of $\{U_\alpha : \alpha \in \Delta\}$ which cover Y . Hence, space Y is a \tilde{g} -compact space.

The proof of other cases are similar. □

Definition 2.13. *A space (X, τ) is said to be:*

- (i) *S -closed [23] (resp. \tilde{g} -closed compact [14]) if every regular closed (resp. \tilde{g} -closed) cover of X has a finite subcover;*
- (ii) *countably S -closed-compact [5] (resp. countably \tilde{g} -closed compact [14]) if every countable cover of X by regular closed (resp. \tilde{g} -closed) sets has a finite subcover;*
- (iii) *S -Lindelof [12] (resp. \tilde{g} -closed Lindelof [14]) if every cover of X by regular closed (resp. \tilde{g} -closed) sets has a countable subcover.*

Theorem 2.14. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a completely \tilde{g} -irresolute surjective function. Then the following statements hold:*

- (i) *If X is S -closed, then Y is \tilde{g} -closed compact.*
- (ii) *If X is S -Lindelof, then Y is \tilde{g} -closed Lindelof.*
- (iii) *If X is countably S -closed-compact, then Y is countably \tilde{g} -closed compact.*

Proof. It can be obtained similarly as the previous Theorem. \square

Definition 2.15. A space X is said to be strongly \tilde{g} -normal (resp. mildly \tilde{g} -normal) if for each pair of distinct \tilde{g} -closed (resp. regular closed) sets A and B of X , there exist disjoint \tilde{g} -open sets U and V such that $A \subset U$ and $B \subset V$.

It is evident that every strongly \tilde{g} -normal space is mildly \tilde{g} -normal.

Recall that a function $f : X \rightarrow Y$ is called \tilde{g}^* -closed [2] if the image of each \tilde{g} -closed set of X is a \tilde{g} -closed set in Y .

In [2], the following theorem is proved.

Theorem 2.16. If a mapping $f : X \rightarrow Y$ is \tilde{g}^* -closed, then for each subset B of Y and each \tilde{g} -open set U of X containing $f^{-1}(B)$, there exists a \tilde{g} -open set V in Y containing B such that $f^{-1}(V) \subset U$.

Theorem 2.17. If $f : X \rightarrow Y$ is completely \tilde{g} -irresolute, \tilde{g}^* -closed function from a mildly \tilde{g} -normal space X onto a space Y , then Y is strongly \tilde{g} -normal.

Proof. Let A and B be two disjoint \tilde{g} -closed subsets of Y . Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular closed subsets of X . Since X is mildly \tilde{g} -normal space, there exist disjoint \tilde{g} -open sets U and V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Then by Theorem 2.16, there exist \tilde{g} -open sets $G = Y \setminus f(X \setminus U)$ and $H = Y \setminus f(X \setminus V)$ such that $A \subset G$, $f^{-1}(G) \subset U$; $B \subset H$, $f^{-1}(H) \subset V$. Clearly, G and H are disjoint \tilde{g} -open subsets of Y . Hence, Y is strongly \tilde{g} -normal. \square

Now, we define the following.

Definition 2.18. A space X is said to be strongly \tilde{g} -regular if for each \tilde{g} -closed set F and each point $x \notin F$, there exists disjoint \tilde{g} -open sets U and V in X such that $x \in U$ and $F \subset V$.

Definition 2.19. A space X is called almost \tilde{g} -regular if for each regular closed subset F and every point $x \notin F$, there exist disjoint \tilde{g} -open sets U and V such that $x \in U$ and $F \subset V$.

Theorem 2.20. If f is a completely \tilde{g} -irresolute, \tilde{g}^* -closed injection of an almost \tilde{g} -regular space X onto a space Y , then Y is strongly \tilde{g} -regular space.

Proof. Let F be a \tilde{g} -closed subset of Y and let $y \notin F$. Then, $f^{-1}(F)$ is regular closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Since X is almost \tilde{g} -regular space, there exists disjoint \tilde{g} -open sets U and V in X such

that $f^{-1}(y) \in U$ and $f^{-1}(F) \subset V$. By Theorem 2.16, there exist \tilde{g} -open sets $G = Y \setminus f(X \setminus U)$ such that $f^{-1}(G) \subset U$, $y \in G$ and $H = Y \setminus f(X \setminus V)$ such that $f^{-1}(H) \subset V$, $F \subset H$. Clearly, G and H are disjoint \tilde{g} -open subsets of Y . Hence, Y is strongly \tilde{g} -regular space. \square

Definition 2.21. A space (X, τ) is said to be \tilde{g} - T_1 [19] (resp. r - T_1 [6]) if for each pair of distinct points x and y of X , there exist \tilde{g} -open (resp. regular open) sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 2.22. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely \tilde{g} -irresolute injective function and Y is \tilde{g} - T_1 , then X is r - T_1 .

Proof. Suppose that Y is \tilde{g} - T_1 . For any two distinct points x and y of X , there exist \tilde{g} -open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since f is injective completely \tilde{g} -irresolute function, we have X is r - T_1 . \square

Definition 2.23. A space (X, τ) is said to be \tilde{g} - T_2 [19] for each pair of distinct points x and y in X , there exist disjoint \tilde{g} -open sets A and B in X such that $x \in A$ and $y \in B$.

Theorem 2.24. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely \tilde{g} -irresolute injective function and Y is \tilde{g} - T_2 , then X is T_2 .

Proof. Similar to the proof of Theorem 2.22. \square

3 Weakly \tilde{g} -irresolute functions

Definition 3.1. A function $f : X \rightarrow Y$ is said to be weakly \tilde{g} -irresolute if for each point $x \in X$ and each $V \in \tilde{G}O(Y, f(x))$, there exists a $U \in \tilde{G}O(X, x)$ such that $f(U) \subset \tilde{g}\text{-Cl}(V)$.

It is evident that every \tilde{g} -irresolute function is weakly \tilde{g} -irresolute but the converse is not true.

Example 3.2. Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Clearly, f is weakly \tilde{g} -irresolute but not \tilde{g} -irresolute.

Theorem 3.3. For a function $f : X \rightarrow Y$, the following statements are equivalent:

- (i) f is weakly \tilde{g} -irresolute;

(ii) $f^{-1}(V) \subset \tilde{g}\text{-Int}(f^{-1}(\tilde{g}\text{-Cl}(V)))$ for every $V \in \tilde{G}O(Y)$;

(iii) $\tilde{g}\text{-Cl}(f^{-1}(V)) \subset f^{-1}(\tilde{g}\text{-Cl}(V))$ for every $V \in \tilde{G}O(Y)$.

Proof. (i) \Rightarrow (ii): Suppose that $V \in \tilde{G}O(Y)$ and let $x \in f^{-1}(V)$. It follows from (i) that $f(U) \subset \tilde{g}\text{-Cl}(V)$ for some $U \in \tilde{G}O(X, x)$. Therefore, we have $U \subset f^{-1}(\tilde{g}\text{-Cl}(V))$ and $x \in U \subset \tilde{g}\text{-Int}(f^{-1}(\tilde{g}\text{-Cl}(V)))$. This shows that $f^{-1}(V) \subset \tilde{g}\text{-Int}(f^{-1}(\tilde{g}\text{-Cl}(V)))$.

(ii) \Rightarrow (iii): Suppose that $V \in \tilde{G}O(Y)$ and $x \notin f^{-1}(\tilde{g}\text{-Cl}(V))$. Then $f(x) \notin \tilde{g}\text{-Cl}(V)$. There exists $G \in \tilde{G}O(Y, f(x))$ such that $G \cap V = \emptyset$. Since $V \in \tilde{G}O(Y)$, we have $\tilde{g}\text{-Cl}(G) \cap V = \emptyset$ and hence $\tilde{g}\text{-Int}(f^{-1}(\tilde{g}\text{-Cl}(G))) \cap f^{-1}(V) = \emptyset$. By (ii), we have $x \in f^{-1}(G) \subset \tilde{g}\text{-Int}(f^{-1}(\tilde{g}\text{-Cl}(G))) \in \tilde{G}O(X)$. Therefore, we obtain $x \notin \tilde{g}\text{-Cl}(f^{-1}(V))$. This shows that $\tilde{g}\text{-Cl}(f^{-1}(V)) \subset f^{-1}(\tilde{g}\text{-Cl}(V))$.

(iii) \Rightarrow (i): Let $x \in X$ and $V \in \tilde{G}O(Y, f(x))$. Then, $x \notin f^{-1}(\tilde{g}\text{-Cl}(Y - \tilde{g}\text{-Cl}(V)))$. Since $Y - \tilde{g}\text{-Cl}(V) \in \tilde{G}O(Y)$, by (iii), we have $x \notin \tilde{g}\text{-Cl}(f^{-1}(Y - \tilde{g}\text{-Cl}(V)))$. Hence there exists $U \in \tilde{G}O(X, x)$ such that $U \cap f^{-1}(Y - \tilde{g}\text{-Cl}(V)) = \emptyset$. Therefore, we obtain $f(U) \cap (Y - \tilde{g}\text{-Cl}(V)) = \emptyset$ and hence $f(U) \subset \tilde{g}\text{-Cl}(V)$. This shows that f is weakly \tilde{g} -irresolute. \square

Theorem 3.4. *A function $f : X \rightarrow Y$ is weakly \tilde{g} -irresolute if the graph function, defined by $g(x) = (x, f(x))$ for each $x \in X$, is weakly \tilde{g} -irresolute.*

Proof. Let $x \in X$ and $V \in \tilde{G}O(Y, f(x))$. Then $X \times V$ is a \tilde{g} -open set of $X \times Y$ containing $g(x)$. Since g is weakly \tilde{g} -irresolute, there exists $U \in \tilde{G}O(X, x)$ such that $g(U) \subset \tilde{g}\text{-Cl}(X \times V) \subset X \times \tilde{g}\text{-Cl}(V)$. Therefore, we have $f(U) \subset \tilde{g}\text{-Cl}(V)$. \square

Theorem 3.5. [19] *A space X is $\tilde{g}\text{-}T_2$ if and only if for any pair of distinct points x, y of X there exists \tilde{g} -open sets U and V such that $x \in U$, and $y \in V$ and $\tilde{g}\text{-Cl}(U) \cap \tilde{g}\text{-Cl}(V) = \emptyset$.*

Theorem 3.6. *If Y is a $\tilde{g}\text{-}T_2$ space and $f : X \rightarrow Y$ is a weakly \tilde{g} -irresolute injection, then X is $\tilde{g}\text{-}T_2$.*

Proof. Let x, y be any two distinct points of X . Since f is injective, we have $f(x) \neq f(y)$. Since Y is $\tilde{g}\text{-}T_2$, by Theorem 3.5 there exists $V \in \tilde{G}O(Y, f(x))$ and $W \in \tilde{G}O(Y, f(y))$ such that $\tilde{g}\text{-Cl}(V) \cap \tilde{g}\text{-Cl}(W) = \emptyset$. Since f is weakly \tilde{g} -irresolute, there exists $G \in \tilde{G}O(X, x)$ and $H \in \tilde{G}O(X, y)$ such that $f(G) \subset \tilde{g}\text{-Cl}(V)$ and $f(H) \subset \tilde{g}\text{-Cl}(W)$. Hence we obtain $G \cap H = \emptyset$. This shows that X is $\tilde{g}\text{-}T_2$. \square

Definition 3.7. A function $f : X \rightarrow Y$ is said to have a strongly \tilde{g} -closed graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \tilde{G}O(X, x)$ and $V \in \tilde{G}O(Y, y)$ such that $(\tilde{g}\text{-Cl}(U) \times \tilde{g}\text{-Cl}(V)) \cap G(f) = \emptyset$.

Theorem 3.8. If Y is a $\tilde{g}\text{-}T_2$ space and $f : X \rightarrow Y$ is weakly \tilde{g} -irresolute, then $G(f)$ is strongly \tilde{g} -closed.

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$ and by Theorem 3.5 there exists $V \in \tilde{G}O(Y, f(x))$ and $W \in \tilde{G}O(Y, y)$ such that $\tilde{g}\text{-Cl}(V) \cap \tilde{g}\text{-Cl}(W) = \emptyset$. Since f is weakly \tilde{g} -irresolute, there exists $U \in \tilde{G}O(X, x)$ such that $f(\tilde{g}\text{-Cl}(U)) \subset \tilde{g}\text{-Cl}(V)$. Therefore, we obtain $f(\tilde{g}\text{-Cl}(U)) \cap \tilde{g}\text{-Cl}(W) = \emptyset$ and hence $(\tilde{g}\text{-Cl}(U) \times \tilde{g}\text{-Cl}(W)) \cap G(f) = \emptyset$. This shows that $G(f)$ is strongly \tilde{g} -closed in $X \times Y$. \square

Theorem 3.9. If a function $f : X \rightarrow Y$ is weakly \tilde{g} -irresolute, injective and $G(f)$ is strongly \tilde{g} -closed, then X is $\tilde{g}\text{-}T_2$.

Proof. Let x, y be a pair of distinct points of X . Since f is injective, $f(x) \neq f(y)$ and $(x, f(y)) \notin G(f)$. Since $G(f)$ is strongly \tilde{g} -closed there exists $G \in \tilde{G}O(X, x)$ and $V \in \tilde{G}O(Y, f(y))$ such that $f(\tilde{g}\text{-Cl}(G)) \cap \tilde{g}\text{-Cl}(V) = \emptyset$. Since f is weakly \tilde{g} -irresolute, there exists $H \in \tilde{G}O(X, y)$ such that $f(H) \subset \tilde{g}\text{-Cl}(V)$. Hence we have $f(\tilde{g}\text{-Cl}(G)) \cap f(H) = \emptyset$; hence $G \cap H = \emptyset$. This shows that X is $\tilde{g}\text{-}T_2$. \square

Recall that a topological space X is said to be \tilde{g} -connected [16] if it cannot be written as the union of two nonempty disjoint \tilde{g} -open sets.

Theorem 3.10. If a function $f : X \rightarrow Y$ is a weakly \tilde{g} -irresolute surjection and X is \tilde{g} -connected, then Y is \tilde{g} -connected.

Proof. Suppose that Y is not \tilde{g} -connected. There exist nonempty \tilde{g} -open sets V and W of Y such that $V \cup W = Y$ and $V \cap W = \emptyset$. Since f is weakly \tilde{g} -irresolute $f^{-1}(V), f^{-1}(W) \in \tilde{G}O(X)$. Moreover, we have $f^{-1}(V) \cup f^{-1}(W) = X$, $f^{-1}(V) \cap f^{-1}(W) = \emptyset$, and $f^{-1}(V)$ and $f^{-1}(W)$ are nonempty. Therefore, X is not \tilde{g} -connected. \square

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