Faculty of Sciences and Mathematics, University of Niš, Serbia

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COMPOSITION OPERATORS FROM BLOCH TYPE SPACES TO $F(p,q,s)$ SPACES

Xiangling Zhu

Abstract

Suppose that φ is an analytic self-map of the unit disk, the compactness of the composition operator C_φ from the Bloch type space into the space $F(p,q,s)$ is investigated .

1 Introduction

Let D be the open unit disk in the complex plane $\mathbb{C}$ and ∂D the unit circle. Denote by $H(D)$ the class of all functions analytic on D. An $f \in H(D)$ is said to belong to the Bloch type space, or $\alpha-$Bloch space $\mathcal{B}^\alpha$ if

$$B(f) = \sup_{z\in D}(1-|z|^2)^\alpha |f'(z)| < \infty.$$

Note that $\mathcal{B}^\alpha$ is a Banach space with the norm which is given by $\|f\|_{\mathcal{B}^\alpha} = |f(0)| + B(f)$. When $\alpha = 1$, $\mathcal{B} = \mathcal{B}^1$ is the well known Bloch space. Let $\mathcal{B}_0^\alpha$, called the little Bloch type space, denote the subspace of $\mathcal{B}^\alpha$ consisting of those $f \in \mathcal{B}^\alpha$ for which $\lim_{|z|\to 1}(1-|z|^2)^\alpha |f'(z)| = 0$.

The one-to-one holomorphic functions that map D onto itself have the form $\lambda\varphi_a$, where $\lambda \in \partial D$ and φ_a is the basic conformal automorphism

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defined by $\varphi_a = \frac{a-z}{1-\bar{a}z}$ for $a \in D$. It is easy to check that the following equalities hold

$$\varphi_a \circ \varphi_a(z) = z, \quad |\varphi_a'(z)| = \frac{1-|a|^2}{|1-\bar{a}z|^2}, \quad 1-|\varphi_a(z)|^2 = (1-|z|^2)|\varphi_a'(z)|.$$

For $a \in D$, let $g(z,a)$ be Green's function for D with logarithmic singularity at a, i.e. $g(z,a) = \log \frac{1}{|\varphi_a(z)|}$. Let $0 < p, s < \infty$, $-2 < q < \infty$. A function $f \in H(D)$ is said to belong to $F(p,q,s)$(see [14]) if

$$\|f\|_{p,q,s}^p = \sup_{a \in D} \int_D |f'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z) < \infty,$$

and $f \in F_0(p,q,s)$ if $f \in H(D)$ and

$$\lim_{|a| \to 1} \int_D |f'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z) = 0.$$

$F(p,q,s)$ is a Banach space under the norm $\|f\|_{F(p,q,s)}^p = |f(0)| + \|f\|_{p,q,s}^p$. $F(p,q,s)$ is called general function space because it can get many function spaces if it takes special parameters of p,q,s. For example, $F(p,q,s) = \mathcal{B}^{\frac{q+2}{p}}$ and $F_0(p,q,s) = \mathcal{B}_0^{\frac{q+2}{p}}$ for $s > 1$; $F(p,q,s) \subset \mathcal{B}^{\frac{q+2}{p}}$ and $F_0(p,q,s) \subset \mathcal{B}_0^{\frac{q+2}{p}}$ for $0 < s \le 1$; $F(2,0,s) = Q_s$ and $F_0(2,0,s) = Q_{s,0}$; $F(2,0,1) = BMOA$ and $F_0(2,0,1) = VMOA$; If $q + s \le -1$, then $F(p,q,s)$ is the space of constant functions.

Let φ be an analytic self-map of D. Then the composition operator C_φ with symbol φ is defined by

$$C_\varphi f = f \circ \varphi$$

for $f \in H(D)$. Littlewood's subordination principle gives that C_φ is a bounded linear operator on the classical Hardy and Bergman spaces. More information about the study of composition operators can be found in [2, 16].

In [15], Zhao has characterized the boundedness and compactness of composition operators between the Bloch type spaces and the Hardy and Besov spaces. Smith and Zhao have characterized the boundedness of $C_\varphi :$ $\mathcal{B} \to Q_p$, $C_\varphi : \mathcal{B}_0 \to Q_{p,0}$ and $C_\varphi : \mathcal{B} \to Q_{p,0}$ in [9]. In [11], Wulan has characterized the compactness of composition operators between the

Bloch space and the Q_K space. Some related results can be founded in [1, 4, 5, 6, 8, 12].

In this paper we study the composition operators from the Bloch type space $\mathcal{B}^\alpha$ into the space $F(p,q,s)$. For a subarc $I \in \partial D$, let

$$S(I) = \{r\zeta \in D : 1 - |I| < r < 1, \zeta \in I\}.$$

If $|I| \geq 1$, then we set $S(I) = D$. For $r \in (0,1)$, let $D_r = \{z \in D : |\varphi(z)| > r\}$. The characteristic function of a set $E \subset D$ is denoted by I_E. Jiang and He in [3] studied the boundedness and compactness of composition operator from the Bloch type space $\mathcal{B}^\alpha$ into the space $F(p,q,s)$. The main results in [3] can be stated as follows.

Theorem A. *Let φ be an analytic self-map of D, $0 < \alpha, p, s < \infty$, $-2 < q < \infty$ and $q + s > -1$. The following statements are equivalent:*

(i) $C_\varphi : \mathcal{B}^\alpha \to F(p,q,s)$ is bounded;

(ii) For $p \geq 2$, $C_\varphi : \mathcal{B}_0^\alpha \to F(p,q,s)$ is bounded;

(iii)

$$\sup_{a\in D} \int_D \frac{|\varphi'(z)|^p}{(1-|\varphi(z)|^2)^{p\alpha}} (1-|z|^2)^q g^s(z,a)\, dA(z) < \infty. \tag{1}$$

Theorem B. *Let φ be an analytic self-map of D, $0 < \alpha, p, s < \infty$, $-2 < q < \infty$ and $q + s > -1$. The following statements are equivalent:*

(i) $C_\varphi : \mathcal{B}^\alpha \to F(p,q,s)$ is a compact operator;

(ii) $C_\varphi : \mathcal{B}_0^\alpha \to F(p,q,s)$ is a compact operator;

(iii) $\varphi \in F(p,q,s)$ and

$$\lim_{r\to 1} \sup_{I\subset \partial D} |I|^{-s} \int_{S(I)} I_{D_r} \frac{|\varphi'(z)|^p}{(1-|\varphi(z)|^2)^{p\alpha}} (1-|z|^2)^{q+s}\, dA(z) = 0. \tag{2}$$

The above compactness condition is very difficult to verify. In this paper, we give another characterization of the compactness of C_φ from Bloch type space $\mathcal{B}^\alpha$ into the space $F(p,q,s)$.

Throughout this paper, C always denote positive constant and may be different at different occurrences.

2 Main Results and Proofs

In this section, we give the main results and the proofs of this paper by using the methods of [11]. For this purpose, we need some lemmas. The following criterion for compactness follows by standard arguments similar, for example, to those outlined in Proposition 3.11 of [2].

Lemma 1. *Let φ be an analytic self-map of D, $0 < \alpha, p, s < \infty$, $-2 < q < \infty$ and $q + s > -1$. Then $C_\varphi : \mathcal{B}^\alpha \to F(p, q, s)$ is a compact operator if and only if $C_\varphi : \mathcal{B}^\alpha \to F(p, q, s)$ is bounded and for any bounded sequence (f_n) in $\mathcal{B}^\alpha$ with $(f_n) \to 0$ uniformly on compact sets as $n \to \infty$, $\|C_\varphi f_n\|_{F(p,q,s)} \to 0$, as $n \to \infty$.*

Lemma 2. *Let φ be an analytic self-map of D, $0 < \alpha, p, s < \infty$, $-2 < q < \infty$ and $q + s > -1$. If $C_\varphi : \mathcal{B}^\alpha(\mathcal{B}^\alpha_0) \to F(p, q, s)$ is a compact operator, then for any $\varepsilon > 0$ there exists a δ, $0 < \delta < 1$, such that for all f in $\mathbf{B}_{\mathcal{B}^\alpha}(\mathbf{B}_{\mathcal{B}^\alpha_0})$, the unit ball of $\mathcal{B}^\alpha(\mathcal{B}^\alpha_0)$,*

$$\sup_{a\in D} \int_{|\varphi(z)|>r} |(f \circ \varphi)'(z)|^p (1 - |z|^2)^q g^s(z, a) dA(z) < \varepsilon$$

holds whenever $\delta < r < 1$.

Proof. We only prove the case of $\mathcal{B}^\alpha_0$. The proof for $\mathcal{B}^\alpha$ is similar, hence we omit the details. Assume that $C_\varphi : \mathcal{B}^\alpha_0 \to F(p, q, s)$ is compact. For $f \in \mathbf{B}_{\mathcal{B}^\alpha_0}$, let $f_t(z) = f(tz)$ for $t \in (0, 1)$ and $z \in D$. Then $f_t \to f$ uniformly on compact subsets of D as $t \to 1$. Since C_φ is compact, then by Lemma 1 we see that $\|C_\varphi f_t - C_\varphi f\|_{p,q,s} \to 0$ as $t \to 1$. Thus, for given $\varepsilon > 0$, there is a $t \in (0, 1)$ such that

$$\sup_{a\in D} \int_D |f_t'(\varphi(z)) - f'(\varphi(z))|^p |\varphi'(z)|^p (1 - |z|^2)^q g^s(z, a) dA(z) < \varepsilon.$$

By the triangle inequality, for $r \in (0,1)$, we have

$$\begin{aligned}
&\sup_{a\in D}\int_{|\varphi(z)|>r}|f'(\varphi(z))|^p|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)\\
\le\ &\sup_{a\in D}\int_{|\varphi(z)|>r}|f_t'(\varphi(z))-f'(\phi(z))|^p|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)\\
+\ &\sup_{a\in D}\int_{|\varphi(z)|>r}|f_t'(\varphi(z))|^p|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)\\
\le\ &\varepsilon+\|f_t'\|_\infty^p\sup_{a\in D}\int_{|\varphi(z)|>r}|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z).
\end{aligned}$$

Now, we prove that for given $\varepsilon > 0$ and $\|f_t'\|_\infty^p > 0$ there exists a $\delta \in (0,1)$ such that if $\delta < r < 1$

$$\|f_t'\|_\infty^p\sup_{a\in D}\int_{|\varphi(z)|>r}|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)<\varepsilon.$$

Let $f_n(z) = n^{\alpha-1}z^n$. it is easy to see that $f_n \in \mathcal{B}_0^\alpha$ and converges to zero uniformly on compact subsets of D. Since C_φ is a compact operator, we have $\lim_{n\to\infty}\|n^{\alpha-1}\varphi^n\|_{p,q,s} \to 0$ as $n \to \infty$. That is, for any given $\varepsilon > 0$ and $\|f_t'\|_\infty^p > 0$, there exists a integer $N > 1$ such that

$$\|f_t'\|_\infty^p\int_{|\varphi(z)|>r}n^{p\alpha}|\varphi(z)|^{p(n-1)}|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)<\varepsilon, \tag{4}$$

whenever $n \ge N$. Given $r \in (0,1)$, (4) yields

$$N^{\alpha p}r^{pN-p}\int_{|\varphi(z)|>r}|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)<\varepsilon.$$

Taking $r = N^{-\frac{\alpha}{N-1}}$, we get

$$\|f_t'\|_\infty^p\sup_{a\in D}\int_{|\varphi(z)|>r}|\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)<\varepsilon.$$

Hence we have already proved that for any $\varepsilon > 0$ and for $f \in \mathbf{B}_{\mathcal{B}_0^\alpha}$, there exists a $\delta = \delta(\varepsilon, f)$ such that

$$\sup_{a\in D}\int_{|\varphi(z)|>r}|(f\circ\varphi)'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z)<\varepsilon$$

holds whenever $\delta < r < 1$.

We finish the proof of Lemma 2 by showing that the $\delta = \delta(\varepsilon, f)$, in fact, is independent of $f \in \mathbf{B}_{\mathcal{B}_0^\alpha}$. Since $C_\varphi(\mathbf{B}_{\mathcal{B}_0^\alpha})$ is a relatively compact subset of $F(p,q,s)$, there are a finite collection of functions $f_1, f_2, \cdots, f_n$ in $\mathbf{B}_{\mathcal{B}_0^\alpha}$ such that for any $\varepsilon > 0$ and $f \in \mathbf{B}_{\mathcal{B}_0^\alpha}$, there is a k, $k = 1, 2, \cdots, n$, satisfying

$$\sup_{a\in D} \int_D |f'(\varphi(z)) - f_k'(\varphi(z))|^p |\varphi'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z) < \varepsilon.$$

On the other hand, if $\max_{1\le k\le n} \delta(\varepsilon, f_k) = \delta < r < 1$, we have for all $k = 1, 2, \cdots, n$,

$$\sup_{a\in D} \int_{|\varphi(z)|>r} |f_k'(\varphi(z))|^p |\varphi'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z) < \varepsilon.$$

By using the triangle inequality, we get

$$\sup_{a\in D} \int_{|\varphi(z)|>r} |(f\circ\varphi)'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z) < \varepsilon$$

whenever $\delta < r < 1$. The proof is completed.

Lemma 3.[17] *Suppose that n_k is an increasing sequence of positive integers with Hadamard gaps, that is, $n_{k+1}/n_k \ge \lambda > 1$ for all k. Let $0 < p < \infty$. Then there is a constant $M > 0$ depending on p and λ such that*

$$M^{-1}\left(\sum_{k=1}^N |a_k|^2\right)^{1/2} \le \left(\frac{1}{2\pi}\int_0^{2\pi}\left|\sum_{k=1}^N a_k e^{in_k\theta}\right|^p d\theta\right)^{1/p} \le M\left(\sum_{k=1}^N |a_k|^2\right)^{1/2}$$

for any scalars $a_1, \ldots, a_N$ and $N = 1, 2, \ldots$.

We are now ready to state and prove the main results in this section.

Theorem 1. *Let φ be an analytic self-map of D, $0 < p, \alpha, s < \infty$, $-2 < q < \infty$ and $q + s > -1$. The following statements are equivalent:*

(i) $C_\varphi : \mathcal{B}^\alpha \to F(p,q,s)$ is a compact operator;

(ii) $C_\varphi : \mathcal{B}_0^\alpha \to F(p,q,s)$ is a compact operator;

(iii) $\varphi \in F(p,q,s)$ and

$$\lim_{r\to 1}\sup_{a\in D}\int_{|\varphi(z)|>r} \frac{|\varphi'(z)|^p}{(1-|\varphi(z)|^2)^{p\alpha}} (1-|z|^2)^q g^s(z,a) dA(z) = 0. \qquad (3)$$

Proof. $(i) \Rightarrow (ii)$ It is trivial since $\mathcal{B}_0^\alpha \subset \mathcal{B}^\alpha$.

$(ii) \Rightarrow (iii)$ Suppose that $C_\varphi : \mathcal{B}_0^\alpha \to F(p,q,s)$ is compact. By choosing $f = z \in \mathcal{B}_0^\alpha$ we have $\varphi \in F(p,q,s)$. Next, we choose the function

$$f(z) = \sum_{k=1}^{\infty} 2^{k(\alpha-1)} z^{2^k},$$

we see that $f(z) \in \mathcal{B}^\alpha$ from [13]. Set $g(z) = f(z)/\|f\|_{\mathcal{B}^\alpha}$, choose a sequence $\{\lambda_n\}$ in D which converges to 1 as $n \to \infty$, and let $g_n(z) = g(\lambda_n z)$ for all $n \in \mathbb{N}$. For $0 \le \theta \le 2\pi$, set $g_{n,\theta}(z) = g_n(e^{i\theta} z)$. It is easy to see that $g_{n,\theta} \in \mathbf{B}_{\mathcal{B}_0^\alpha}$. Replace f by $g_{n,\theta}$ in Lemma 2 and then integrate against $d\theta$, by Fubini's Theorem and Lemma 3 we obtain

$$\begin{aligned} \varepsilon &\ge \frac{1}{2\pi} \int_{|\varphi(z)|>r} \Big(\int_0^{2\pi} |g_n'(e^{i\theta}\varphi(z))|^p d\theta \Big) |\varphi'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z) \\ &\ge C \int_{|\varphi(z)|>r} \Big(\sum_{k=1}^{\infty} 2^{2\alpha k} |\lambda_n \varphi(z)|^{2(2^k-1)} \Big)^{p/2} |\lambda_n \varphi'(z)|^p (1-|z|^2)^q g^s(z,a) dA(z). \end{aligned} \tag{4}$$

Notice that(see [3] or [15])

$$\sum_{k=1}^{\infty} 2^{2\alpha k} |\lambda_n \varphi(z)|^{2(2^k-1)} > \frac{C(\alpha)}{(1-|\lambda_n \varphi(z)|^2)^{2\alpha}}. \tag{5}$$

Here $C(\alpha)$ is only depend on α. Therefore, for $\delta < r < 1$ and for sufficient large n, (4) and (5) give

$$\sup_{a\in D} \int_{|\varphi(z)|>r} \frac{|\lambda_n \varphi'(z)|^p}{(1-|\lambda_n \varphi(z)|^2)^{p\alpha}} (1-|z|^2)^q g^s(z,a) dA(z) < C\varepsilon.$$

By Fatou's Lemma we obtain

$$\lim_{r\to 1} \sup_{a\in D} \int_{|\varphi(z)|>r} \frac{|\varphi'(z)|^p}{(1-|\varphi(z)|^2)^{p\alpha}} (1-|z|^2)^q g^s(z,a) dA(z) = 0.$$

$(iii) \Rightarrow (i)$ Suppose that $\varphi \in F(p,q,s)$ and (3) holds. Then it is easy to check that $C_\varphi : \mathcal{B}^\alpha \to F(p,q,s)$ is bounded. Let $\{f_n\} \subset \mathbf{B}_{\mathcal{B}_0^\alpha}$. We only need to show that $\{C_\varphi f_n\}$ has a subsequence that converges in $F(p,q,s)$. Since $\mathbf{B}_{\mathcal{B}_0^\alpha}$ is a normal family, by passing to a subsequence, we may assume, without loss of generality, that $\{f_n\}$ converges to 0 uniformly on compact subsets of D. By the Cauchy's estimate, we see that $\{f_n'\}$ also converges to 0 uniformly on compact subsets of D. We must show that $\{C_\varphi f_n\}$ converges to 0 in the topology of the norm of $\|\cdot\|_{F(p,q,s)}$. Given $\varepsilon \in (0,1)$, by (3), there is an $r \in (0,1)$ such that for all the functions f_n and all $a \in D$,

$$\int_{|\varphi(z)|>r} |f_n'(\varphi(z))\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z) < \varepsilon. \tag{6}$$

Since $D_r = \{z \in D : |z| \leq r\}$ is a compact subset of D, $\{f_n'\}$ also converges to 0 uniformly on D_r. Therefore, there exists an integer $N > 1$ such that as $n \geq N$,

$$\int_{|\varphi(z)|\leq r} |f_n'(\varphi(z))\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z) < \varepsilon\|\varphi\|^p_{p,q,s}. \tag{7}$$

Therefore, by (6) and (7),

$$\int_D |f_n'(\varphi(z))\varphi'(z)|^p(1-|z|^2)^q g^s(z,a)dA(z) < \varepsilon(1+\|\varphi\|^2_{p,q,s})$$

when $n \geq N$. That is, $\|C_\varphi f_n\|_{p,q,s} \to 0$ as $n \to \infty$. Therefore $\|C_\varphi f_n\|_{F(p,q,s)} \to 0$ as $n \to \infty$. By Lemma 1, we see that $C_\varphi : \mathcal{B}^\alpha \to F(p,q,s)$ is a compact operator.

Corollary 1. *Let φ be an analytic self-map of D. Then the following statements are equivalent:*

(i) $C_\varphi : \mathcal{B} \to \mathcal{B}$ is a compact operator;

(ii) $C_\varphi : \mathcal{B}_0 \to \mathcal{B}$ is a compact operator;

(iii) $\varphi \in \mathcal{B}$ and

$$\lim_{r\to 1}\sup_{a\in D}\int_{|\varphi(z)|>r} \frac{|\varphi'(z)|^p}{(1-|\varphi(z)|^2)^p}(1-|z|^2)^{p-2}g^s(z,a)dA(z) = 0$$

for all $p > 0$ and all $s > 1$;

(iv) $\varphi \in \mathcal{B}$ and

$$\lim_{r\to 1}\sup_{a\in D}\int_{|\varphi(z)|>r} \frac{|\varphi'(z)|^p}{(1-|\varphi(z)|^2)^p}(1-|z|^2)^{p-2}g^s(z,a)dA(z) = 0$$

for each $p > 0$ and each $s > 1$.

Proof. Since $\mathcal{B} = F(p, p-2, s)$ for any $p > 0$ and any $s > 1$(see Theorem 1.3 in [14]), the resul is a direct consequence of Theorem 1.

Remark 1. The compactness of composition operator on Bloch space was characterized in [7]. In [10], Tjani proved that $C_\varphi : \mathcal{B} \to \mathcal{B}$ is compact if and only if $\lim_{|a|\to 1} \|C_\varphi \varphi_a\|_{\mathcal{B}} = 0$. Another related result can be found in [11].

Remark 2. From the proof of Theorem 1 and the proof of Theorem 1.1 in [3], we find that by using Lemma 3, we can remove the restrict condition $p \geq 2$ in Theorem A, i.e. $p \geq 2$ in Theorem 1.1 in [3].

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Department of Mathematics, Shantou University, Shantou, 515063, GuangDong, China
Department of Mathematics, Jiaying University, Meizhou, 514015, GuangDong, China
E-mail: jyuzxl@163.com