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ON SOME CHARACTERIZATIONS OF VIVIDLY AND BLURLY $(1,2)$-β -IRRESOLUTE MAPPINGS

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Abstract

The aim of this paper is to introduce and characterize the vividly $(1,2)$-β-irresolute mapping and blurly $(1,2)$-β-irresolute mapping . We also define $(1,2)$-β-T_2 spaces and $(1,2)$-semi-preregular spaces. These spaces are characterized by a new class of open sets, called $(1,2)$-semipre-θ-open sets.

1 Introduction

In 1983, Abd El-Monsef et al.[1] defined β-open sets and Andrijevic [2] called these sets as semi-preopen sets. The notion of semi-pre θ-open sets was introduced by Noiri [6] in 2003. The concept of $(1,2)$-semi-preopen sets were defined and investigated by Raja Rajeswari and Lellis Thivagar [7] in 2005. The notion of $(1,2)$-semi-preirresolute mapping what we call as $(1,2)$-β-irresolute mapping, was introduced by Navalagi et al.[5]. In this paper,we define the vividly $(1,2)$-β-irresolute mappings and blurly $(1,2)$-β-irresolute mappings. Also we introduce and investigate some properties of $(1,2)$-semipre-θ-open sets in bitopological spaces and characterize the vivid $(1,2)$-β-irresolute mappings and blur $(1,2)$-β-irresolute mappings. Also the $(1,2)$-β-T_2 spaces and the $(1,2)$-semi-preregular spaces are defined and characterized by the class of $(1,2)$-semipre-θ-open sets.

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2 Preliminaries

The interior and the closure of a subset A of a topological space (X, τ) are denoted by $int(A)$ and $cl(A)$ respectively.

A subset A of a topological space (X, τ) is said to be semi-preopen [2] if $A \subset cl(int(cl(A)))$ and semi-preclosed if its complement in X is semi-preopen. The semi-preclosure of a subset A of X, denoted by $spcl(A)$, is the intersection of all the semi-preclosed sets containing A and A is semi-preclosed if $A = spcl(A)$. The semi-preinterior of a subset A of X, denoted by $spint(A)$ is the union of all the semi-preopen sets contained in A and A is semi-preopen if $A = spint(A)$. The family of all semi-preopen sets of X is denoted by $SPO(X)$.

A point $x \in X$ is called semipre-θ-cluster point [6] if $A \cap spcl(U) \neq \emptyset$ for each semipre-open set U containing x. The semipre-θ-cluster points of A is called the semipre-θ-closure of A and is denoted by $spcl_\theta(A)$. A subset A is semipre-θ-closed if $spcl_\theta(A) = A$. The family of all the semipre-θ-open sets of a space X is denoted by $SP\theta O(X)$.

The complement of a semipre-θ-closed set in X is semipre-θ-open. The semipre-θ-interior of A, denoted by $spint_\theta(A)$ is defined as follows.
$spint_\theta(A) = \{x \in X : x \in U \subset spcl(U) \subset A$ for some semi-preopen set U of $X\}$.

Definition 1 *A topological space X is said to be β-T_2 [6] if for x, $y \in X$, $x \neq y$, there exist disjont semi-preopen sets U, V such that $x \in U$ and $y \in V$.*

Definition 2 *A map $f:(X, \tau) \to (Y, \sigma)$ is called β-irresolute [4] if $f^{-1}(V)$ is semi-preopen for every semi-preopen set V in Y.*

In the following sections by X, Y and Z, we mean a bitopological space (X, τ_1, τ_2), (Y, σ_1, σ_2) and $(Z, \varrho_1, \varrho_2)$, respectively.

Definition 3 *A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-open [3] if $A \in \tau_1 \cup \tau_2$ and $\tau_1\tau_2$-closed if its complement in X is $\tau_1\tau_2$-open.*

Definition 4 *A subset A of a space X is said to be an $(1,2)$-semi-preopen set [7] if $A \subset \tau_1\tau_2$-$cl(\tau_1$- $int(\tau_1\tau_2$-$cl(A)))$ and $(1,2)$-semi-preclosed if its complement in X is $(1,2)$-semi-preopen.*

The family of all

(i). $(1,2)$-semi-preopen sets in X is denoted by $(1,2)$-$SPO(X)$.

(ii). $(1,2)$-semi-preopen sets containing $x \in X$ is denoted by $(1,2)$-$SPO(X,x)$.

(iii). $(1,2)$-semi-preclosed sets in X is denoted by $(1,2)$-$SPC(X)$.

Definition 5 *For any subset A of a bitopological space X, the $(1,2)$-semi-preclosure[7] of A denoted by $(1,2)$-$spcl(A)$ is the intersection of all the $(1,2)$-semi-preclosed sets containing A. The $(1,2)$-semi-preinterior of a subset A of X is the union of all the $(1,2)$-semi-preopen sets contained in A, and is denoted by $(1,2)$-$spint(A)$ and A is $(1,2)$-semi-preopen if $(1,2)$-$spint(A)$ $= A$.*

Remark 6 *It was observed that a subset A of a bitopological space X is $(1,2)$-semi-preclosed if $(1,2)$-$spcl(A) = A$. If $A \subset B$, then $(1,2)$-$spcl(A)$ $\subset (1,2)$-$spcl(B)$.*

Definition 7 *A map $f{:}X \to Y$ is called $(1,2)$-β-irresolute [5] if $f^{-1}(V)$ is $(1,2)$-semi-preopen for every $(1,2)$-semi-preopen set V in Y.*

3 $(1,2)$-semi-preregular sets and $(1,2)$-semipre-θ-open sets

In this section we define the $(1,2)$-semi-preregular sets and $(1,2)$-semipre-θ-open sets and investigate some of their properties.

Lemma 8 *The following hold for a subset A of X.*

(i). $(1,2)$-$spint(A) = A \cap \tau_1\tau_2$-$cl(\tau_1$- $int(\tau_1\tau_2$-$cl(A)))$.

(ii). $(1,2)$-$spcl(A) = A \cup \tau_1\tau_2$-$int(\tau_1$- $cl(\tau_1\tau_2$-$int(A)))$.

(iii). *$x \in (1,2)$-$spcl(A)$ if and only if $A \cap U \neq \emptyset$ for every $U \in (1,2)$-$SPO(X,x)$.*

(iv). $(1,2)$-$spcl(X \setminus A) = X \setminus (1,2)$-$spint(A)$.

Definition 9 *A subset A of a space X is said to be $(1,2)$-semi-preregular (briefly $(1,2)$-sp-regular) if it is both $(1,2)$-semi-preopen and $(1,2)$-semi-preclosed.*

The family of all

(i). $(1,2)$-semi-preregular sets in X is denoted by $(1,2)$-$SPR(X)$.

(ii). $(1,2)$-semi-preregular sets containing $x \in X$ is denoted by $(1,2)$-$SPR(X,x)$.

Theorem 10 *Let A be a subset of X. Then*

(i). $A \in (1,2)$-$SPO(X)$ if and only if $(1,2)$-$spcl(A) \in (1,2)$-$SPR(X)$.

(ii). $A \in (1,2)$-$SPC(X)$ if and only if $(1,2)$-$spint(A) \in (1,2)$-$SPR(X)$.

Proof. (i). **Necessity.** Let $A \in (1,2)$-$SPO(X)$. Then $A \subset \tau_1\tau_2$-cl (τ_1-int($\tau_1\tau_2$-$cl(A)$)) and so $(1,2)$-$spcl(A) \subset (1,2)$-$spcl(\tau_1\tau_2$-$cl(\tau_1$- $int(\tau_1\tau_2$-$cl(A)))) \subset (1,2)$-$spcl(\tau_1\tau_2$-$cl(\tau_1$ -$int(\tau_1\tau_2$-$cl((1,2)spcl(A)))))$ and hence $(1,2)$-$spcl(A) \subset \tau_1\tau_2$-$cl(\tau_1$- $int(\tau_1\tau_2$-$cl((1,2)$-$spcl(A))))$. Hence $(1,2)$-$spcl(A)$ is $(1,2)$-semi-preopen and it is $(1,2)$-semi-preclosed. Thus $(1,2)$-$spcl(A) \in (1,2)$-$SPR(X)$.
Sufficiency. Let $(1,2)$-$spcl(A) \in (1,2)$-$SPR(X)$. Then $(1,2)$-$spcl(A)$ is $(1,2)$-semi-preopen and $(1,2)$-semi-preclosed. Therefore, $A \subset (1,2)$-$spcl(A)$ $\subset \tau_1\tau_2$-$cl(\tau_1$- $int(\tau_1\tau_2$-$cl((1,2)$-$spcl(A))) \subset \tau_1\tau_2$-$cl(\tau_1$- $int(\tau_1\tau_2$-$cl(\tau_1\,\tau_2$-$cl(A))))$ $= \tau_1\tau_2$ -$cl(\tau_1$-$int(\tau_1\tau_2$-$cl(A)))$. Hence A is $(1,2)$-semi-preopen.

(ii). Follows from (i) and Lemma 8. ■

Theorem 11 *For a subset A of a space X, the following are equivalent.*

(i). $A \in (1,2)$-$SPR(X)$.

(ii). $A = (1,2)$-$spint((1,2)$-$spcl(A))$.

(iii). $A = (1,2)$-$spcl((1,2)$-$spint(A))$.

Proof. $(i) \Rightarrow (ii)$. If $A \in (1,2)$-$SPR(X)$ then A is $(1,2)$-semi-preclosed and $(1,2)$-$spcl(A) = A$ and therefore, $(1,2)$-$spint((1,2)$-$spcl(A)) = A$ since A is $(1,2)$-semi-preopen.

$(ii) \Rightarrow (i)$. Since $(1,2)$-$spcl(A)$ is $(1,2)$-semi-preclosed, by Theorem 10, $(1,2)$-$spint((1,2)$-$spcl(A)) \in (1,2)$-$SPR(X)$ and then $A \in (1,2)$-$SPR(X)$.

(i) $\Rightarrow$ *(iii)*. Follows from the fact that A is $(1,2)$-semi-preopen and $(1,2)$-semi-preclosed.

(iii) $\Rightarrow$ *(i)*. Since $(1,2)$-$spint(A)$ is $(1,2)$-semi-preopen and by Theorem 10, $(1,2)$-$spcl((1,2)$-$spint(A)) \in (1,2)$-$SPR(X)$, then $A \in (1,2)$-$SPR(X)$. ■

The $(1,2)$-semipre-θ-interior and $(1,2)$-semipre-θ-closure of a subset A of X are denoted by $(1,2)$-$spint_\theta(A)$ and $(1,2)$-$spcl_\theta(A)$ are defined as follows.
$(1,2)$-$spint_\theta(A) = \{x \in X{:}x \in U \subset (1,2)$-$spcl(U) \subset A$ for some $(1,2)$-semi-preopen set U of $X\}$ and
$(1,2)$-$spcl_\theta(A) = \{x \in X{:}\ (1,2)$-$spcl(U) \cap A \neq \emptyset$ for every $(1,2)$-semi-preopen set containing $x\}$

Remark 12 *Let A be a subset of X. Then*

(i). *A is $(1,2)$-semipre-θ-open (briefly $(1,2)$-sp-θ-open) if and only if $A = (1,2)$-$spint_\theta(A)$ and $(1,2)$-semipre-θ-closed (briefly $(1,2)$-sp-θ-closed) if and only if $A = (1,2)$-$spcl_\theta(A)$.*

(ii). $X\setminus (1,2)$-$spint_\theta(A) = (1,2)$-$spcl_\theta(X \setminus A)$ *and* $(1,2)$-$spint_\theta(X \setminus A) = X\setminus (1,2)$-$spcl_\theta(A)$.

(iii). $(1,2)$-$spint_\theta(A)$ *is $(1,2)$-sp-θ-open and* $(1,2)$-$spcl_\theta(A)$ *is $(1,2)$-sp-θ-closed.*

Theorem 13 *For any two subsets A, B of X, the following statements hold.*

(i). $(1,2)$-$spint_\theta((1,2)$-$spint_\theta(A)) \subset (1,2)$-$spint_\theta(A)$.

(ii). *If $A \subset B$, then* $(1,2)$-$spint_\theta(A) \subset (1,2)$-$spint_\theta(B)$.

(iii). $(1,2)$-$spint_\theta(A) \cup (1,2)$-$spint_\theta(B) \subset (1,2)$-$spint_\theta\ (A \cup B)$.

(iv). $(1,2)$-$spint_\theta(A \cap B) \subset (1,2)$-$spint_\theta(A) \cap (1,2)$-$spint_\theta(B)$.

Theorem 14 *For a subset A of X, the following properties hold.*

(i). *If $A \in (1,2)$-$SPO(X)$, then* $(1,2)$-$spcl(A) = (1,2)$-$spcl_\theta(A)$.

(ii). *$A \in (1,2)$-$SPR(X)$, if and only if A is $(1,2)$-sp-θ-open and $(1,2)$-sp-θ-closed.*

Proof. (i). For any $A \subset X$, it is observed that $(1,2)$-*spcl*(A) $\subset (1,2)$-*spcl*$_\theta(A)$. Let $A \in (1,2)$-$SPO(X)$ and $x \notin (1,2)$-*spcl*(A). Then, there exists $V \in (1,2)$-$SPO(X,x)$ such that $V \cap A = \emptyset$. Since $A \in (1,2)$-$SPO(X)$, $(1,2)$-*spcl*$(V) \cap A = \emptyset$. Hence $x \notin (1,2)$-*spcl*$_\theta(A)$. Therefore, $(1,2)$-*spcl*$_\theta(A) \subset (1,2)$-*spcl*(A).

(ii). Let $A \in (1,2)$-$SPR(X)$. Then A is $(1,2)$-semi-preopen and $(1,2)$-semi-preclosed and by (i), A is $(1,2)$-*sp*-θ-closed. Since $X \setminus A$ is $(1,2)$-semi-preopen and $(1,2)$-semi-preclosed, $X \setminus A$ is $(1,2)$-*sp*-θ-closed and hence A is $(1,2)$-*sp*-θ-open.

Conversely, if A is $(1,2)$-*sp*-θ-open, then $A = (1,2)$-*spint*$_\theta(A) \subset (1,2)$-*spint*(A) and therefore, A is $(1,2)$-semi-preopen. If A is $(1,2)$-*sp*-θ-closed, then $(1,2)$-*spcl*$(A) \subset (1,2)$-*spcl*$_\theta(A) = A$ and hence A is $(1,2)$-semi-preclosed. Thus we obtain $A \in (1,2)$-$SPR(X)$. ■

Theorem 15 *If A_α is $(1,2)$-sp-θ-closed in X for each $\alpha \in \Delta$, then $\bigcap_{\alpha\in\Delta} A_\alpha$ is $(1,2)$-sp-θ-closed.*

Proof. For each $\alpha \in \Delta$, if A_α is $(1,2)$-*sp*-θ-closed, then $(1,2)$-*spcl*$_\theta(A_\alpha)$ $= A_\alpha$. We have $(1,2)$-*spcl*$_\theta$ $(\bigcap_{\alpha\in\Delta} A_\alpha) \subset \bigcap_{\alpha\in\Delta}(1,2)$-*spcl*$_\theta A_\alpha = \bigcap_{\alpha\in\Delta}(A_\alpha)$. It is obvious that $\bigcap_{\alpha\in\Delta}(A_\alpha) \subset (1,2)$-*spcl*$_\theta$ $(\bigcap_{\alpha\in\Delta} A_\alpha)$. Hence $\bigcap_{\alpha\in\Delta} A_\alpha$ is $(1,2)$-*sp*-θ-closed. ■

Remark 16 *The union of two $(1,2)$-sp-θ-closed sets is not $(1,2)$-sp-θ-closed, in general as shown in the following example.*

Example 17 *Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X\}$ and $\tau_2 = \{\emptyset, \{b,c\}, X\}$. Then the sets $\{b\}$, $\{c\}$ are $(1,2)$-sp-θ-closed but $\{b,c\}$ is not $(1,2)$-sp-θ-closed.*

Remark 18 *If A_α is $(1,2)$-sp-θ-open in X for each $\alpha \in \Delta$, then $\bigcup_{\alpha\in\Delta} A_\alpha$ is $(1,2)$-sp-θ-open in X.*

Remark 19 •

(i). Every $(1,2)$-semi-preregular set is $(1,2)$-sp-θ-open.

(ii). Every $(1,2)$-sp-θ-open set is $(1,2)$-semi-preopen.

Remark 20 *The statements (i) and (ii) of Remark 19 are not reversible as shown in the following examples.*

Example 21 *Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X\}$ and $\tau_2 = \{\emptyset, \{b,c\}, X\}$. Then the set $\{b,c\}$ is $(1,2)$-sp-θ-open but it is not $(1,2)$-semi-preregular.*

Example 22 *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, X\}$ *and* $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. *Then* $\{a\}$ *is* $(1,2)$-*semi-preopen but not* $(1,2)$-*sp*-θ-*open.*

4 Blurly $(1,2)$-β-Irresolute Mappings

In this section we introduce the notion of blurly $(1,2)$-β-irresolute mappings and study some properties.

Definition 23 *A map* $f{:}X \to Y$ *is called blurly* $(1,2)$-β-*irresolute if for each point* $x \in X$ *and each* $V \in (1,2)$-$SPO(X, f(x))$, *there exists a* $U \in (1,2)$-$SPO(X, x)$ *such that* $f(U) \subset (1,2)$-$spcl(V)$.

Remark 24 *Every* $(1,2)$-β-*irresolute map is blurly* $(1,2)$-β-*irresolute but the converse is not true.*

Example 25 *Let* X *be the space as in Example 21, and let* $Y = \{p, q, r\}$, $\sigma_1 = \{\emptyset, \{p\}, \{p, q\}, Y\}$ *and* $\sigma_2 = \{\emptyset, \{p\}, X\}$. *Define a function* $f{:}X \to Y$ *as* $f(a) = p$, $f(b) = r$ *and* $f(c) = q$. *Then* f *is blurly* $(1,2)$-β-*irresolute but not* $(1,2)$-β-*irresolute.*

Definition 26 *A space* X *is said to be* $(1,2)$-β-T_2 *if for each pair of distinct points* $x, y \in X$, *there exist* $U \in (1,2)$-$SPO(X, x)$ *and* $V \in (1,2)$-$SPO(X, y)$ *such that* $U \cap V = \emptyset$.

Lemma 27 *A space* X *is* $(1,2)$-β-T_2 *if and only if for each pair of distinct points* $x, y \in X$, *there exist* $U \in (1,2)$-$SPO(X, x)$ *and* $V \in (1,2)$-$SPO(X, y)$ *such that* $(1,2)$-$spcl(U) \cap (1,2)$-$spcl(V) = \emptyset$.

Proof. Follows from Theorem 10. ■

Theorem 28 *If* $f{:}X \to Y$ *and* $g{:}Y \to Z$ *are* $(1,2)$-β-*irresolute, then the composition* $g \circ f$ *is* $(1,2)$-β-*irresolute.*

Theorem 29 *For a function* $f{:}X \to Y$, *the following properties are equivalent.*

(i). f *is blurly* $(1,2)$-β-*irresolute.*

(ii). $f^{-1}(V) \subset (1,2)$-$spint(f^{-1}((1,2)$-$spcl(V)))$ *for every* $V \in (1,2)$-$SPO(Y)$.

(iii). $(1,2)$-$spcl(f^{-1}(V)) \subset f^{-1}((1,2)$-$spcl(V))$ *for every* $V \in (1,2)$-$SPO(Y)$.

Proof. $(i) \Rightarrow (ii)$. Let $V \in (1,2)$-$SPO(Y)$ and $x \in f^{-1}(V)$. Since f is blurly $(1,2)$-β-irresolute, $f(U) \subset (1,2)$-$spcl(V)$ for some $U \in (1,2)$-$SPO(X,x)$. Therefore, $U \subset f^{-1}((1,2)$-$spcl(V))$ and $x \in U \subset (1,2)$-$spint$ $(f^{-1}((1,2)$- $spcl(V)))$. Hence$f^{-1}(V) \subset (1,2)$-$spint(f^{-1}((1,2)$- $spcl(V)))$.
$(ii) \Rightarrow (iii)$. Let $V \in (1,2)$-$SPO(Y)$ and $x \notin f^{-1}((1,2)$-$spcl(V))$. Then $f(x) \notin (1,2)$-$spcl(V)$. Therefore, there exists $W \in (1,2)$-$SPO(Y, f(x))$ such that $W \cap V = \emptyset$. Since $V \in (1,2)$-$SPO(Y)$, $(1,2)$-$spcl(W) \cap V = \emptyset$ and hence $(1,2)$-$spint(f^{-1}((1,2)$-$spcl(W))) \cap f^{-1}(V) = \emptyset$. Then by (ii), we have $x \in f^{-1}(W) \subset (1,2)$-$spint(f^{-1}((1,2)$-$spcl(W))) \in (1,2)$-$SPO(X)$. Therefore, $x \notin (1,2)$-$spcl(f^{-1}(V))$. Hence, $(1,2)$-$spcl(f^{-1}(V)) \subset f^{-1}((1,2)$-$spcl(V))$.

$(iii) \Rightarrow (i)$. Let $x \in X$ and $V \in (1,2)$-$SPO(Y, f(x))$. Then by Theorem 10, $(1,2)$-$spcl(V) \in (1,2)$-$SPR(Y)$ and $x \notin f^{-1}((1,2)$-$spcl(Y \setminus (1,2)$-$spcl(V)))$. Since $Y \setminus (1,2)$-$spcl(V) \in (1,2)$-$SPO(Y)$, by (iii), we have $x \notin (1,2)$-$spcl(f^{-1}(Y \setminus (1,2)$-$spcl(V)))$. Hence there exists $U \in (1,2)$-$SPO(X,x)$ such that $U \cap f^{-1}(Y \setminus (1,2)$-$spcl(V)) = \emptyset$. Therefore, $f(U) \cap (Y \setminus (1,2)$-$spcl(V)) = \emptyset$ and so $f(U) \subset (1,2)$-$spcl(V)$. ■

Theorem 30 *If $f{:}X \to Y$ is $(1,2)$-β-irresolute and V is $(1,2)$-sp-θ-open in Y, then $f^{-1}(V)$ is $(1,2)$-sp-θ-open in X.*

Proof. Let V be $(1,2)$-sp-θ-open in Y and $x \in f^{-1}(V)$. Then there exists $W \in (1,2)$-$SPO(Y)$ such that $f(x) \in W \subset (1,2)$-$spcl(W) \subset V$. Since f is $(1,2)$-β-irresolute, $f^{-1}(W) \in (1,2)$-$SPO(X)$ and $(1,2)$-$spcl(f^{-1}(W)) \subset f^{-1}((1,2)$-$spcl(W))$. Therefore, we have $x \in f^{-1}(W) \subset (1,2)$-$spcl(f^{-1}(W)) \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is $(1,2)$-sp-θ-open in X. ■

Theorem 31 *For a function $f{:}X \to Y$, the following are equivalent.*

(i). f *is blurly* $(1,2)$-β-*irresolute.*

(ii). $(1,2)$-$spcl(f^{-1}(B)) \subset f^{-1}((1,2)$-$spcl_\theta(B))$ *for every subset* B *of* Y.

(iii). $f((1,2)$-$spcl(A)) \subset (1,2)$-$spcl_\theta(f(A))$ *for every subset* A *of* X.

(iv). $f^{-1}(F) \in (1,2)$-$SPC(X)$ *for every* $(1,2)$-*sp*-θ-*closed subset* F *of* Y.

(v). $f^{-1}(V) \in (1,2)$-$SPO(X)$ *for every* $(1,2)$-*sp*-θ-*open set* V *of* Y.

Proof. $(i) \Rightarrow (ii)$. Let B be any subset of Y and $x \notin f^{-1}((1,2)$-$spcl_\theta(B))$. Then $f(x) \notin (1,2)$-$spcl_\theta(B)$ and there exists $V \in (1,2)$-

$SPO(Y, f(x))$ such that $(1,2)$-$spcl(V) \cap B = \emptyset$. Since f is blurly $(1,2)$-β-irresolute, there exists $U \in (1,2)$-$SPO(X,x)$ such that $f(U) \subset (1,2)$-$spcl(V)$. Hence $f(U) \cap B = \emptyset$ and $U \cap f^{-1}(B) = \emptyset$. Thus we obtain $x \notin (1,2)$-$spcl(f^{-1}(B))$.

(ii) $\Rightarrow$ *(iii)*. Let A be any subset of X. By (ii), $(1,2)$-$spcl(A) \subset (1,2)$-$spcl(f^{-1}(f(A))) \subset f^{-1}((1,2)$-$spcl_\theta(f(A)))$ and so $f((1,2)$-$spcl(A)) \subset (1,2)$-$spcl_\theta f((A))$.

(iii) $\Rightarrow$ *(iv)*. Let F be $(1,2)$-sp-θ-closed in Y. Then, by (iii), $f((1,2)$-$spcl(f^{-1}(F))) \subset (1,2)$-$spcl_\theta(f(f^{-1}(F))) \subset (1,2)$-$spcl_\theta(F) = F$. Therefore, $(1,2)$-$spcl(f^{-1}(F)) \subset f^{-1}(F)$ and therefore, $(1,2)$-$spcl(f^{-1}(F)) = f^{-1}(F)$.

(iv) $\Rightarrow$ *(v)*. Obvious.

(v) $\Rightarrow$ *(i)*. Let $x \in X$ and $V \in (1,2)$-$SPO(Y, f(x))$. By Theorem 10 and Theorem 14, $(1,2)$-$spcl(V)$ is $(1,2)$-sp_θ-open in Y. Set $U = f^{-1}((1,2)$-$spcl(V))$. Then by our assumption, $U \in (1,2)$-$SPO(X,x)$ and $f(U) \subset (1,2)$-$spcl(V)$. hence f is blurly $(1,2)$-β-irresolute. ■

Theorem 32 *For a function $f{:}X \to Y$ the following are equivalent.*

(i). f is blurly $(1,2)$-β-irresolute.

(ii). For each $x \in X$ and each $V \in (1,2)$-$SPO(Y, f(x))$, there exists $U \in (1,2)$-$SPO(X,x)$ such that $f((1,2)$-$spcl(U)) \subset (1,2)$-$spcl(V)$.

(iii). $f^{-1}(F) \in (1,2)$-$SPR(X)$ for every $F \in (1,2)$-$SPR(Y)$.

Proof. *(i)* $\Rightarrow$ *(ii)*. Let $x \in X$ and $V \in (1,2)$-$SPO(Y, f(x))$. Then by Theorem 10 and Theorem 14, $(1,2)$-$spcl(V)$ is $(1,2)$-sp-θ-open and $(1,2)$-sp-θ-closed. If we let $U = f^{-1}((1,2)$-$spcl(V))$ by Theorem 29, $U \in (1,2)$-$SPR(X)$. Thus U is $(1,2)$-semi-preopen and $(1,2)$-semi-preclosed and therefore, $f((1,2)$-$spcl(U)) \subset (1,2)$-$spcl(V)$.

(ii) $\Rightarrow$ *(iii)*. Let $F \in (1,2)$-$SPR(Y)$ and $x \in f^{-1}(F)$. Then $f(x) \in F$ and hence by our assumption, there exists $U \in (1,2)$-$SPO(X,x)$ such that $f((1,2)$-$spcl(U)) \subset F$. Thus we have $x \in U \subset (1,2)$-$spcl(U) \subset f^{-1}(F)$ and hence $f^{-1}(F) \in (1,2)$-$SPO(X)$. Now $Y \setminus F \in (1,2)$-$SPR(Y)$, $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F) \in (1,2)$-$SPR(X)$. Thus $f^{-1}(F)$ is $(1,2)$-semi-preclosed and

hence $f^{-1}(F) \in (1,2)$-$SPR(X)$.

$(iii) \Rightarrow (i)$. Let $x \in X$ and $V \in (1,2)$-$SPO(Y, f(x))$. Then $(1,2)$-$spcl(V)$ $\in$ $(1,2)$-$SPR(Y, f(x))$ by Theorem 10, and $f^{-1}((1,2)$-$spcl(V))$ $\in (1,2)$ $SPR(X, x)$. If we let $U = f^{-1}(1,2)$-$spcl(V)$, then $U \in (1,2)$-$SPO(X, x)$ and $f(U) \subset (1,2)$-$spcl(V)$. Therefore, f is blurly $(1,2)$-β -irresolute. ■

Theorem 33 *For a function $f{:}X \to Y$ the following are equivalent.*

(i). f *is blurly* $(1,2)$-β*-irresolute.*

(ii). $f^{-1}(V) \subset (1,2)$-$spint_\theta(f^{-1}((1,2)$-$spcl_\theta(V)))$ *for every* $V \in (1,2)$-$SPO(Y)$.

(iii). $(1,2)$-$spcl_\theta(f^{-1}(V)) \subset f^{-1}((1,2)$-$spcl_\theta(V))$ *for every* $V \in (1,2)$-$SPO(Y)$.

Proof. Proof is similar to that of Theorem 29. ■

Theorem 34 *For a function $f{:}X \to Y$ the following are equivalent.*

(i). f *is blurly* $(1,2)$-β*-irresolute.*

(ii). $(1,2)$-$spcl_\theta(f^{-1}(B)) \subset f^{-1}((1,2)$-$spcl_\theta(B))$ *for every subset* B *of* Y.

(iii). $f((1,2)$-$spcl_\theta(A)) \subset (1,2)$-$spcl_\theta(f(A))$ *for every subset* A *of* X.

(iv). $f^{-1}(F)$ *is* $(1,2)$*-sp-*θ*-closed for every* $(1,2)$*-sp-*θ*-closed subset* F *of* Y.

(v). $f^{-1}(V)$ *is* $(1,2)$*-sp-*θ*-open for every* $(1,2)$*-sp-*θ*-open set* V *of* Y.

Proof. Proof is similar to that of Theorem 31. ■

Definition 35 *A space X is said to be $(1,2)$-semi-preregular if for each $F \in (1,2)$-$SPC(X)$ and each $x \notin F$, there exist disjoint $(1,2)$-semi-preopen sets U and V such that $x \in U$ and $F \subset V$.*

Lemma 36 *For a space X the following properties are equivalent.*

(i). X *is* $(1,2)$*-semi-preregular.*

(ii). *For each* $U \in (1,2)$-$SPO(X)$ *and each* $x \in U$*, there exists* $V \in (1,2)$-$SPO(X)$ *such that* $x \in V \subset (1,2)$-$spcl(V) \subset U$.

(iii). For each $U \in (1,2)$-$SPO(X)$ and each $x \in U$, there exists $V \in (1,2)$-$SPR(X)$ such that $x \in V \subset U$.

Proof. Follows from Theorem 10. ■

Theorem 37 *Let Y be an $(1,2)$-semi-preregular space. Then a function $f{:}X \to Y$ is blurly $(1,2)$-β-irresolute if and only if it is $(1,2)$-β-irresolute.*

Proof. Let f be blurly $(1,2)$-β-irresolute and V be $(1,2)$-semi-preopen in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. Therefore, by Lemma 36, there exists $W \in (1,2)$-$SPO(Y)$ such that $f(x) \in W \subset (1,2)$-$spcl(W) \subset V$. Since f is blurly $(1,2)$-β-irresolute, there exists $U \in (1,2)$-$SPO(X,x)$ such that $f(U) \subset (1,2)$-$spcl(W)$. Thus we have $x \in U \subset f^{-1}(V)$ and $f^{-1}(V) \in (1,2)$-$SPO(X)$. Hence f is $(1,2)$-β-irresolute.

The converse follows from Remark 24. ■

Theorem 38 *If Y is $(1,2)$-β-T_2 and $f{:}X \to Y$ is a blurly $(1,2)$-β-irresolute injective map, then X is $(1,2)$-β-T_2.*

Proof. Let x, y be two distinct points of X. Since f is injective, $f(x) \neq f(y)$. Since Y is $(1,2)$-β-T_2, by Lemma 27, $(1,2)$-$spcl(U) \cap (1,2)$-$spcl(V) = \emptyset$. Since f is blurly $(1,2)$-β-irresolute, there exist $P \in (1,2)$-$SPO(X,x)$ and $Q \in (1,2)$-$SPO(X,y)$ such that $f(P) \subset (1,2)$-$spcl(V)$ and $f(Q) \subset (1,2)$-$spcl(W)$. Therefore, $P \cap Q = \emptyset$. Hence X is $(1,2)$-β-T_2. ■

5 Vividly $(1,2)$-β-Irresolute Mappings

In this section we introduce vividly $(1,2)$-β-irresolute mappings.

Definition 39 *A map $f{:}X \to Y$ is called vividly $(1,2)$-β-irresolute if for each point $x \in X$ and each $V \in (1,2)$-$SPO(X, f(x))$, there exists a $U \in (1,2)$-$SPO(X,x)$ such that $f((1,2)$-$spcl(U)) \subset V$.*

Remark 40 *Every vividly $(1,2)$-β-irresolute map is $(1,2)$-β-irresolute but the converse is not true as shown in the following example.*

Example 41 *Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, \{a\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, X\}$ and $Y = \{p,q,r\}$, $\sigma_1 = \{\emptyset, \{p, q\}, Y\}$ and $\sigma_2 = \{\emptyset, \{p\}, Y\}$. Define a function $f{:}X \to Y$ as $f(a) = p$, $f(b) = q$, $f(c) = r$. Then the function is $(1,2)$-β-irresolute but it is not vividly $(1,2)$-β-irresolute since for $a \in X$, $f(a) = p \in \{p\}$ and for any $U \in (1,2)$-$SPO(X,a)$ $(1,2)$-$spcl(U) = X \not\subset \{p\}$.*

Remark 42 *¿From the above discussions we obtain vividly* $(1,2)$-*β-irresolute* $\Rightarrow$ $(1,2)$-*β-irresolute* $\Rightarrow$ *blurly* $(1,2)$-*β-irresolute and none of them is reversible.*

Theorem 43 *For a function* $f{:}X \to Y$ *the following are equivalent.*

(i). f *is vividly* $(1,2)$-*β-irresolute.*

(ii). For each $x \in X$ *and each* $V \in (1,2)$-$SPO(Y, f(x))$*, there exists* $U \in (1,2)$-$SPO(X, x)$ *such that* $f((1,2)\text{-}spcl_\theta(U)) \subset V$.

(iii). For each $x \in X$ *and each* $V \in (1,2)$-$SPO(Y, f(x))$*, there exists* $U \in (1,2)$-$SPR(X, x)$ *such that*$f(U) \subset V$.

(iv). For each $x \in X$ *and each* $V \in (1,2)$-$SPO(Y, f(x))$*, there exists an* $(1,2)$-*sp-θ-open set* U *in* X *containing* x *such that* $f(U) \subset V$.

(v). $f^{-1}(V)$ *is* $(1,2)$-*sp-θ-open for every* $V \in (1,2)$-$SPO(Y)$.

(vi). $f^{-1}(F)$ *is* $(1,2)$-*sp-θ-closed for every* $F \in (1,2)$-$SPC(Y)$.

(vii). $f((1,2)\text{-}spcl_\theta(A)) \subset (1,2)\text{-}spcl(f(A))$ *for every subset* A *of* X.

(viii). $(1,2)\text{-}spcl_\theta(f^{-1}(B)) \subset f^{-1}((1,2)\text{-}spcl(B))$ *for every subset* B *of* Y.

Proof. $(i) \Rightarrow (ii)$. Follows from Theorem 14.
$(ii) \Rightarrow (iii)$. Follows from Theorem 10.
$(iii) \Rightarrow (iv)$. Follows from Theorem 10.
$(iv) \Rightarrow (v)$. Let $V \in (1,2)$-$SPO(Y)$. If $x \in f^{-1}(V)$, then $f(x) \in V$ and there exists an $(1,2)$-*sp*-θ-open set U in X containing x such that $f(U) \subset V$. Therefore, $x \in U \subset f^{-1}(V)$ and hence $f^{-1}(V)$ is the union of $(1,2)$-*sp*-θ-open sets. Thus $f^{-1}(V)$ is $(1,2)$-*sp*-θ-open in X since the union of $(1,2)$-*sp*-θ-open sets is $(1,2)$-*sp*-θ-open.
$(v) \Rightarrow (vi)$. Obvious.
$(vi) \Rightarrow (vii)$. Let $A \subset X$.Since $(1,2)\text{-}spcl(f(A))$ is $(1,2)$-semi-preclosed in Y by (vi), $f^{-1}((1,2)\text{-}spcl(f(A)))$ is $(1,2)$-*sp*-θ-closed in X and $(1,2)\text{-}spcl_\theta(A) \subset (1,2)\text{-}spcl_\theta\ f^{-1}(f(A)) \subset (1,2)\text{-}spcl_\theta(f^{-1}((1,2)\text{-}spcl(f(A)))) = f^{-1}((1,2)\text{-}spcl(f(A)))$. Therefore, $f((1,2)\text{-}spcl_\theta(A)) \subset (1,2)\text{-}spcl(f(A))$.
$(vii) \Rightarrow (viii)$. Let $B \subset Y$. Then $f((1,2)\text{-}spcl_\theta(f^{-1}(B))) \subset (1,2)\text{-}spcl(f(f^{-1}(B))) \subset (1,2)\text{-}spcl(B)$ and hence $(1,2)\text{-}spcl_\theta(f^{-1}(B)) \subset f^{-1}((1,2)\text{-}spcl(B))$.
$(viii) \Rightarrow (i)$. Let $x \in X$ and $V \in (1,2)$-$SPO(Y, f(x))$. By (viii), $(1,2)\text{-}spcl_\theta(f^{-1}(Y \backslash V)) \subset f^{-1}((1,2)\text{-}spcl(Y \backslash V)) = f^{-1}(Y \backslash V)$. Therefore, $f^{-1}(Y \backslash V)$ is

$(1,2)$-sp-θ-closed in X and $f^{-1}(V)$ is $(1,2)$-sp-θ-open in X and it contains x. Hence there exists $U \in (1,2)$-$SPO(X,x)$ such that $(1,2)$-$spcl(U) \subset f^{-1}(V)$ and $f((1,2)\text{-}spcl(U)) \subset V$. ■

Theorem 44 *Every* $(1,2)$-β-*irresolute map* $f{:}X \to Y$ *is vividly* $(1,2)$-β-*irresolute if and only if* X *is* $(1,2)$-*semi-preregular.*

Proof. Necessity Let $f{:}X \to Y$ be the identity function. Then f is $(1,2)$-β-irresolute and by the hypothesis, it is vividly $(1,2)$-β-irresolute. If $x \in U \in (1,2)$-$SPO(X)$, then $f(x) = x \in U$, there exists $V \in (1,2)$-$SPO(X,x)$ such that $f((1,2)\text{-}spcl(V)) \subset U$. Therefore, we have $x \in V \subset (1,2)$-$spcl(V) \subset U$. Hence by Lemma 36, X is $(1,2)$-semi-preregular.
Sufficiency. If $x \in X$ and $V \in (1,2)$-$SPO(X, f(x))$, then $f^{-1}(V)$ is $(1,2)$-semi-preopen in X containing x. Since X is $(1,2)$-semi-preregular, there exists $U \in (1,2)$-$SPO(X)$ such that $x \in U \subset (1,2)spcl(U) \subset f^{-1}(V)$. Therefore, $f(1,2)$-$spcl(U) \subset V$, f is vividly $(1,2)$-β-irresolute. ■

Theorem 45 *Let* $f{:}X \to Y$ *and* $g{:}Y \to Z$ *be two functions. Then the following properties hold.*

(i). If f *is vividly* $(1,2)$-β-*irresolute and* g *is* $(1,2)$-β-*irresolute, then* $g \circ f$ *is vividly* $(1,2)$-β-*irresolute.*

(ii). If f *is* $(1,2)$-β-*irresolute and* g *is vividly* $(1,2)$-β-*irresolute, then* $g \circ f$ *is vividly* $(1,2)$-β-*irresolute.*

Proof. (i). Obvious.
(ii). Follows from Theorem 43 and Theorem 29. ■

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